

# Interkinetic nuclear migration in the zebrafish retina as a diffusive process

Afnan Azizi<sup>1†</sup>, Anne Herrmann<sup>2†</sup>, Yinan Wan<sup>3</sup>, Salvador J. R. Buse<sup>1</sup>, Philipp J. Keller<sup>3</sup>, Raymond E. Goldstein<sup>2\*</sup>, William A. Harris<sup>1\*</sup>

**\*For correspondence:**

[wah20@cam.ac.uk](mailto:wah20@cam.ac.uk) (WAH);  
[r.e.goldstein@damtp.cam.ac.uk](mailto:r.e.goldstein@damtp.cam.ac.uk)  
(REG)

<sup>†</sup>These authors contributed equally to this work

<sup>1</sup>Department of Physiology, Development and Neuroscience, University of Cambridge, Cambridge CB2 3DY, United Kingdom; <sup>2</sup>Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Cambridge CB3 0WA, United Kingdom; <sup>3</sup>Howard Hughes Medical Institute, Janelia Research Campus, Ashburn, VA 20147, USA

**Abstract** A major hallmark of neural development is the oscillatory movement of nuclei between the apical and basal surfaces of the neuroepithelium during the process of interkinetic nuclear migration (IKNM). Here, we employ long-term, rapid lightsheet and two-photon imaging of zebrafish retinas *in vivo* during early development to uncover the physical processes that govern the behavior of nuclei during IKNM. These images allow the capture of reliable tracks of nuclear movements and division during early retinogenesis for many tightly packed nuclei. These tracks are then used to create and test a theory of retinal IKNM as a diffusive process across a nuclear concentration gradient generated by the addition of new nuclei at the apical surface. The analysis reveals the role of nuclear packing at the apical surface on the migration dynamics of nuclei, provides a robust quantitative explanation for the distribution of nuclei across the retina, and may have implications for stochastic fate choice in this system.

## Introduction

The vertebrate nervous system arises from a pseudostratified epithelium within which elongated proliferating cells contact both the apical and basal surfaces. Within these cells, striking nuclear movements take place during the proliferative phase of neural development. More than 80 years ago, these movements, termed interkinetic nuclear migration (IKNM), were shown to occur in synchrony with their cell cycle (*Sauer, 1935*). Under normal conditions, nuclei of proliferating cells undergo mitosis (M) exclusively at the apical surface. During the first gap phase (G1) of the cell cycle, nuclei migrate away from this surface to reach more basal positions by S-phase, when DNA is replicated. In the second gap phase (G2), nuclei migrate rapidly toward the apical surface where they divide again (*Del Bene, 2011; Sauer, 1935; Baye and Link, 2007; Leung et al., 2011; Kosodo et al., 2011; Norden et al., 2009*). The molecular mechanisms that drive the rapid nuclear movement in G2 have been investigated in a number of tissues (*Norden, 2017*). For instance, in the mammalian cortex they are thought to involve microtubules as well as various microtubule motors and actomyosin (*Xie et al., 2007; Tsai et al., 2007*). In the zebrafish retina, it appears to be the actomyosin complex alone that moves the nuclei to the apical surface during G2 (*Norden et al., 2009; Leung et al., 2011*). In contrast, the nuclear movements during the majority of the cell cycle, in G1 and S phases, have been less thoroughly examined. Although similar molecular motors have

41 been implicated (*Schenk et al., 2009; Tsai et al., 2010*), the underlying processes remain elusive.  
42 Importantly, IKNM is known to affect morphogenesis and cell differentiation in neural tissues  
43 (*Spear and Erickson, 2012*), as retinas with perturbed IKNM are known to develop prematurely and  
44 to display abnormalities in cell composition (*Del Bene et al., 2008*). Given this regulatory involve-  
45 ment of IKNM in retinal cell differentiation, a deeper understanding of the nuclear movements  
46 remains a major prerequisite for insights into the development of neural systems. On a phenomeno-  
47 logical level, the movements of nuclei during the G1 and S phases have been shown to resemble a  
48 stochastic process in the zebrafish retina (*Norden et al., 2009; Leung et al., 2011*). During these  
49 periods, individual nuclei switch between apical and basal movements at random intervals, leading  
50 to considerable variability in the maximum basal position they reach during each cell cycle (*Baye  
51 and Link, 2007*). Similarly, in the mammalian cerebral cortex, the considerable internuclear variabil-  
52 ity in IKNM leads to nuclear positions scattered throughout the entire neuroepithelium in S-phase  
53 (*Sidman et al., 1959; Kosodo et al., 2011*). The high variability in the movements of nuclei during  
54 G1 and S phases of the cell cycle suggests that passive, rather than active, molecular processes are  
55 a main driver of basal migration. This hypothesis was supported by experiments demonstrating  
56 similarly variable basalward-biased migration of nuclear-sized microbeads inserted in between cells  
57 during IKNM in the mouse cortex (*Kosodo et al., 2011*). Various possible explanations for these  
58 passive processes have been put forward. These suggestions include the possibility of direct energy  
59 transfer from rapidly moving G2 nuclei (*Norden et al., 2009*), as well as nuclear movements caused  
60 by apical crowding (*Kosodo et al., 2011; Okamoto et al., 2013*). Here, we present experiments to  
61 test these hypotheses.

62 Our work relies on the tracks of closely packed nuclei of zebrafish retinal progenitor cells. The  
63 retina of the oviparous zebrafish is easily accessible to light microscopy throughout embryonic  
64 development (*Avanesov and Malicki, 2010*) and has been used for several studies of the movements  
65 of nuclei during IKNM (*Baye and Link, 2007; Del Bene et al., 2008; Norden et al., 2009; Sugiyama  
66 et al., 2009; Leung et al., 2011*). We find evidence for IKNM being driven by apical crowding and  
67 therefore further develop this idea into a mathematical model. Given the seemingly stochastic  
68 nature of individual nuclear trajectories, we base the model on a comparison between IKNM and a  
69 simple diffusion process. The model reveals the remarkable and largely overlooked importance of  
70 simple physical constraints imposed by the overall tissue architecture and allows us to describe  
71 accurately the global distribution of nuclei as a function of time within the retinal tissue. In this  
72 way, we describe IKNM as a tissue-wide rather than a single-cell phenomenon. In the future, this  
73 description might shed light on other aspects of progenitor cell biology, such as cell cycle exit and  
74 fate.

## 75 Results

### 76 Generating image sets with high temporal resolution

77 We imaged fluorescently-labeled nuclei of whole retinas of developing zebrafish at 2 min intervals,  
78 an optimal time period given the difficulty to track nuclei accurately over long times and the  
79 increased photobleaching with shorter intervals. We compared movies of retinas imaged at 2  
80 min and at 20 s intervals over a period of 2 hours and found that the improvement in temporal  
81 resolution made no difference to our analyses. This suggests that it is unlikely that within each 2  
82 min interval there were important intervening movements that might complicate the analysis.

83 To follow the nuclei of all cells within a portion of the retina we used H2B-GFP transgenic  
84 lines with GFP expression exclusively in the nuclei (Figure 1A). In order to achieve the desired  
85 temporal resolution without sacrificing image quality, fluorescence bleaching and sample drift must  
86 be minimized as much as possible. The retinas of H2B-GFP embryos were imaged using either  
87 a single-angle lightsheet microscope (see Figure 1B for a schematic) or an upright two-photon  
88 scanning microscope. Both of these methods yield images with minimal bleaching compared  
89 to other microscopic techniques (*Svoboda and Yasuda, 2006; Stelzer, 2015*). However, while the

90 single-angle lightsheet can generate large stacks of images, it is very sensitive to lateral drift due to  
91 a small area of high resolution imaging. Therefore, some datasets were produced using two-photon  
92 microscopy, which, despite the limitations of scanning time, could produce areas of high resolution  
93 images of sufficient size.

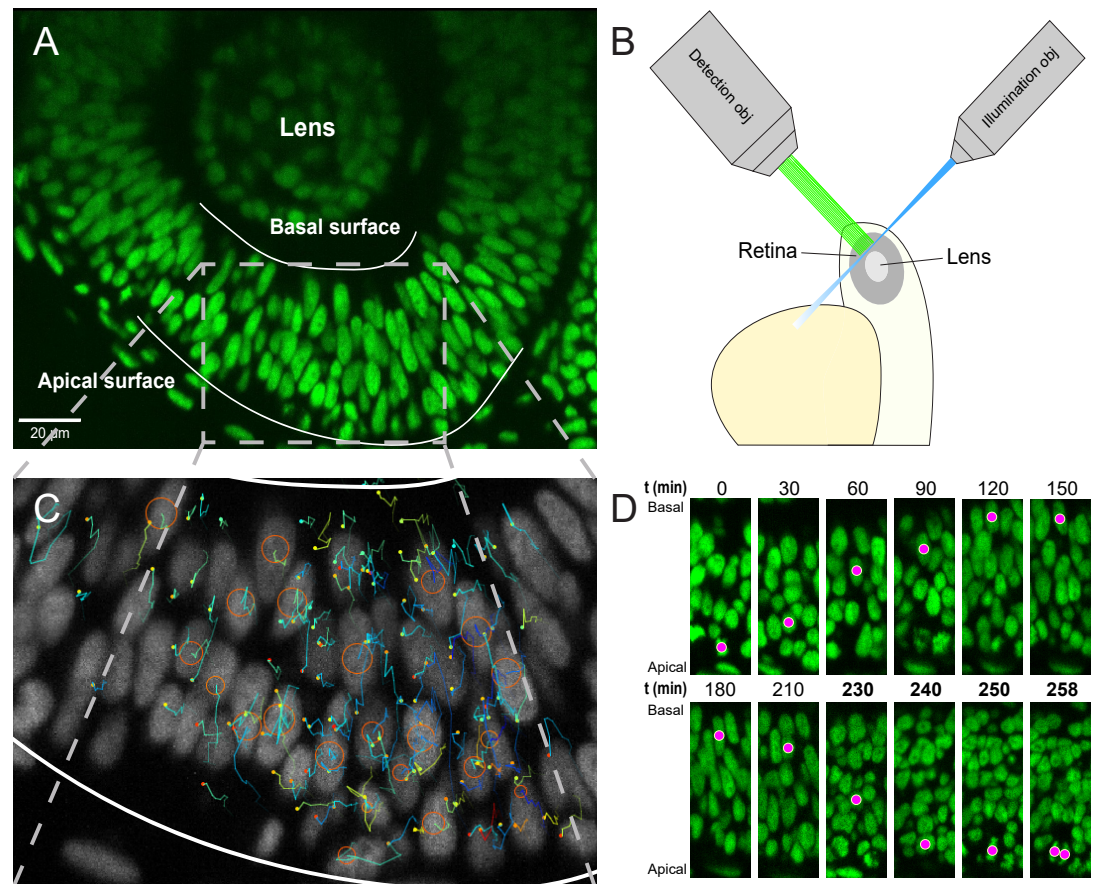
94 Both lightsheet and two-photon microscopes produced images of at least half the retina with  
95 a depth of at least 50  $\mu\text{m}$  over several hours in 2 min intervals. The images were processed  
96 using a suite of algorithms (*Amat et al., 2015*) to compress them to a lossless format, Keller Lab  
97 Block (KLB), correct global and local drift, and normalize signal intensities for further processing.  
98 Automated segmentation and tracking of the nuclei were carried out through a previously published  
99 computational pipeline that takes advantage of watershed techniques and persistence-based  
100 clustering (PBC) agglomeration to create segments and Gaussian mixture models with Bayesian  
101 inference to generate tracks of nuclei through time (*Amat et al., 2014, 2015*). Two main parameters  
102 greatly affect tracking results, overall background threshold and PBC agglomeration threshold. To  
103 obtain best automated tracking results, ground truth tracks were created for a section of the retina  
104 over 120 min and were compared to tracks generated over a range of these two parameters. The  
105 best combination of the two parameters was chosen as the one with highest tracking fidelity and  
106 lowest amount of oversegmentation over that interval.

107 The most optimal combination of parameters yielded an average linkage accuracy, from each  
108 time point to the next, of approximately 65%. Hence, extensive manual curation and correction of  
109 tracks were required. Tracking by Gaussian mixture models (TGMM) software generates tracks that  
110 can be viewed and modified using the Massive Multi-view Tracker (MaMuT) plugin of the Fiji software  
111 (*Wolff et al., 2018; Schindelin et al., 2012*). A region of the retina with the best fluorescence signal  
112 was chosen and all tracks within that region were examined and any errors were corrected. The  
113 tracks consist of sequentially connected sets of 3D coordinates representing the centers of each  
114 nucleus (Figure 1C), with which their movement across the tissue can be mapped over time. For  
115 example, Figure 1D shows IKNM of a single nucleus tracked from its birth, at the apical surface of  
116 the retina, to its eventual division into two daughter cells.

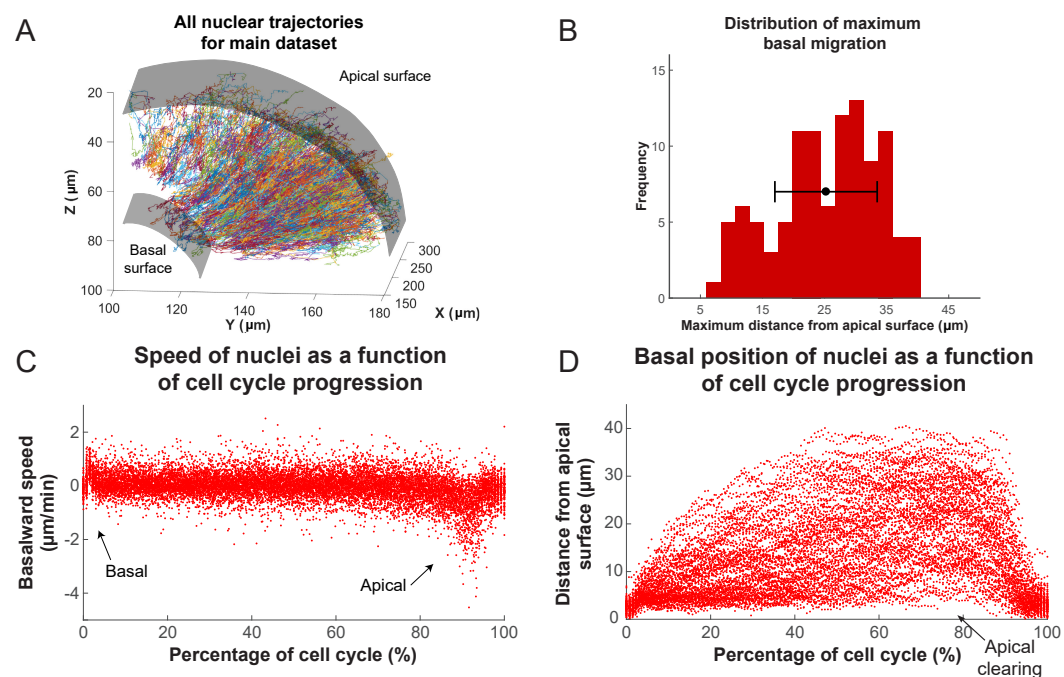
### 117 **Analysis of nuclear tracks**

118 This process yielded tracks for hundreds of nuclei, across various samples, over time intervals of at  
119 least 200 min. We used custom-written MATLAB scripts to analyze these tracks. The aggregated  
120 tracks of the main dataset, in Cartesian coordinates, for all tracked lineages is shown in Figure 2A.  
121 Single tracks for any given time interval can be extracted and analyzed from this collection. In order  
122 to transform the Cartesian coordinates of the tracks into an apicobasal coordinate system, we drew  
123 contour curves at the apical surface of the retina (e.g. see Figure 1A) separating RPC nuclei from the  
124 elongated nuclei of the pigmented epithelium. We then calculated curves of best fit (second degree  
125 polynomials) in both the XY and YZ planes. Assuming that the apical cortex is perpendicular to the  
126 apicobasal axis of each cell, displacement vectors of the nuclei at each time point can be separated  
127 into apicobasal and lateral components. Since, in IKNM, the apicobasal motion is that of interest,  
128 we used this component for our remaining analyses.

129 Figure 2C,D shows the speed and position of tracked nuclei of the same dataset, over the  
130 duration of their cell cycle, for all cells that went through a full cell cycle. While all nuclei behave  
131 similarly minutes after their birth (early G1) and before their division (G2), their speed of movement  
132 and displacement is highly variable for the majority of the time that they spend in the cell cycle  
133 (Figure 2C,D). Most daughter nuclei move away from the apical surface, within minutes from being  
134 born, with a clear basalward bias in their speed distribution (Figure 2C). This abrupt basal motion of  
135 newly divided nuclei has also been recently observed by others (*Shinoda et al., 2018; Barrasso et al.,*  
136 *2018*). However, immediately after this brief period, nuclear speeds become much more equally  
137 distributed between basalward and apicalward, with a mean value near 0. Such a distribution is  
138 indicative of random, stochastic motion, which in turn leads to a large variability in the position of  
139 nuclei within the tissue (away from the apical surface) during the cell cycle (Figure 2B).



**Figure 1.** Imaging and tracking fluorescently labeled nuclei. **(A)** A transgenic H2B-GFP embryonic retina imaged using lightsheet microscopy at ~30 hpf. The lens, as well as apical and basal surfaces are indicated. **(B)** A schematic representation of single-angle lightsheet imaging of the retina. Laser light is focused into a sheet of light by the illumination objective and scans the retina. Fluorescent light is then collected by the perpendicular detection objective. **(C)** Track visualization and curation using the MaMuT plugin of Fiji. All tracks within a region of the retina are curated and visualized. Circles and dots represent centers of nuclei, and lines show their immediate (10 previous steps) track. **(D)** The position of a single nucleus within the retinal tissue from its birth to its eventual division. The magenta dot indicates the nucleus tracked at various time points during its cell cycle. The last 4 panels are at shorter time intervals to highlight the rapid movement of the nucleus prior to mitosis.

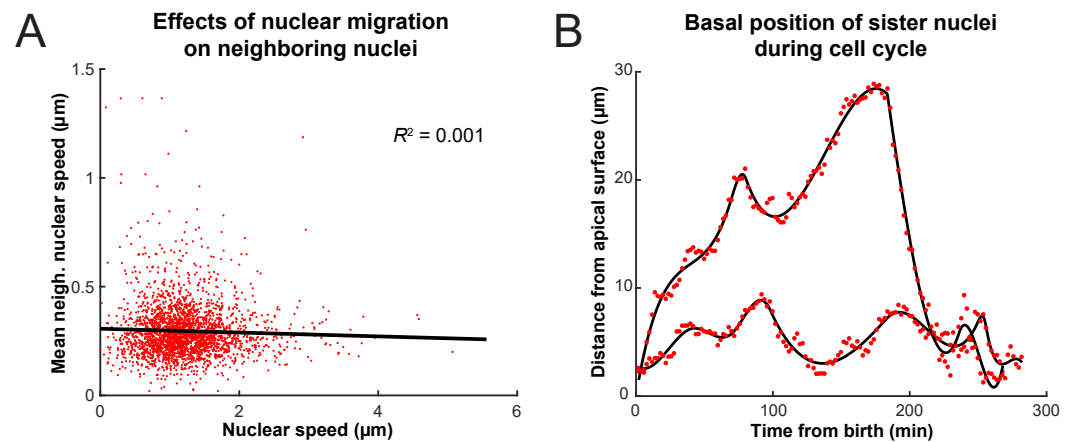


**Figure 2.** Analysis of nuclear tracks during IKNM. **(A)** Extracted trajectories of nuclei in 3 dimensions. All curated tracks of the main dataset over 400 minutes in the region shown in Figure 1C are presented. **(B)** The distribution of maximum distances reached away from the apical surface by nuclei during their completed cell cycles. The mean and one standard deviation are shown. **(C)** The speed distribution of nuclei over complete cell cycles. The cell cycle lengths of all nuclei were normalized and superimposed to highlight the early basal burst of speed, as well as pre-division apical rapid migration. The speeds between these two periods are normally distributed. **(D)** Position of nuclei as measured by their distance from the apical surface over normalized cell cycle time. Even though all nuclei start and end their cell cycle near the apical surface, they move out across the retina to take positions in all available spaces, creating an apical clearing as indicated.

140 Interestingly, except during mitosis, we find an apical clearing of a few microns for dividing  
 141 cells (Figure 2D). We checked to see if this was an artifact of measuring the distance to nuclear  
 142 centers due to nuclear shape, as nuclei are rounded during M phase but are more elongated along  
 143 the apicobasal axis at other times. We found no significant difference between average length of  
 144 nuclear long axis when measured for nuclei right before their division compared to nuclei chosen  
 145 randomly from any other time point within the cell cycle, indicating that this clearing is likely to  
 146 have a biological explanation, such as the preferential occupancy of M phase nuclei to the apical  
 147 surface during IKNM.

### 148 Basal movement of nuclei is driven like a diffusive process

149 Previous work has shown that when RPCs are pharmacologically inhibited from replicating their  
 150 DNA, their nuclei neither enter G2 nor exhibit rapid persistent apical migration that normally occurs  
 151 during the G2 phase of the cell cycle (Leung et al., 2011; Kosodo et al., 2011). A more surprising  
 152 result of these experiments is that the stochastic movements of nuclei in G1 and S phases also slow  
 153 down considerably during such treatment (Leung et al., 2011). It was, therefore, suspected that the  
 154 migration of nuclei of cells in G2 toward the apical surface jostles those in other phases (Norden  
 155 et al., 2009). We therefore searched our tracks for evidence of such direct kinetic interactions  
 156 among nuclei by correlating the speed and direction of movement of single nuclei with their nearest  
 157 neighbors. These neighbors were chosen such that their centers fell within a cylindrical volume  
 158 of a height and base diameter twice the length of long and short axes, respectively, of an average  
 159 nucleus. Figure 3A shows the lack of correlation between the speed of movement of nuclei and the  
 160 average speed of their neighbors. We further categorized the neighboring nuclei by their position



**Figure 3. (A)** Average speed of nuclei neighboring a nucleus of interest as a function of the speed of that nucleus. **(B)** The positions of two sister nuclei at each time point imaged (red circles) over their complete cell cycle. The black lines are spline curves indicating the general trend of their movements.

161 in relation to the nucleus of interest (along the apicobasal axis), their direction of movement, and  
162 whether they were moving in the same direction of the nucleus of interest or not. None of the  
163 resulting eight categories of neighboring nuclei showed a correlation in their average speed with  
164 the speed of the nucleus of interest. Furthermore, we considered the movement of neighboring  
165 nuclei one time point (2 min) before or one time point after the movement of the nucleus of interest.  
166 Yet, we still found no correlation between these time-delayed and original speeds. These results  
167 suggest that there does not appear to be much transfer of kinetic energy between neighboring  
168 nuclei.

169 Another hypothesis advanced for variability in basal IKNM is that the nuclear movements are  
170 driven by apical crowding (Kosodo *et al.*, 2011; Okamoto *et al.*, 2013). How apical crowding might  
171 result in basal IKNM can be understood by comparing IKNM to a diffusive process. In diffusion, a  
172 concentration gradient drives the average movement of particles from areas of high to areas of low  
173 concentration. However, despite the average movement being directed, each individual particle's  
174 trajectory is a random walk (Reif, 1965). Similarly, during IKNM a gradient in nuclear concentration  
175 is generated because nuclei divide exclusively at the apical surface. If basal IKNM were comparable  
176 to diffusion, this nuclear concentration gradient would be expected to result in a net movement  
177 of nuclei away from the area of high nuclear crowding at the apical side of the neuroepithelium  
178 (Miyata *et al.*, 2015; Okamoto *et al.*, 2013). Indeed, in IKNM we find that each individual nucleus'  
179 trajectory resembles a random walk (Norden *et al.*, 2009). Therefore, for the cells in the G1 and S  
180 phases (which account for more than 90% of the cell cycle time in our system), IKNM has, at least  
181 on a phenomenological level, the main features of a diffusive process.

182 To test further whether we can indeed describe IKNM using a model of diffusion, we first  
183 asked what would happen to the concentration gradient if we blocked the cell cycle in S phase,  
184 which inhibits both the apical movement of the nuclei in G2 and mitosis at the apical surface. If the  
185 comparison to diffusion were valid, we expect the blockage to abolish the build-up and maintenance  
186 of the concentration gradient. We, therefore, compared the normally evolving distribution of nuclei  
187 in control retinas with those measured from retinas where the cell cycle was arrested at S-phase  
188 using a combination of hydroxyurea (HU) and aphidicolin (AC) (Norden *et al.*, 2009; Icha *et al.*,  
189 2016). We counted the number of nuclei in a three dimensional section of the retina containing  
190 approximately 100 nuclei, at equal time intervals, starting with 120 min after drug treatment. The  
191 delay ensured that almost all cell divisions, from nuclei that had already completed the S phase at  
192 the time of treatment, had taken place. As expected from the diffusion model (Figure 4D), over the  
193 course of 160 min, the mean of the nuclear distribution moved further towards the basal surface in

194 treated retinas, and the concentration difference between the apical and basal surfaces diminished  
195 (Figure 4A,C). In contrast, in control retinas the mean of the nuclear distribution moved towards the  
196 apical surface (Figure 4B,C) as the gradient continued to build up. Hence, these results support the  
197 suitability of a diffusive model to describe the basal nuclear migration during IKNM.

### 198 **An analytical diffusion model of IKNM**

199 To investigate whether a diffusion model would also provide a useful quantitative description  
200 for IKNM, we formalized the process of IKNM in mathematical terms. This formalization again  
201 focuses on the crowding of nuclei at the apical side of the tissue. Crowding can be thought of, in  
202 mathematical terms, as creating a gradient in nuclear concentration  $c$  along the apicobasal direction  
203 of the retina. In contrast, we assumed no dependence of the nuclear concentration on the lateral  
204 position within the tissue. Thus we employed the diffusion equation for the nuclear concentration  
205  $c(r, t)$  as a function only of the apicobasal distance  $r$  and time  $t$ . The retina can be approximated as  
206 one half of a spherical shell around the lens, and thus we use spherical polar coordinates with the  
207 origin of the coordinate system at the center of the lens, the basal surface at  $r = b$  and the apical  
208 surface at  $r = a$  (Figure 5B). We first study the simplest diffusion equation for this system, in which  
209 there is a diffusion constant  $D$  independent of position, time, and  $c$  itself, namely

$$\frac{\partial c(r, t)}{\partial t} = D \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial c(r, t)}{\partial r} \right). \quad (1)$$

210 By analyzing the experimental data we seek to determine  $D$ . This equation provided the basis for  
211 our mathematical description of IKNM in terms of a diffusion process.

212 In addition to Equation 1, we also needed to specify the boundary conditions adequate to  
213 describing IKNM. As mentioned above, we focused our description of IKNM on the apical crowding  
214 of nuclei. Since nuclei only divide close to the apical surface of the tissue, we treat mitosis as creating  
215 an effective influx of nuclei through the apical boundary. To quantify this influx, we extracted the  
216 number of cells  $N(t)$  as a function of time. As during the stages of development examined here cells  
217 are neither dying nor exiting the cell cycle (*Biehler et al., 2001*), we assumed that the number  
218 of cell divisions is always proportional to the number of currently existing cells. This assumption  
219 predicts an exponential increase in the number of cells or nuclei, over time, also recently found by  
220 *Matejčić et al. (2018)*:

$$N(t) = N_0 e^{t/\tau}, \quad (2)$$

221 where  $N_0$  is the initial number of nuclei and  $\tau = T_p / \ln 2$ , with  $T_p$  the average cell cycle length. Fig-  
222 ure 5A shows the agreement between the theoretically predicted curve  $N(t)$  with the experimentally  
223 obtained numbers of nuclei over time. Having obtained  $N_0$  and  $T_p$  from our experimental data, the  
224 predicted curve does not have any remaining free parameters and thus no fitting is necessary. Thus,  
225 the obtained description for the number of nuclei over time, Equation 2, was used to formulate the  
226 influx boundary condition for our mathematical model

$$D \frac{\partial c}{\partial r} \Big|_{r=a} = \frac{1}{S} \frac{\partial N(t)}{\partial t} = \frac{N_0}{S\tau} e^{t/\tau}, \quad (3)$$

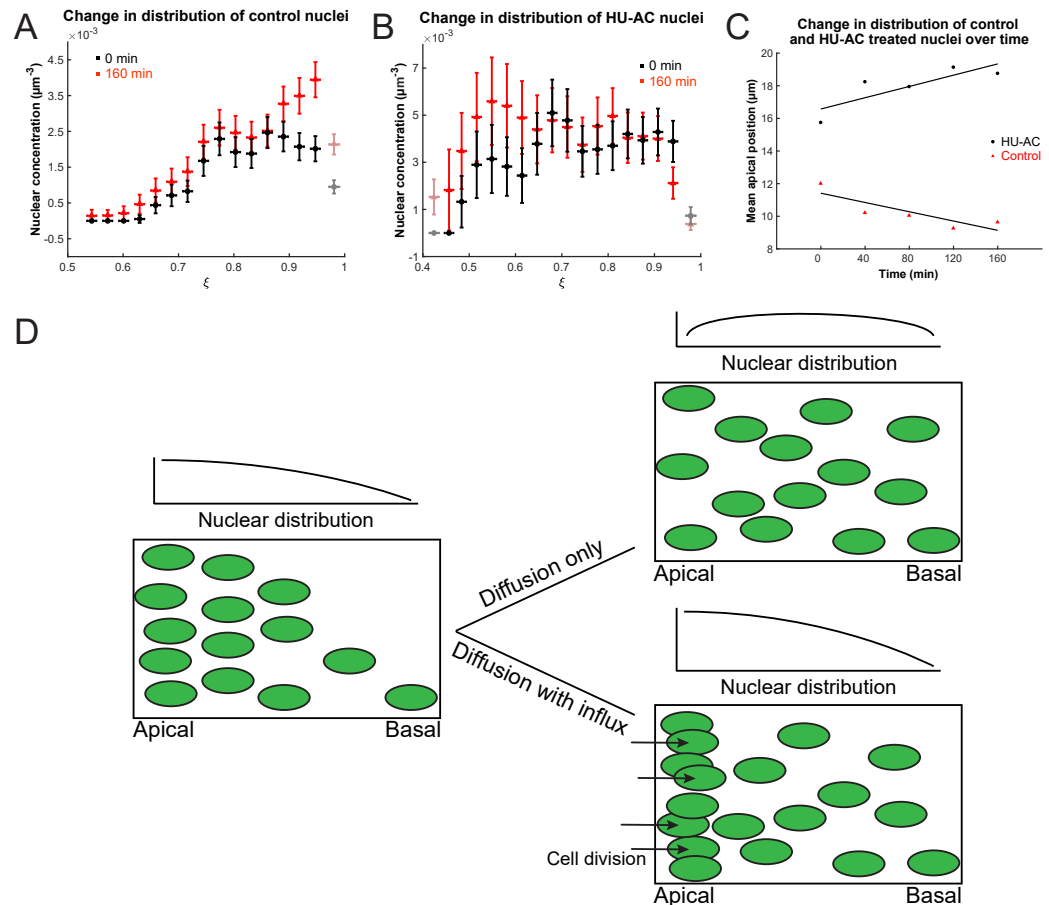
227 with  $S$  the apical surface area of our domain of interest. In contrast to the apical side of the tissue,  
228 there is no creation (or depletion) of nuclei at the basal side (*Matejčić et al., 2018*), and hence a  
229 no-flux boundary condition,

$$\frac{\partial c}{\partial r} \Big|_{r=b} = 0. \quad (4)$$

230 Equations 1, 3 and 4 fully specify this simplest mathematical model of IKNM.

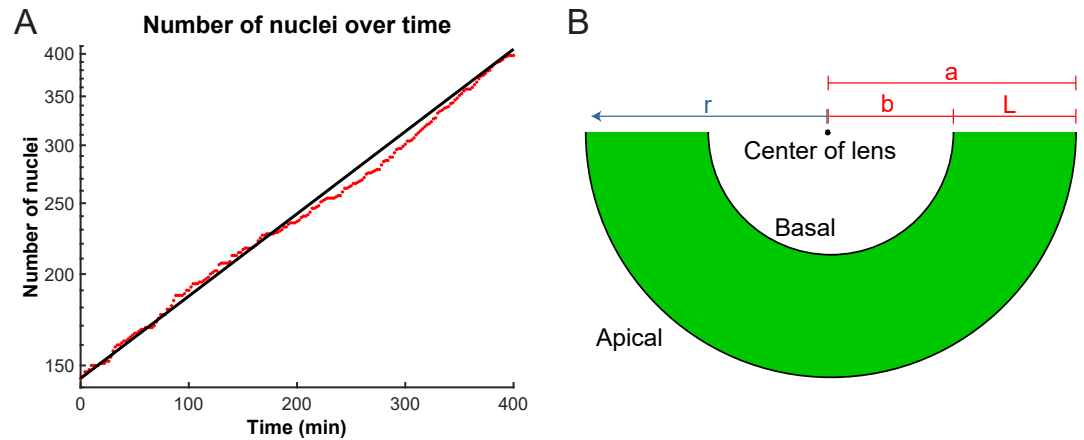
231 From these equations we can derive an expression for the concentration of nuclei  $c(r, t)$  in the  
232 retinal tissue. To this end, we introduced dimensionless variables for space and time,

$$\xi = \frac{r}{a}, \quad s = \frac{Dt}{a^2} \quad (5)$$



**Figure 4.** Nuclear concentration gradient across the apicobasal axis of the retina. The concentration of nuclei is higher near the apical surface compared to the basal surface. **(A)** In the control retina the nuclear concentration gradient builds up over time. **(B)** Blocking apical migration and division of nuclei, by inhibiting S phase progression, leads to a shift in the distribution of nuclei towards the basal surface in the HU-AC treated retina. **(C)** The shift in the distribution of nuclei under HU-AC treatment when compared to the untreated retina. The number of nuclei away from the apical surface increases consistently over time in the absence of cell division, but remains the same when new nuclei are constantly added at the apical surface. **(D)** A schematic of how a diffusion model would work in the context of IKNM in the retina. A concentration gradient of nuclei (left) would drive the net movement of nuclei from the apical surface to the basal surface. However, without maintenance of the gradient, the drive for this net migration is lost (top right). In the retina, the gradient is maintained through cell divisions at the apical surface, modeled as a one way influx across the apical surface (bottom right), continuously driving the net movement basally.





**Figure 5. (A)** Number of nuclei grows exponentially during the proliferative stage of the retinal development. A line can be fit to the log-lin graph of nuclear numbers as a function of time to extract the doubling time (cell cycle length) in this period. **(B)** A schematic of the retina indicating the variables used in the diffusion model of IKNM.  $a$ : distance from center of lens to apical surface;  $b$ : distance from center of lens to basal surface;  $L$ : thickness of the retina;  $r$ : distance from center of lens for each particle.

233 and further define  $\rho = b/a < 1$ . The exact solution for the nuclear concentration, whose detailed  
 234 derivation is given in the Appendix, is

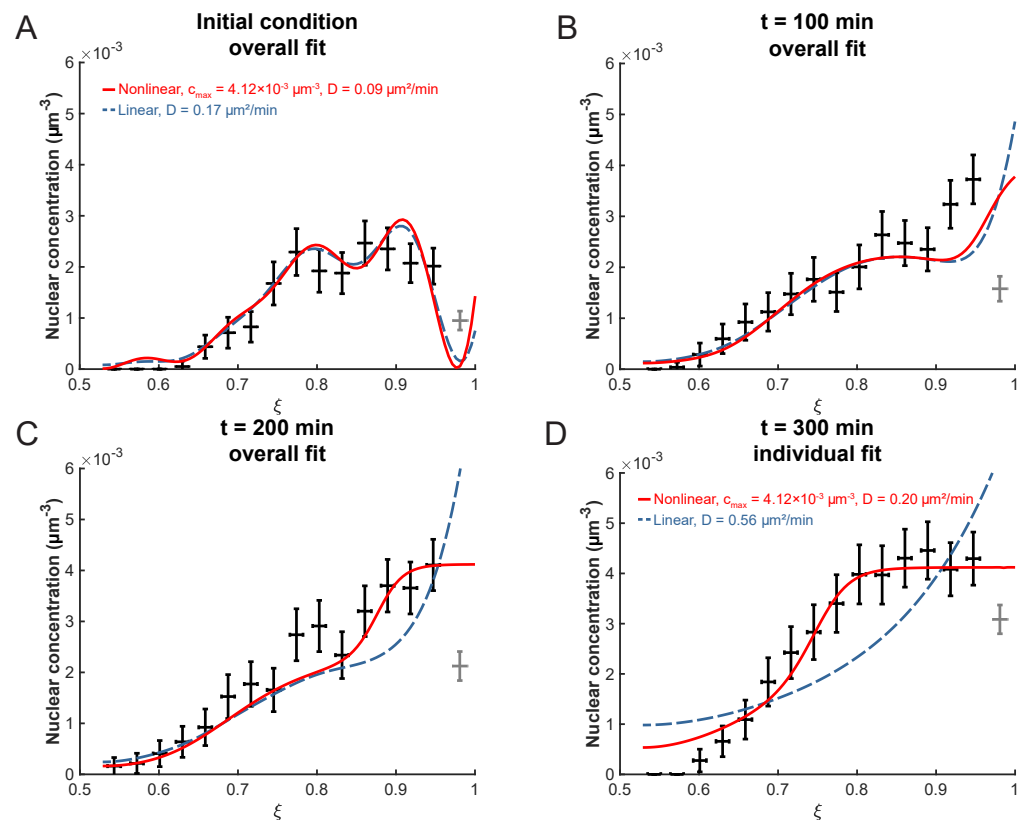
$$c(\xi, s) = \sum_{i=1}^{\infty} \left( h_i e^{-\lambda_i^2 s} + \frac{\alpha_i f_0}{\sigma + \lambda_i^2} e^{\sigma s} \right) H_i(\xi) + \frac{1}{1-\rho} \left( \frac{1}{2} \xi^2 - \rho \xi + g_0 \right) f_0 e^{\sigma s}. \quad (6)$$

235 The first terms within parentheses describes the decay over time of the initial condition  $c_{\text{exp}}(\xi, s = 0)$ .  
 236 Here,  $\lambda_i$  are the eigenvalues and  $H_i(\xi)$  the eigenfunctions of the radial diffusion problem, and the  
 237 coefficients  $h_i$  are determined from the experimental initial conditions (see Methods). The second  
 238 terms within the sum and the final term on the right hand side of Equation 6 are constructed such  
 239 that the solution fulfills the boundary conditions 3 and 4. In the last term, the constant  $g_0$  was  
 240 obtained using the constraint that the volume integral of the initial concentration yields the initial  
 241 number of nuclei  $N_0$ .  $f_0$ ,  $\sigma$  and  $\alpha_i$  emerge within the calculation of the solution and are specified in  
 242 the Appendix. Thus, the effective diffusion constant  $D$  in Equations 1 and 6 is the only unknown in  
 243 the model.

### 244 **The linear model is accurate at early times**

245 As mentioned before, the only parameter in the solution 6 is the effective diffusion constant  $D$ . To  
 246 determine this from the data, the experimentally obtained distribution of nuclei in the retinal tissue  
 247 was first converted into a concentration profile. Then, the optimal  $D$ -value, henceforth termed  
 248  $D^*$ , was obtained using a minimal- $\chi^2$  approach. The value obtained within the linear model for a  
 249 binning width of  $3 \mu\text{m}$  and an apical exclusion width of  $4 \mu\text{m}$  is  $D_{\text{lin}}^* = 0.17 \pm 0.07 \mu\text{m}^2/\text{min}$ . Using this,  
 250 we can examine the decay times of the different modes in the first term of Equation 6. The slowest  
 251 decaying modes are the ones with the smallest eigenvalues  $\lambda_i$  and we find that the longest three  
 252 decay times are  $\mathcal{T}_1 \approx 1325 \text{ min}$ ,  $\mathcal{T}_2 \approx 350 \text{ min}$  and  $\mathcal{T}_3 \approx 158 \text{ min}$ . This shows that indeed all three  
 253 terms of Equation 6 are relevant on the timescale of our experiment and need to be taken into  
 254 account when calculating the concentration profile. The corresponding plots of  $c(\xi, s)$  are shown in  
 255 Figure 6A-C. As can be seen from this figure, the diffusion model fits the data very well at early times,  
 256  $t \leq 200 \text{ min}$ . However, for  $t \geq 200 \text{ min}$  the model does not fit the data as well; the experimentally  
 257 observed nuclear concentration levels off at a value between  $4.00$  and  $4.50 \times 10^{-3} \mu\text{m}^{-3}$  (Figure 6D),  
 258 an aspect that is not captured by the model of linear diffusion.

259 One particular aspect of the biology that the linear model neglects is the spatial extent of the  
 260 nuclei. In a linear diffusion model, particles are treated as point-like and non-interacting. However,



**Figure 6.** (A) The initial experimental concentration profile of nuclei at  $t = 0$  min as well as the calculated initial condition curves (see Methods Equation 11) for the linear (red solid line) and nonlinear (blue dashed line) models. B,C,D) The fit of the models to experimental distribution of nuclei after 100 min (B), 200 min (C), and 300 min (D) are shown. For the first three graphs, the best fit over all 100 intervening time points were used with the corresponding diffusion constants shown in (A). For  $t = 300$  min, the best fit at that time point only was used with the corresponding diffusion constants indicated.

261 our microscopy images (see Figure 1A) clearly indicate that the nuclei have finite incompressible  
 262 volumes, so that their dense arrangement within the retinal tissue would lead to steric interactions  
 263 once the nuclear concentration is sufficiently high, and moreover that the packing density of nuclei  
 264 can not exceed a maximum value dictated by their geometry. Next, we examine whether accounting  
 265 for these effects leads to a more accurate theory.

### 266 Nonlinear extension to the model

267 If we write the diffusion equation 1 in the form

$$\frac{\partial c}{\partial t} = D \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 c \frac{\partial}{\partial r} \left[ \frac{\partial}{\partial c} (c \ln c) \right] \right\}, \quad (7)$$

268 we can identify the term  $c \ln c$  as proportional to the entropy  $\mathcal{S}$  of an ideal gas, and its derivative  
 269 with respect to  $c$  as a chemical potential. In an ideal gas, all particles are treated as point-like and  
 270 without mutual interactions. In order to include the spatial extent of particles, we estimate the  
 271 entropy using the model of a lattice gas, a system in which space is divided into discrete sites which  
 272 can either be empty or occupied by a single gas particle. Due to the discrete lattice, particles cannot  
 273 get closer than the lattice spacing from each other, and there is a maximum possible concentration  
 274  $c_{\max}$  (Huang, 1987). In this system the entropy takes the form

$$\mathcal{S}_{\text{lattice gas}} \propto c \ln c + (c_{\max} - c) \ln (c_{\max} - c). \quad (8)$$

**Table 1.** List of best-fit diffusion constants  $D^*$ , their standard deviations and probabilities for the studied conditions.

	$D_{\text{nonlin}}^*$ ( $\mu\text{m}^2/\text{min}$ )	$\sigma_D$ ( $\mu\text{m}^2/\text{min}$ )	$P_\chi(\chi^2; \nu)$
Normal	0.09	0.05	0.49 - 0.51
Normal (repeat sample)	0.10	0.06	0.47 - 0.48
High T	0.13	0.08	0.42
Low T	0.06	0.05	0.69 - 0.7

275 Substituting this expression for the term  $c \ln c$  in 7, we obtain the nonlinear diffusion equation

$$\frac{\partial c}{\partial t} = D \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{c_{\text{max}}}{c_{\text{max}} - c} \frac{\partial c}{\partial r} \right). \quad (9)$$

276 Adjusting the boundary conditions at the apical side accordingly leads to

$$D \frac{c_{\text{max}}}{c_{\text{max}} - c} \frac{\partial c}{\partial r} \Big|_{r=a} = \frac{N_0}{S\tau} e^{t/\tau}, \quad (10)$$

277 while the basal boundary condition remains the same as Equation 4. Together, Equation 9 and the  
 278 boundary conditions in Equations 10 and 4 represent an extension to the diffusion model for IKNM,  
 279 which now accounts for steric interactions between the nuclei. The maximum concentration  $c_{\text{max}}$   
 280 incorporated in this model was obtained, as described in the Methods, by considering a range of  
 281 nuclear radii and the maximum possible packing density for aligned ellipsoids (*Donev et al., 2004*).

282 Similar to fitting the linear model, we also need to establish a description of the initial condition.  
 283 To make both models consistent with each other, we employ the linear model's initial condition,  
 284 Equation 6 at  $s = 0$  with  $h_i$  as obtained from Equation 11, as an initial condition for this nonlinear  
 285 model as well (Figure 6A). The concentration profile in the nonlinear model and its derivative were  
 286 obtained numerically using the MATLAB pdepe solver. Fitting this concentration profile to the data  
 287 was again by means of a minimal- $\chi^2$  approach. When the optimization took data points up to  
 288  $t = 200$  min into account, we find  $D_{\text{nonlin}}^* = 0.09 \pm 0.05 \mu\text{m}^2/\text{min}$  (Figure 6, Table 1). As can be seen,  
 289 by choosing  $c_{\text{max}}$  correctly, an excellent fit to the data can be obtained. These results show that  
 290 a lattice-gas based diffusion model is indeed suitable to describe time evolution of the nuclear  
 291 concentration profile in zebrafish retina tissue during IKNM over several hours of development.

### 292 **Incubation temperature has direct effects on IKNM**

293 The diffusion model may also address mechanistic questions about IKNM in retinas growing under  
 294 varying experimental conditions. Zebrafish embryos are often grown at different temperatures to  
 295 manipulate their growth rate (*Kimmel et al., 1995; Reider and Connaughton, 2014*), but it has been  
 296 unclear how the nuclei in the retina behave at these different temperatures. To examine this issue,  
 297 we grew the embryos at the normal temperature of 28.5 °C overnight and then incubated them  
 298 at lower temperature (LT) of 25 °C or higher temperature (HT) of 32 °C during imaging. We could  
 299 directly measure the change in average cell cycle length from experimental data and found that  
 300 in HT, it is 205.5 min, while in LT, it is a much longer 532.78 min. We were then able to use these  
 301 values in the model to investigate whether the change in temperature influences the processes  
 302 that determine the effective diffusion constant of the nuclei. The resulting values for  $D_{\text{nonlin}}^*$  are  
 303 summarised in Table 1. Based on these values, two-sided  $t$ -tests (see Methods) confirmed that  
 304 there is no significant difference between the  $D$ -values obtained from the two normal condition  
 305 data sets. In contrast,  $D$ -values for the LT and HT data sets were significantly different from the  
 306 normal ones, with  $p \leq 0.01$ . These results indicate, that aside from its effect on cell cycle length,  
 307 incubation temperature is likely to influence IKNM directly by altering the mobility of nuclei, here  
 308 represented by the effective diffusion constant  $D$ .

## 309 Discussion

310 In this work, we have shown that high density nuclear trajectories can be used to tease apart the  
311 possible physical processes behind the apparently stochastic movement of nuclei during interkinetic  
312 nuclear migration. Firstly, we generated these trajectories using long-term imaging and tracking of  
313 nuclei with high spatial and temporal resolution within a 3-dimensional segment of the zebrafish  
314 retina. Analysis of speed and positional distributions of more than a hundred nuclei revealed a  
315 large degree of variability in their movements during G1 and S phases. Although this variability had  
316 been observed before, previous experiments had only considered sparsely labeled nuclei within  
317 an otherwise unlabeled environment (*Baye and Link, 2007; Norden et al., 2009; Leung et al., 2011*).  
318 Thus, our results provide an important account of the variability of IKNM on a whole tissue level. In  
319 effect, the variability of IKNM means that nuclear trajectories appear stochastic during the majority  
320 of the cell cycle. Previously, it had been suggested that the origins of this apparent stochasticity lay  
321 in the transfer of kinetic energy between nuclei in G2 exhibiting rapid apical migration to nuclei  
322 in G1 and S phases of the cell cycle, much as a person with an empty beer glass may nudge away  
323 other customers to get to the bar (*Norden et al., 2009*). However, we found no evidence for direct  
324 transfer of kinetic energy between nuclei and their immediate neighbors. Recently *Shinoda et al.*  
325 (*2018*) have also provided evidence that suggests direct collisions do not contribute to basal IKNM.

326 Another possibility is that the stochastic trajectories of G1 and S nuclei could be a result of  
327 passive displacements, arising from a diffusive process depending on a nuclear concentration  
328 gradient between the apical and basal sides of the tissue (*Miyata et al., 2015*). This gradient  
329 could be formed by nuclear divisions taking place exclusively at the apical surface. We confirmed  
330 the presence of such a gradient by calculating the nuclear concentration along the apicobasal  
331 dimension within the retinal tissue at various time points. Further, to probe the source of the  
332 gradient, we treated the zebrafish retina with HU-AC to stop the cell cycle in S phase. While  
333 we observed the build-up of the nuclear concentration gradient over time in the control retina,  
334 the nuclear distribution flattened when cell division was inhibited with HU-AC treatment. These  
335 phenomenological similarities between IKNM and diffusion suggested the diffusive model. This  
336 model includes two key features: most importantly, it focuses on the crowding of nuclei at the apical  
337 surface of the tissue, here included as the apical boundary condition. Additionally, in the nonlinear  
338 extension of the model, it incorporates a maximum possible nuclear concentration. This addition  
339 provided a striking overall improvement to the fits to experimental data over periods of many hours.  
340 The resulting difference in the obtained  $D$ -values between the linear and nonlinear versions of our  
341 model can be understood heuristically when closely examining the difference between Eqs. 1 and 9.  
342 The latter introduces the new term  $c_{\max}/(c_{\max} - c)$  which one could think of loosely as corresponding  
343 to an effective, concentration dependent diffusion constant  $\tilde{D} = Dc_{\max}/(c_{\max} - c)$ . In general  $\tilde{D}$  will  
344 vary across the tissue thickness and, since  $c > 0$  for most of the retinal tissue,  $\tilde{D} > D$ . Therefore,  
345 averaging across the retina tissue,  $\tilde{D}$  might actually be in very good agreement with the  $D$ -value  
346 found in the linear model. However, the fact the linear model fails to describe, and which leads to a  
347 better representation of the data using the nonlinear model, is that the mobility of the nuclei is  
348 likely to be concentration dependent.

349 The underlying processes causing IKNM during the G1 and S phases of the cell cycle in pseudos-  
350 trated epithelia have been largely elusive. Several partially competing ideas have been put forward,  
351 ranging from the active involvement of cytoskeletal transport processes to passive mechanisms of  
352 direct energy transfer or movements driven by apical nuclear crowding (*Schenk et al., 2009; Tsai*  
353 *et al., 2010; Norden et al., 2009; Kosodo et al., 2011*). The fact that inanimate microbeads migrate  
354 much like nuclei during IKNM in the mouse cerebral cortex (*Kosodo et al., 2011*) suggests that active,  
355 unidirectional intracellular transport mechanisms are not directly responsible for these stochastic  
356 movements. Instead, we showed that a passive diffusive process which takes steric interactions  
357 between nuclei into account produces an excellent representation of the time evolution of the actual  
358 nuclear distribution within the retinal tissue during early development. Consequently, our work

359 builds on earlier models of apical crowding based on *in silico* simulations of IKNM (*Kosodo et al.*,  
360 **2011**). Having said this, it remains to understand the general scale of the diffusion constant ( $D \sim 0.1$   
361  $\mu^2/\text{min}$ ) from microscopic considerations, perhaps analogous to those used to relate random walks  
362 to diffusion (*Goldstein, 2018*). In addition, our work revealed the remarkable importance of simple  
363 physical constraints imposed by the overall tissue architecture, which could not be explored in pre-  
364 vious studies which tracked sparse nuclei, and thus lacked the means to explore the effect of such  
365 3-dimensional arrangements. Hence, we paid special attention to the spherical shape of the retina  
366 and the concentration of nuclei in that space. Examining the evolution in distribution of nuclei over  
367 time unveils the importance of spatial restriction due to the curvature of the tissue. Additionally, the  
368 size of the nuclei in comparison to the neuroprogenitor cells leads to the emergence of a maximum  
369 nuclear concentration which must be taken into account to accurately model IKNM.

370 By inhibiting cell cycle progression or changing temperature, we used our model to shed some  
371 light on some of the properties of and mechanisms of the stochastic movements of nuclei during  
372 IKNM. From our results and previous studies, we knew that cell cycle length is affected by change in  
373 incubation temperature (*Kimmel et al., 1995; Reider and Connaughton, 2014*). However, our results  
374 also indicate a significant influence of temperature on the mobility of nuclei and thus the underlying  
375 processes controlling their movement. For example, the speed and dynamic properties of both  
376 the microtubule and actomyosin systems are dependent on temperature and could in part explain  
377 the changes in the diffusion constant that we see as a function of temperature (*Hartshorne et al.*,  
378 **1972; Hong et al., 2016**) as the diffusion constant may be influenced by stochastic associations with  
379 motor proteins or the physical properties of the epithelium. However, a much closer examination  
380 of molecular mechanisms driving stochastic nuclear movements is required to better understand  
381 the connections between these phenomena, as we are far from understanding the nature of forces  
382 involved in this process. Furthermore, the diffusion constant reported here contains all types of  
383 nuclear movement during IKNM as it is derived from the changing nuclear concentration profile  
384 over time. However, it is not immediately clear what the contribution the rapid apical migration to  
385 this overall diffusion constant may be. Nonetheless, despite the large displacement during rapid  
386 apical migration at G2, this phase only accounts for about 8% of the cell cycle (*Leung et al., 2011*).  
387 Therefore, given this small portion of the cell cycle when rapid migration can happen and the good  
388 agreement of our calculated diffusion constant with those previously reported in the literature for  
389 individual nuclei (*Leung et al., 2011*), the proposed model appears to describe tissue-wide IKNM  
390 quite well.

391 The physiological consequences of nuclear arrangements and the IKNM movements associated  
392 with all pseudostratified epithelia are not well understood. Our results provide a quantitative  
393 description of the stochastic distribution of the nuclei across the retina. This distribution has  
394 been implicated in stochastic cell fate decision making of progenitor cells during differentiation  
395 (*Clark et al., 2012; Baye and Link, 2007; Hiscock et al., 2018*). Our observations would fit with  
396 previous suggestions that a signalling gradient, such as a Notch gradient, exists across the retina  
397 and location-dependent exposure to it is important for downstream decision-making (*Murciano*  
398 *et al., 2002; Del Bene et al., 2008; Hiscock et al., 2018; Aggarwal et al., 2016*). Thus, our results  
399 not only have important implications for understanding the organisation of developing vertebrate  
400 tissues, but may also provide a starting point for further exploration of the connection between  
401 variability in nuclear positions and cell fate decision making in neuroepithelia.

## 402 **Methods and Materials**

### 403 **Animals and Transgenic Lines**

404 All animal work was approved by Local Ethical Review Committee of the University of Cambridge  
405 and performed in accordance with a Home Office project license PL80/2198. All zebrafish were  
406 maintained and bred at 26.5 °C. All embryos were incubated at 28.5 °C before imaging sessions. At  
407 10 hours post fertilization (hpf), 0.003% phenylthiourea (PTU) (sigma) was added to the medium to

408 stop pigmentation in the eye.

### 409 **Lightsheet microscopy**

410 Images of retinal development for the main dataset were obtained using lightsheet microscopy.  
411 Double transgenic embryos, Tg(bactin2:H2B-GFP::ptf1a:DsRed) were dechorionated at 24 hpf and  
412 screened positive for the fluorescent transgenic markers prior to the imaging experiment. The  
413 embryo selected for imaging was then embedded in 0.4% low gelling temperature agarose (Type  
414 VII, Sigma-Aldrich) prepared in the imaging buffer (0.3x Danialu's solution with 0.2% tricaine and  
415 0.003% PTU (*Godinho, 2011*)) within an FEP tube with 25  $\mu\text{m}$  thick walls (Zeus), with an eye facing  
416 the camera and the illumination light shedding from the ventral side. The tube was held in place by  
417 a custom-designed glass capillary (3 mm outer diameter, 20 mm length; Hilgenberg GmbH). The  
418 capillary itself was mounted vertically in the imaging specimen chamber filled with the imaging  
419 buffer. To ensure normal development, a perfusion system was used to pump warm water into the  
420 specimen chamber, maintaining a constant temperature of 28.5 °C at the location of the specimen.

421 Time-lapse recording of retinal development was performed using a SiMView light-sheet micro-  
422 scope (*Tomer et al., 2012*) with one illumination and one detection arm. Lasers were focused by  
423 Nikon 10x/0.3 NA water immersion objectives. Images were acquired with Nikon 40x/0.8 NA water  
424 immersion objective and Hamamatsu Ocras Flash 4.0 sCMOS camera. GFP was excited with scanned  
425 light sheets using a 488 nm laser, and detected through a 525/50 nm band pass detection filter  
426 (Semrock). Image stacks were acquired with confocal slit detection (*Baumgart and Kubitschek,*  
427 *2012*) with exposure time of 10 ms per frame, and the sample was moved in 0.812  $\mu\text{m}$  steps  
428 along the axial direction. For each time point, two 330 x 330 x 250  $\mu\text{m}^3$  image stacks with a 40  
429  $\mu\text{m}$  horizontal offset were acquired to ensure the coverage of the entire retina. The images were  
430 acquired every 2 min from 30 hpf to 72 hpf. The position of the sample was manually adjusted  
431 during imaging to compensate for drift. The two image stacks in the same time point were fused  
432 together to keep the combined image with the best resolution. An algorithm based on phase  
433 correlation was subsequently used to estimate and correct for the sample drift over time. The  
434 processing pipeline was implemented with MATLAB (MathWorks).

### 435 **Two photon microscopy**

436 Images for the repetition dataset and all other conditions were obtained using a TriM Scope II  
437 2-photon microscope (LaVision BioTec). A previously established Tg(H2B-GFP) line, generated by  
438 injecting a DNA construct of H2B-GFP driven from the actin promoter (*He et al., 2012*), was used for  
439 all these experiments. Embryos were dechorionated and screened for expression of GFP at 24 hpf.  
440 An embryo was then embedded in 0.9% UltraPure low melting point agarose (Invitrogen) prepared  
441 in E3 medium containing 0.003% PTU and 0.2% tricaine. The agarose and embryo were placed  
442 laterally within a 3D printed half cylinder of transparent ABS plastic, 0.8 mm in diameter, attached  
443 to the bottom of a petri dish, such that one eye faced the detection lens of the microscope. The  
444 petri dish was then filled with an incubation solution of E3 medium, PTU, and tricaine in the same  
445 concentrations as above. For the experiment involving cell cycle arrest, hydroxyurea and aphidicolin  
446 (Abcam) were added to the incubation solution right before imaging, to a final concentration of 20  
447 mM and 150  $\mu\text{M}$ , respectively. The imaging chamber was maintained at a temperature of 25 °C,  
448 28.5 °C, or 32 °C, as required, using a precision air heater (The Cube, Life Imaging Services).

449 Green fluorescence was excited using an Insight DeepSee laser (Spectra-Physics) at 927 nm.  
450 The emission of the fluorophore was detected through an Olympus 25x/1.05 NA water immersion  
451 objective, and all the signal within the visible spectrum was recorded by a sensitive GaAsP detector.  
452 Image stacks with step size of 1  $\mu\text{m}$  were acquired with exposure time of 1.35 ms per line averaged  
453 over two scans. The images were recorded every 2 min for 10-15 hours starting at 26-28 hpf. The  
454 same post processing procedure for data compression and drift correction was used on these raw  
455 images as on those from lightsheet imaging.

## 456 **Obtaining experimental input values for the model**

457 The radial coordinates  $r_n$  of nuclei were calculated by subtracting  $l_n$  from  $a$ , wherein  $l_n$  is the distance  
458 from the center of a nucleus  $n$  to the apical surface and  $a$  is the distance from the center of the  
459 lens to the apical surface. We estimated a total uncertainty of  $\Delta r = \pm 3 \mu\text{m}$  for each single distance  
460 measurement of  $r_n$ . This value is a result of uncertainty in detecting the center of the nucleus and  
461 in establishing the position of the apical surface.

462 Because each nuclear position has an error bar  $\Delta r$ , binning the data leads to an uncertainty  
463 in the bin count. In order to calculate this uncertainty, we considered the probability distribution  
464 of a nucleus' position. In the simplest case, this probability is uniform within the width of the  
465 positional error bar and zero elsewhere. The probability,  $p_{n,\text{bin}}$ , of finding a given nucleus  $n$  within a  
466 given bin, is proportional to the size of the overlap of probability distribution and bin. It follows  
467 that the expectation value for the number of nuclei within a bin is given as  $\mathbb{E}(N_{\text{bin}}) = \sum_n p_{n,\text{bin}}$ .  
468 Correspondingly,  $\text{Var}(N_{\text{bin}}) = \sum_n p_{n,\text{bin}}(1 - p_{n,\text{bin}})$  is the variance of the number of nuclei within this  
469 bin. Thus, the error bar of the bin count is  $\sigma_{y,\text{bin}} = \sqrt{\text{Var}(N_{\text{bin}})}$ . The nuclear distribution profile  
470  $N(r, t)$  is not expected to be uniform or linear, therefore the expectation value  $\mathbb{E}(N_{\text{bin}})$  does not  
471 correspond to the number of nuclei at the center of the bin. Since the position of the expectation  
472 value is unknown *a priori*, it is still plotted at the center of the bin with an error bar denoting its  
473 positional uncertainty. Here we assume this error bar to be the square-root of the bin size  $\Delta r_{\text{bin}}$ , i.e.  
474  $\sigma_{x,\text{bin}} = \sqrt{\Delta r_{\text{bin}}}$ .

475 In order to obtain the experimental nuclear concentration profile  $c(r, t)$ , and its error bars, from  
476 the distribution of nuclei  $N(r, t)$ , the volume of the retina also has to be taken into account, since  
477  $c = N/V$ . The total retinal volume within which nuclei tracking took place was estimated directly  
478 from the microscopy images. To this end, we outlined the area of observation in each image slice  
479 using the Fiji software and multiplied this area with the distance between successive images. Given  
480 the total volume,  $V_{\text{total}}$ , we proceeded to calculate the volume per bin, which depends on the radii at  
481 the inner and outer bin surfaces. In general, the volume of part of a sphere, e.g. a spherical sector,  
482 is given as  $V_{\text{sector}} = \frac{1}{3}\Omega r_{\text{sector}}^3$ , where  $\Omega$  denotes the solid angle. Knowing the apical and basal tissue  
483 radii,  $r = a$  and  $r = b$ , one can thus calculate  $\Omega$  as  $\Omega = 3V_{\text{total}}/(a^3 - b^3)$ . This gives the volume of each  
484 bin as  $V_{\text{bin}} = \frac{1}{3}\Omega (r_{\text{bin,outer}}^3 - r_{\text{bin,inner}}^3)$ , where  $r_{\text{bin,outer}}$  and  $r_{\text{bin,inner}}$  denote the outer and inner radii of  
485 a bin, respectively. Similarly, we calculated the effective surface area  $S$  through which the influx of  
486 nuclei occurs (see Equation 3) from the solid angle  $\Omega$ . This surface area is simply given as  $S = \Omega a^2$ .

487 To retrieve the average cell cycle time  $T_p$  for each of the data sets, we used two different  
488 approaches. In the case of the main data set, sufficient number of nuclear tracks consisting of  
489 a whole cell cycle were present. Thus we directly calculated the average cell cycle duration from  
490 these tracks. For the other datasets, we make use of the fact that the number of nuclei follows an  
491 exponential growth law depending on  $T_p$  (see Equation 2). Knowing the initial number of tracked  
492 nuclei  $N_0$  for each data set, we obtained  $T_p$  from fitting the following equation to the number of  
493 nuclei as a function of time in a log-lin plot:  $\ln N(t) = \ln N_0 + t/\tau = \ln N_0 + (\ln 2/T_p)t$ . Then  $T_p$  was  
494 deduced from the slope of this fit.

495 In order to determine the maximum nuclear concentration  $c_{\text{max}}$  for the nonlinear model, we  
496 first randomly selected 100 nuclei from our dataset of tracked nuclei and measured the size of their  
497 longest diameter in both XY and YZ planes. From these measurements we established that the  
498 size of the principal semi-axis of each nucleus is likely to lie in the range of about  $3 \mu\text{m}$  to  $5 \mu\text{m}$ ,  
499 where the nuclear shape is regarded to be ellipsoidal. This led to the range of possible maximum  
500 concentrations  $c_{\text{max}}$ , although we did not measure the precise nuclear volume. The lower limit for  
501 the nuclear volume is set by the volume of a sphere of radius  $3 \mu\text{m}$ , the upper limit by a sphere of  
502 radius  $5 \mu\text{m}$ . Taking into account the maximum possible packing density of nuclei, which for aligned  
503 ellipsoids is the same as that of spheres (Donev *et al.*, 2004),  $\frac{\pi}{3\sqrt{2}} \approx 0.74$ , we obtained a range of  
504  $1.41 \times 10^{-3} \mu\text{m}^{-3} \leq c_{\text{max}} \leq 6.55 \times 10^{-3} \mu\text{m}^{-3}$ .

## 505 Obtaining the initial condition

506 We determined the prefactors  $h_i$  from the experimental nuclear distribution at the start of the  
 507 experiment,  $c_{\text{exp}}(\xi, 0)$ . For convenience, we chose to determine first  $\tilde{h}_i = h_i + \alpha_i f_0 / (\sigma + \lambda_i^2)$  and then  
 508 obtained  $h_i$  by subtracting  $\alpha_i f_0 / (\sigma + \lambda_i^2)$  from the results. The  $\tilde{h}_i$  can be calculated from the data,  
 509 using Equation 6 for  $s = 0$ , as

$$\tilde{h}_i = \sum_m \xi_m^2 H_i(\xi_m) c_{\text{exp}}(\xi_m, 0) \Delta \xi_m - \frac{f_0}{1 - \rho} \int_0^1 \xi^2 H_i(\xi) \left( \frac{1}{2} \xi^2 - \rho \xi + g_0 \right) d\xi, \quad (11)$$

510 where  $m$  denotes the  $m$ -th binned data point,  $\xi_m$  its position and  $\Delta \xi_m$  the width of bin  $m$ . As in  
 511 Equation 6, the index  $i$  denotes the  $i$ -th eigenfunction or -mode.

## 512 The concentration profile in the nonlinear model

513 The non-linear concentration profile was determined numerically from the same initial condition as  
 514 used for the linear model, Equation 6, at  $s = 0$  with  $\tilde{h}_i$  as in Equation 11. Time evolution of the initial  
 515 condition, according to Equation 9, was performed using the pdepe solver in MATLAB.

## 516 Fitting the model

517 The range of sizes of the nuclear principal semi-axes was used to determine the range of data to be  
 518 included in our fits. Any data closer than 3  $\mu\text{m}$  to 5  $\mu\text{m}$  from the apical or basal tissue surfaces was  
 519 not taken into account for fitting because the center of a nucleus cannot be any closer to a surface  
 520 than the nuclear radius. Thus, all data collection very close to the apical or basal tissue surfaces  
 521 must have been due to the above mentioned measurement uncertainties  $\Delta r$ .

522 In principle, the full solution for  $c(\xi, s)$  is composed of infinitely many modes. However, in  
 523 practice, we truncated this series and only included the first 8 modes in our fits. This is due to the  
 524 fact that we have a finite set of data points, so adding too many modes could lead to over-fitting.  
 525 Fits with a wide range of numbers of modes were found to result in the same optimal  $D$ -values.

526 For fitting, we first rescaled the data in accordance with the non-dimensionalisation of the  
 527 theoretical variables  $r$  and  $t$  (see Equation 5). Thus we obtain  $c_{\text{exp}}(\xi, s)$  from  $c_{\text{exp}}(r, t)$ . Then both  
 528 models were fitted to the experimental data using a minimal- $\chi^2$  approach. The goodness of fit  
 529 parameter  $\chi^2 = \sum_m (c_{\text{exp}}(\xi, s) - c(\xi, s))^2 / \sigma_m^2$ , where  $\sum_m$  denotes the summation over all bins  $m$ . Since  
 530 binning resulted in uncertainties  $\sigma_{y,\text{bin}}$  and  $\sigma_{x,\text{bin}}$  in the  $y$ - and  $x$ -directions, both had to be taken  
 531 into account when calculating  $\sigma_m$  and  $\chi^2$ . The combined contribution of  $x$ - and  $y$ - uncertainties is:  
 532  $\sigma_m^2 = \sigma_{y,m}^2 + \sigma_{y,\text{indirect},m}^2$  with  $\sigma_{y,\text{indirect},m} = \sigma_{x,m} (dc(\xi, s)/d\xi) \Big|_{\xi=\xi_m}$  (Bevington and Robinson, 2003). In our  
 533 fits, the value  $\chi^2$  was calculated for a large range of possible diffusion constants  $D$ , from  $D = 0.01$   
 534  $\mu\text{m}^2/\text{min}$  to  $D = 10 \mu\text{m}^2/\text{min}$ . By finding the value of  $D$  for which  $\chi^2$  became minimal for a given  
 535 data set and time point, we established our optimal fit.

536 The minimal- $\chi^2$  approach furthermore enabled us to determine the optimal binning width  $\Delta r_{\text{bin}}$   
 537 or  $\Delta \xi_{\text{bin}}$  and width of data exclusion for the fits. In order to do so, fits of the normal data set were  
 538 performed for different data binning widths and exclusion sizes of 3  $\mu\text{m}$  to 5  $\mu\text{m}$ . For each of these  
 539 fits the  $\chi^2$ -value and the number of degrees of freedom  $\nu$ , i.e. the number of data points minus  
 540 the number of free fit parameters (here number of data points minus 1), were registered. From  
 541  $\chi^2$  and  $\nu$  we calculated the reduced  $\chi^2$  value,  $\chi_\nu^2 = \chi^2 / \nu$  (Bevington and Robinson, 2003). Using  $\nu$   
 542 and  $\chi_\nu^2$ , the probability  $P_\chi(\chi^2; \nu)$  of exceeding  $\chi$  for a given fit can be estimated, which should be  
 543 approximately 0.5 (Bevington and Robinson, 2003). Therefore, we found our optimal data binning  
 544 width of 3  $\mu\text{m}$  to 4  $\mu\text{m}$  as the width that resulted in a  $P_\chi(\chi^2; \nu)$  as close to 0.5 as possible for all the  
 545 different time points when fitting the nonlinear model. The exact choice of exclusion width was  
 546 found not to influence the fitting result for the nonlinear model.

547 In addition to finding the optimal  $D$ -value for individual time points, we also modified the  
 548 minimal- $\chi^2$  routine to find the value of  $D$  that fits a whole data set (i.e. all time points simultaneously)  
 549 in the best possible way. In order to do so, we summed the  $\chi^2$ -values obtained for each  $D$  over all  
 550 time points, in this way producing a  $\sum_t \chi^2(D)$ -curve. The minimum of this curve indicates  $D^*$  for



551 the whole time series. Furthermore, dividing  $\sum_i \chi^2(D)$  by the number of time points included in the  
552 optimization yields an average  $\chi^2$ - and reduced  $\chi^2$ -value corresponding to this  $D^*$ . In addition, the  
553 width of this time averaged curve at  $\chi^2 = \chi_{\min}^2 + 1$  indicates the standard deviation of the optimal  
554  $D$ -value,  $\sigma_D$ . By approximating the minimum with a quadratic curve, we obtain an estimate for this  
555 standard deviation as  $\sigma_D = \Delta_D \sqrt{2 \left( \chi_{D^*-\Delta_D}^2 - 2\chi_{D^*}^2 + \chi_{D^*+\Delta_D}^2 \right)}$  (Bevington and Robinson, 2003) where  
556  $\Delta_D$  is the step size between individual fitted  $D$ -values, here  $\Delta_D = 0.01 \mu\text{m}^2/\text{min}$ . Lastly, based on the  
557 average reduced  $\chi^2$ -values, we also compared several  $c_{\max}$ -values for each data set to find the fit  
558 with probability  $P_\chi(\chi^2; \nu)$  the closest to 0.5 in each case.

559 All fits were performed using custom MATLAB routines.

## 560 **t-tests**

561 To compare results between data sets, the values  $D^*$  and corresponding  $\sigma_D$  from the overall fits  
562 were considered. It should be noted that these values were not obtained by averaging several data  
563 sets of the same experimental condition but instead each value results from one data set only.  
564 However, the sample size for each data set was set to 100 because 100 time points were taken into  
565 account for each overall optimization. These time points might not be completely uncorrelated,  
566 limiting the predictive power of the  $t$ -test. Two sided tests, specifically unequal variances  $t$ -test, also  
567 known as Welch's  $t$ -test, (Precht and Kraft, 2015), were performed in order to determine whether  
568 samples differ significantly from each other.

## 569 **Acknowledgments**

570 AH and REG would like to thank Oliver Y. Feng, Timothy J. Pedley, Michael E. Cates and Salvatore  
571 Torquato for helpful advice and input. This work was supported by the Cambridge Wellcome Trust  
572 PhD Programme in Developmental Biology, the Cambridge Commonwealth, European and Interna-  
573 tional Trust, and Natural Sciences and Engineering Research Council of Canada (AA); Established  
574 Career Fellowship EP/M017982/1 from the Engineering and Physical Sciences Research Council  
575 (REG); and Wellcome Trust Investigator Award (SIA 100329/Z/12/Z) (WAH).

## 576 **References**

- 577 **Aggarwal V**, Dickinson RB, Lele TP. Concentration Sensing by the Moving Nucleus in Cell Fate Determination: A  
578 Computational Analysis. PLoS One. 2016; 11(2):e0149213. doi: [10.1371/journal.pone.0149213](https://doi.org/10.1371/journal.pone.0149213).
- 579 **Amat F**, Höckendorf B, Wan Y, Lemon WC, Katie M, Keller PJ. Efficient processing and analysis of large-scale  
580 light-sheet microscopy data. Nat Protoc. 2015; 10(11):1679–1696. doi: [10.1038/nprot.2015.111](https://doi.org/10.1038/nprot.2015.111).
- 581 **Amat F**, Lemon W, Mossing DP, Katie M, Wan Y, Branson K, Myers EW, Keller PJ. Fast, accurate reconstruc-  
582 tion of cell lineages from large-scale fluorescence microscopy data. Nat Methods. 2014; 11(9):951. doi:  
583 [10.1038/nmeth.3036](https://doi.org/10.1038/nmeth.3036).
- 584 **Avanesov A**, Malicki J. Analysis of the retina in the zebrafish model. Methods Cell Biol. 2010; 100:153–204. doi:  
585 [10.1016/b978-0-12-384892-5.00006-2](https://doi.org/10.1016/b978-0-12-384892-5.00006-2).
- 586 **Barrasso AP**, Wang S, Tong X, Christiansen AE, Larina IV, Poché RA. Live imaging of developing mouse retinal  
587 slices. Neural Dev. 2018; 13(1):23. doi: [10.1186/s13064-018-0120-y](https://doi.org/10.1186/s13064-018-0120-y).
- 588 **Baumgart E**, Kubitschek U. Scanned light sheet microscopy with confocal slit detection. Opt Express. 2012;  
589 20(19):21805–14.
- 590 **Baye LM**, Link BA. Interkinetic Nuclear Migration and the Selection of Neurogenic Cell Divisions during Vertebrate  
591 Retinogenesis. J Neurosci. 2007; 27(38):10143–10152. doi: [10.1523/JNEUROSCI.2754-07.2007](https://doi.org/10.1523/JNEUROSCI.2754-07.2007).
- 592 **Bevington PR**, Robinson DK. Data Reduction and Error Analysis for the Physical Sciences. 3rd ed. McGraw-Hill;  
593 2003.
- 594 **Biehlmaier O**, Neuhauss SCF, Kohler K. Onset and time course of apoptosis in the developing zebrafish retina.  
595 Cell and Tissue Research. 2001; 306(2):199–207. doi: [10.1007/s004410100447](https://doi.org/10.1007/s004410100447).

- 596 **Clark BS**, Cui S, Miesfeld JB, Klezovitch O, Vasioukhin V, Link BA. Loss of Llg1 in retinal neuroepithelia reveals  
597 links between apical domain size, Notch activity and neurogenesis. *Development*. 2012; 139(9):1599–1610.  
598 doi: [10.1242/dev.078097](https://doi.org/10.1242/dev.078097).
- 599 **Del Bene F**. Interkinetic nuclear migration: cell cycle on the move. *Embo J*. 2011; 30(9):1676–1677. doi:  
600 [10.1038/emboj.2011.114](https://doi.org/10.1038/emboj.2011.114).
- 601 **Del Bene F**, Wehman AM, Link BA, Baier H. Regulation of Neurogenesis by Interkinetic Nuclear Migration  
602 through an Apical-Basal Notch Gradient. *Cell*. 2008; 134(6):1055–1065. doi: [10.1016/j.cell.2008.07.017](https://doi.org/10.1016/j.cell.2008.07.017).
- 603 **Donev A**, Stillinger FH, Chaikin PM, Torquato S. Unusually dense crystal packings of ellipsoids. *Physical Review*  
604 *Letters*. 2004; 92(25):255506. doi: [10.1103/PhysRevLett.92.255506](https://doi.org/10.1103/PhysRevLett.92.255506).
- 605 **Godinho L**. Imaging zebrafish development. *Cold Spring Harb protoc*. 2011; p. 879–83. doi:  
606 [10.1101/pdb.prot5647](https://doi.org/10.1101/pdb.prot5647).
- 607 **Goldstein RE**. Are theoretical results ‘Results?’ *eLife*. 2018; 7:e40018. doi: [10.7554/eLife.40018](https://doi.org/10.7554/eLife.40018).
- 608 **Hartshorne D**, Barns E, Parker L, Fuchs F. The effect of temperature on actomyosin. *Biochim Biophys Acta*  
609 *Bioenerg*. 1972; 267(1):190–202. doi: [10.1016/0005-2728\(72\)90150-8](https://doi.org/10.1016/0005-2728(72)90150-8).
- 610 **He J**, Zhang G, Almeida AD, Cayouette M, Simons BD, Harris WA. How Variable Clones Build an Invariant Retina.  
611 *Neuron*. 2012; 75(5):786–798. doi: [10.1016/j.neuron.2012.06.033](https://doi.org/10.1016/j.neuron.2012.06.033).
- 612 **Hiscock TW**, Miesfeld JB, Mosaliganti KR, Link BA, Megason SG. Feedback between tissue packing and neuroge-  
613 nesis in the zebrafish neural tube. *Development*. 2018; 145(9):dev.157040. doi: [10.1242/dev.157040](https://doi.org/10.1242/dev.157040).
- 614 **Hong W**, Takshak A, Osunbayo O, Kunwar A, Vershinin M. The Effect of Temperature on Microtubule-  
615 Based Transport by Cytoplasmic Dynein and Kinesin-1 Motors. *Biophys J*. 2016; 111(6):1287–1294. doi:  
616 [10.1016/j.bpj.2016.08.006](https://doi.org/10.1016/j.bpj.2016.08.006).
- 617 **Huang K**. *Statistical Mechanics*. 2nd ed. John Wiley & Sons; 1987.
- 618 **Icha J**, Kunath C, Mauricio R, Norden C. Independent modes of ganglion cell translocation ensure correct  
619 lamination of the zebrafish retina. *J Cell Biol*. 2016; 215(2):259–275. doi: [10.1083/jcb.201604095](https://doi.org/10.1083/jcb.201604095).
- 620 **Kimmel C**, Ballard W, Kimmel S, Ullmann B, Schilling T. Stages of embryonic development of the zebrafish. *Dev*  
621 *Dyn*. 1995; 203(3):253–310. doi: [10.1002/aja.1002030302](https://doi.org/10.1002/aja.1002030302).
- 622 **Kosodo Y**, Suetsugu T, Suda M, Yuko M, Toida K, Baba SA, Kimura A, Matsuzaki F. Regulation of interkinetic  
623 nuclear migration by cell cycle-coupled active and passive mechanisms in the developing brain. *Embo J*. 2011;  
624 30(9):1690–1704. doi: [10.1038/emboj.2011.81](https://doi.org/10.1038/emboj.2011.81).
- 625 **Leung L**, Klopper AV, Grill SW, Harris WA, Norden C. Apical migration of nuclei during G2 is a prerequisite for all  
626 nuclear motion in zebrafish neuroepithelia. *Development*. 2011; 139(14):2635–2635. doi: [10.1242/dev.085456](https://doi.org/10.1242/dev.085456).
- 627 **Matejčić M**, Salbreux G, Norden C. A non-cell-autonomous actin redistribution enables isotropic retinal growth.  
628 *Plos Biol*. 2018; 16(8):e2006018. doi: [10.1371/journal.pbio.2006018](https://doi.org/10.1371/journal.pbio.2006018).
- 629 **Miyata T**, Okamoto M, Shinoda T, Kawaguchi A. Interkinetic nuclear migration generates and opposes  
630 ventricular-zone crowding: insight into tissue mechanics. *Front Cell Neurosci*. 2015; 8:473. doi: [10.3389/fn-](https://doi.org/10.3389/fn-)  
631 [cel.2014.00473](https://doi.org/10.3389/fn-cel.2014.00473).
- 632 **Murciano A**, Zamora J, Jesús L, Frade JM. Interkinetic Nuclear Movement May Provide Spatial Clues to the  
633 Regulation of Neurogenesis. *Mol Cell Neurosci*. 2002; 21(2):285–300. doi: [10.1006/mcne.2002.1174](https://doi.org/10.1006/mcne.2002.1174).
- 634 **Norden C**. Pseudostratified epithelia – cell biology, diversity and roles in organ formation at a glance. *J Cell Sci*.  
635 2017; 130(11):jcs.192997. doi: [10.1242/jcs.192997](https://doi.org/10.1242/jcs.192997).
- 636 **Norden C**, Young S, Link BA, Harris WA. Actomyosin Is the Main Driver of Interkinetic Nuclear Migration in the  
637 Retina. *Cell*. 2009; 138(6):1195–1208. doi: [10.1016/j.cell.2009.06.032](https://doi.org/10.1016/j.cell.2009.06.032).
- 638 **Okamoto M**, Namba T, Shinoda T, Kondo T, Watanabe T, Inoue Y, Takeuchi K, Enomoto Y, Ota K, Oda K, Wada  
639 Y, Sagou K, Saito K, Sakakibara A, Kawaguchi A, Nakajima K, Adachi T, Fujimori T, Ueda M, Hayashi S, et al.  
640 TAG-1-assisted progenitor elongation streamlines nuclear migration to optimize subapical crowding. *Nat*  
641 *Neurosci*. 2013; 16(11):nn.3525. doi: [10.1038/nn.3525](https://doi.org/10.1038/nn.3525).
- 642 **Precht M**, Kraft R. *Bio-Statistik 2*. 5th ed. De Gruyter; 2015.

- 643 **Reider M**, Connaughton VP. Effects of Low-Dose Embryonic Thyroid Disruption and Rearing Temperature on the  
644 Development of the Eye and Retina in Zebrafish. *Birth Defects Res B Dev Reprod Toxicol*. 2014; 101(5):347–354.  
645 doi: [10.1002/bdrb.21118](https://doi.org/10.1002/bdrb.21118).
- 646 **Reif F**. Fundamentals of statistical and thermal physics. 1st ed. McGraw-Hill; 1965.
- 647 **Sauer FC**. Mitosis in the neural tube. *J Comp Neurol*. 1935; .
- 648 **Schenk J**, Michaela W, Calegari F, Huttner WB. Myosin II is required for interkinetic nuclear migration of neural  
649 progenitors. *Proc Natl Acad Sci USA*. 2009; 106(38):16487–16492. doi: [10.1073/pnas.0908928106](https://doi.org/10.1073/pnas.0908928106).
- 650 **Schindelin J**, Ignacio A, Frise E, Kaynig V, Longair M, Pietzsch T, Preibisch S, Rueden C, Saalfeld S, Schmid  
651 B, Tinevez J, White D, Hartenstein V, Eliceiri K, Tomancak P, Cardona A. Fiji: an open-source platform for  
652 biological-image analysis. *Nat Methods*. 2012; 9(7):676. doi: [10.1038/nmeth.2019](https://doi.org/10.1038/nmeth.2019).
- 653 **Shinoda T**, Nagasaka A, Inoue Y, Higuchi R, Minami Y, Kato K, Suzuki M, Kondo T, Kawae T, Saito K, Ueno  
654 N, Fukazawa Y, Nagayama M, Miura T, Adachi T, Miyata T. Elasticity-based boosting of neuroepithelial  
655 nucleokinesis via indirect energy transfer from mother to daughter. *Plos Biol*. 2018; 16(4):e2004426. doi:  
656 [10.1371/journal.pbio.2004426](https://doi.org/10.1371/journal.pbio.2004426).
- 657 **Sidman RL**, Miale IL, Feder N. Cell proliferation and migration in the primitive ependymal zone; An autoradio-  
658 graphic study of histogenesis in the nervous system. *Exp Neurol*. 1959; 1(4):322–333. doi: [10.1016/0014-4886\(59\)90024-X](https://doi.org/10.1016/0014-4886(59)90024-X).
- 660 **Spear PC**, Erickson CA. Interkinetic nuclear migration: A mysterious process in search of a function. *Dev Growth*  
661 *Differ*. 2012; 54(3):306–316. doi: [10.1111/j.1440-169X.2012.01342.x](https://doi.org/10.1111/j.1440-169X.2012.01342.x).
- 662 **Stelzer EH**. Light-sheet fluorescence microscopy for quantitative biology. *Nat Methods*. 2015; 12(1):23–26. doi:  
663 [10.1038/nmeth.3219](https://doi.org/10.1038/nmeth.3219).
- 664 **Sugiyama M**, Asako S, Imura T, Fukami K, Kitaguchi T, Kawakami K, Okamoto H, Higashijima Si, Miyawaki  
665 A. Illuminating cell-cycle progression in the developing zebrafish embryo. *Proc Natl Acad Sci USA*. 2009;  
666 106(49):20812–20817. doi: [10.1073/pnas.0906464106](https://doi.org/10.1073/pnas.0906464106).
- 667 **Svoboda K**, Yasuda R. Principles of Two-Photon Excitation Microscopy and Its Applications to Neuroscience.  
668 *Neuron*. 2006; 50(6):823–839. doi: [10.1016/j.neuron.2006.05.019](https://doi.org/10.1016/j.neuron.2006.05.019).
- 669 **Tomer R**, Khairy K, Amat F, Keller P. Quantitative high-speed imaging of entire developing embryos with  
670 simultaneous multiview light-sheet microscopy. *Nat Methods*. 2012; 9(7):755. doi: [10.1038/nmeth.2062](https://doi.org/10.1038/nmeth.2062).
- 671 **Tsai J**, Bremner K, Vallee R. Dual subcellular roles for LIS1 and dynein in radial neuronal migration in live brain  
672 tissue. *Nat Neurosci*. 2007; 10(8):970–979. doi: [10.1038/nn1934](https://doi.org/10.1038/nn1934).
- 673 **Tsai J**, Lian W, Kemal S, Kriegstein AR, Vallee RB. Kinesin 3 and cytoplasmic dynein mediate interkinetic nuclear  
674 migration in neural stem cells. *Nat Neurosci*. 2010; 13(12):1463. doi: [10.1038/nn.2665](https://doi.org/10.1038/nn.2665).
- 675 **Wolff C**, Tinevez J, Pietzsch T, Stamatakis E, Harich B, Guignard L, Preibisch S, Shorte S, Keller PJ, Tomancak P,  
676 Pavlopoulos A. Multi-view light-sheet imaging and tracking with the MaMuT software reveals the cell lineage  
677 of a direct developing arthropod limb. *Elife*. 2018; 7:e34410. doi: [10.7554/eLife.34410](https://doi.org/10.7554/eLife.34410).
- 678 **Xie Z**, Moy LY, Sanada K, Zhou Y, Buchman JJ, Tsai L. Cep120 and TACCs Control Interkinetic Nuclear Migration  
679 and the Neural Progenitor Pool. *Neuron*. 2007; 56(1):79–93. doi: [10.1016/j.neuron.2007.08.026](https://doi.org/10.1016/j.neuron.2007.08.026).

## 680 Appendix

### 681 Full solution of the linear diffusion equation

682 After rescaling space and time as in Equation 5 and introducing  $\rho = a/b < 1$ , Equation 1 and the  
683 boundary conditions 3 and 4 read

$$\begin{aligned} \frac{\partial c(\xi, s)}{\partial s} &= \frac{1}{\xi^2} \frac{\partial}{\partial \xi} \left( \xi^2 \frac{\partial c(\xi, s)}{\partial \xi} \right), \\ \frac{\partial c(\xi, s)}{\partial \xi} \Big|_{\xi=1} &= f_0 e^{\sigma s} = f(s) \quad \text{and} \quad \frac{\partial c(\xi, s)}{\partial \xi} \Big|_{\xi=\rho} = 0, \end{aligned} \quad (12)$$

684 where we have defined  $f_0 = aN_0/DS\tau$  and  $\sigma = a^2/D\tau$ . We transform this homogeneous differential  
 685 equation with inhomogeneous boundary conditions into the problem of solving an inhomogeneous  
 686 differential equation with homogeneous boundary conditions by writing  $c(\xi, s)$  as a sum of two  
 687 contributions,

$$c(\xi, s) = \phi(\xi, s) + \psi(\xi, s), \quad (13)$$

688 where we require  $\phi(\xi, s)$  to satisfy the inhomogeneous boundary conditions

$$\left. \frac{\partial \phi(\xi, s)}{\partial \xi} \right|_{\xi=1} = f_0 e^{\sigma s} \quad \text{and} \quad \left. \frac{\partial \phi(\xi, s)}{\partial \xi} \right|_{\xi=\rho} = 0. \quad (14)$$

689 These conditions are satisfied if  $\phi(\xi, s)$  has the form

$$\phi(\xi, s) = \frac{1}{1-\rho} \left( \frac{1}{2} \xi^2 - \rho \xi + g_0 \right) f_0 e^{\sigma s}. \quad (15)$$

690 where  $g_0$  is a constant of integration to be determined later. The remaining problem to solve for  
 691  $\psi(\xi, s)$  is

$$\frac{\partial \psi(\xi, s)}{\partial s} = \frac{1}{\xi^2} \frac{\partial}{\partial \xi} \left( \xi^2 \frac{\partial \psi(\xi, s)}{\partial \xi} \right) + \frac{f_0 e^{\sigma s}}{1-\rho} \left( 3 - \frac{2\rho}{\xi} - \sigma \left( \frac{1}{2} \xi^2 - \rho \xi + g_0 \right) \right), \quad (16)$$

692 with homogeneous boundary conditions

$$\left. \frac{\partial \psi(\xi, s)}{\partial \xi} \right|_{\xi=1} = 0 \quad \text{and} \quad \left. \frac{\partial \psi(\xi, s)}{\partial \xi} \right|_{\xi=\rho} = 0. \quad (17)$$

693 We can further write  $\psi(\xi, s)$  as the sum of two contributions,

$$\psi(\xi, s) = \psi_h(\xi, s) + \psi_p(\xi, s), \quad (18)$$

694 where  $\psi_h$  is the general solution of the homogeneous problem

$$\begin{aligned} \frac{\partial \psi_h(\xi, s)}{\partial s} &= \frac{1}{\xi^2} \frac{\partial}{\partial \xi} \left( \xi^2 \frac{\partial \psi_h(\xi, s)}{\partial \xi} \right), \\ \left. \frac{\partial \psi_h(\xi, s)}{\partial \xi} \right|_{\xi=1} &= 0 \quad \text{and} \quad \left. \frac{\partial \psi_h(\xi, s)}{\partial \xi} \right|_{\xi=\rho} = 0, \end{aligned} \quad (19)$$

695 and  $\psi_p$  is a particular solution of the full inhomogeneous problem 17. The full solution of the  
 696 homogeneous problem is given as a series of linearly independent eigenfunctions, each of the form  
 697

$$e^{-\lambda^2 s} W(\xi) = e^{-\lambda^2 s} \left( A \frac{\sin \lambda \xi}{\xi} + B \frac{\cos \lambda \xi}{\xi} \right), \quad (20)$$

698 where the eigenvalues  $\lambda$  can be found from simultaneous solution of the boundary conditions,

$$\begin{aligned} A(\lambda \cos \lambda - \sin \lambda) - B(\lambda \sin \lambda + \cos \lambda) &= 0 \\ A \left( \frac{\lambda \cos \lambda \rho}{\rho} - \frac{\sin \lambda \rho}{\rho^2} \right) - B \left( \frac{\lambda \sin \lambda \rho}{\rho} + \frac{\cos \lambda \rho}{\rho^2} \right) &= 0, \end{aligned} \quad (21)$$

699 which yields the transcendental relation

$$\tan \lambda(1-\rho) = \frac{\lambda(1-\rho)}{\lambda^2 \rho + 1}, \quad (22)$$

700 for which each eigenvalue  $\lambda_i$  is a solution corresponding to one of the linearly independent eigen-  
 701 functions (only  $\lambda_i > 0$  need to be taken into account). We can further deduce from the Equation 21  
 702 that  $B_i = \beta_i A_i$ , where

$$\beta_i = \frac{\lambda_i \cos \lambda_i - \sin \lambda_i}{\lambda_i \sin \lambda_i + \cos \lambda_i}, \quad (23)$$

703 and we normalize the obtained expression for  $W_i(\xi)$  from Equation 20

$$H_i(\xi) = \frac{1}{Y_i} \left( \frac{\sin \lambda_i \xi}{\xi} + \beta_i \frac{\cos \lambda_i \xi}{\xi} \right), \quad (24)$$

704 with

$$Y_i^2 = \frac{1}{2}(1 - \rho)(1 + \beta_i^2) - \frac{1}{4\lambda_i}(\sin 2\lambda_i - \sin 2\lambda_i\rho)(1 - \beta_i^2) + \frac{\beta_i}{\lambda_i}(\sin^2 \lambda_i - \sin^2 \lambda_i\rho). \quad (25)$$

705 Thus, the homogeneous solution  $\psi_h$  is

$$\psi_h = \sum_{i=1}^{\infty} h_i H_i(\xi) e^{-\lambda_i^2 s}, \quad (26)$$

706 with prefactors  $h_i$  to be determined from the initial condition.

707 In order to find a particular solution of the inhomogeneous problem, we first rewrite 17 as

$$\frac{\partial \psi(\xi, s)}{\partial s} - \frac{1}{\xi^2} \frac{\partial}{\partial \xi} \left( \xi^2 \frac{\partial \psi(\xi, s)}{\partial \xi} \right) = \mathcal{R}(\xi, s). \quad (27)$$

708 Now, we express  $\mathcal{R}(\xi, s)$ , as well as the unknown inhomogeneous solution  $\psi_i(\xi, s)$  in terms of the  
709 normalized eigenfunctions  $H(\xi, s)$  of the homogeneous problem,

$$\mathcal{R}(\xi, s) = \sum_{i=1}^{\infty} R_i(s) H_i(\xi), \quad (28)$$

710 and

$$\psi_i(\xi, s) = \sum_{i=1}^{\infty} C_i(s) H_i(\xi). \quad (29)$$

711 Substituting these forms into 27, and noting that each term in the series must vanish separately we  
712 obtain

$$\frac{\partial C_i(s)}{\partial s} + \lambda_i^2 C_i(s) - R_i(s) = 0. \quad (30)$$

713 From the form of  $\mathcal{R}(\xi, s)$  it follows that  $R_i(s) = \alpha_i f_0 e^{\sigma s}$  with some purely numerical prefactors  $\alpha_i$ , so  
714 we expect  $C_i(s) \propto p_i e^{\sigma s}$  and find

$$p_i = \frac{\alpha_i f_0}{\sigma + \lambda_i^2}. \quad (31)$$

715 Finally, we determine the  $\alpha_i$  by reconsidering Equation 28. We multiply both sides by  $\xi^2 H_j(\xi)$ , where  
716  $H_j(\xi)$  is one specific but arbitrary eigenfunction of the homogeneous problem, and then integrate  
717 over the whole volume  $V$ . By the orthogonormality of these eigenfunctions we obtain

$$\alpha_j = \int \frac{1}{1 - \rho} \left( 3 - \frac{2\rho}{\xi} - \sigma \left( \frac{1}{2} \xi^2 - \rho \xi + g_0 \right) \right) \xi^2 H_j(\xi) d\xi, \quad (32)$$

718 and all the  $\alpha_i$  can be calculated explicitly. Thus, the full solution of the linear problem is

$$c(\xi, s) = \sum_{i=1}^{\infty} \left( h_i e^{-\lambda_i^2 s} + \frac{\alpha_i f_0}{\sigma + \lambda_i^2} e^{\sigma s} \right) H_i(\xi) + \frac{1}{1 - \rho} \left( \frac{1}{2} \xi^2 - \rho \xi + g_0 \right) f_0 e^{\sigma s}. \quad (33)$$

719 The constant  $g_0$  can now be calculated from the requirement that  $\int c(\xi, s = 0) dV = N_0$ . Here we  
720 make use of the fact that  $\int H_i(\xi) \xi^2 d\xi = 0$  if  $\lambda_i$  satisfies Equation 22, thus

$$g_0 = \frac{(1 - \rho)/\sigma - \frac{1}{10} + \frac{1}{4}\rho + \frac{1}{10}\rho^5 - \frac{1}{4}\rho^5}{\frac{1}{3}(1 - \rho^3)}. \quad (34)$$