A mathematical model of calcium dynamics: Obesity and mitochondria-associated ER membranes

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Abstract

Multiple cellular organelles tightly orchestrate intracellular calcium (Ca^{2+}) dynamics to regulate cellular activities and maintain homeostasis. The interplay between the endoplasmic reticulum (ER), a major store of intracellular Ca^{2+}, and mitochondria, an important source of adenosine triphosphate (ATP), has been the subject of much research, as their dysfunctionality has been linked with metabolic diseases. Interestingly, through out the cell's cytosolic domain, these two organelles share common microdomains called mitochondria-associated ER membranes (MAMs), where their membranes are in close apposition. The role of MAMs is critical for intracellular Ca^{2+} dynamics as they provide hubs for direct Ca^{2+} exchange between the organelles. A recent experimental study reported correlation between obesity and MAM formation in mouse liver cells, and obesity-related cellular changes that are closely associated with the regulation of Ca^{2+} dynamics. We constructed a mathematical model to study the effects of MAM Ca^{2+} dynamics on global Ca^{2+} activities. Through a series of model simulations, we investigated cellular mechanisms underlying the altered Ca^{2+} dynamics in the cells under obesity. We found that the formation of MAMs is negatively correlated with the amplitude of cytosolic Ca^{2+} activities, but positively correlated with that of mitochondrial Ca^{2+} dynamics and the overall frequency of Ca^{2+} oscillations. We

predict that, as the dosage of stimulus gradually increases, liver cells from obese mice will reach the state of saturated cytosolic Ca^{2+} concentration at a lower stimulus concentration, compared to cells from healthy mice.

Author summary

It is well known that intracellular Ca^{2+} oscillations carry encoded signals in their amplitude and frequency to regulate various cellular processes, and accumulating evidence supports the importance of the interplay between the ER and mitochondria in cellular Ca^{2+} homeostasis. Miscommunications between the organelles may be involved in the development of metabolic diseases. Based on a recent experimental study that spotlighted a correlation between obesity and physical interactions of the ER and mitochondria in mouse hepatic cells, we constructed a mathematical model as a probing tool that can be used to computationally investigate the effects of the cellular changes linked with obesity on global cellular Ca^{2+} dynamics. Our model successfully reproduced the experimental study that observed a positive correlation between the ER-mitochondrial junctions and the magnitude of mitochondrial Ca^{2+} responses. We postulate that hepatic cells from lean animals exhibit Ca^{2+} oscillations that are more robust under higher concentrations of stimulus, compared to cells from obese animals.

Introduction

In most multicellular organisms, calcium (Ca^{2+}) is a ubiquitous second messenger that controls a vast array of cellular activities spanning from cell birth to apoptosis [1]. The endoplasmic/sarcoplasmic reticulum (ER/SR) and mitochondria have been the center of attention in the study of intracellular Ca^{2+} dynamics, due to their role as internal Ca^{2+} stores. The SR is mostly found in muscle cells, which are not the subject of this paper, so we only refer to the ER. It has been suggested that dysfunction of Ca^{2+} regulation in the ER and/or mitochondria leads to disrupted cellular homeostasis, and is associated with pathological processes, including metabolic diseases and neurodegenerative diseases [2–7].

Upon agonist stimulation, almost all types of cells exhibit fluctuations in cytosolic

Ca^{2+} concentration, phenomena often referred to as Ca^{2+} oscillations, with signals encoded in oscillation frequencies and amplitudes. Among many cellular compartments, the ER, whose internal Ca^{2+} concentration is three to four orders of magnitude larger than that of the cytosol in resting condition, is considered as the main contributor to the generation of Ca^{2+} oscillations. The ER has several types of Ca^{2+} channels on the membrane that release Ca^{2+} once activated. The most well-studied Ca^{2+} release channels are inositol trisphosphate receptors (IPRs) and ryanodine receptors (RyRs). As a high cytosolic Ca^{2+} concentration is toxic and often leads to cell death, released Ca^{2+} is quickly pumped back into the ER lumen through sarco/endoplasmic reticulum Ca^{2+} ATPase (SERCA) pumps, which consume energy to sequester Ca^{2+} against its concentration gradient. Some Ca^{2+} released from the ER can be taken up by mitochondria through the mitochondrial Ca^{2+} uniporter (MCU), and then released back to the cytosol via the sodium/calcium exchanger (NCX). Thus, it is generally accepted that mitochondria have the ability to modulate oscillation frequencies and amplitudes, and consequently, affect the progression of cellular activities [4].

Having a spatially extended membrane network, the ER is often positioned in close proximity with other cellular organelles and forms membrane contact sites. Such sites between the ER and mitochondria are called mitochondria-associated ER membranes (MAMs), and it has been suggested that they play a critical role in Ca^{2+} exchange between the organelles [4, 5]. Since mitochondrial Ca^{2+} regulation is closely linked with adenosine triphosphate (ATP) synthesis and reactive oxygen species (ROS) production [8], understanding the mechanisms underlying the ER-mitochondrial Ca^{2+} crosstalk is of great scientific and physiological interest. A major advantage of MAM formation is that due to its minuscule size, even a small Ca^{2+} flux into the domain would be amplified, which is convenient for the MCUs which have a low Ca^{2+} affinity, i.e., they require a high concentration of Ca^{2+} in order to activate.

Arruda et al. [9] reported a positive correlation between obesity and the degree of MAM formation. They also found different expression levels of Ca^{2+} channels between liver cells of lean and obese mice. These findings indicate the possibility of obesity-induced changes in Ca^{2+} dynamics in MAMs, and consequently, in the ER as well as mitochondria. Indeed, liver cells from obese animals showed higher baselines of cytosolic Ca^{2+} concentration and mitochondrial Ca^{2+} concentration, compared to cells

from lean mice. Furthermore, Ca^{2+} transients generated from ATP stimulation led to higher concentration peaks in obese mouse mitochondria. Interestingly, this observation was not accompanied by higher peaks in cytosolic Ca^{2+} concentration, i.e., cells from obese and lean mice exhibited similar ATP-induced rises in cytosolic Ca^{2+} concentration.

Computational models of experimental data have been a valuable tool for understanding the dynamics of intracellular Ca^{2+}. Most models have focused on either the ER Ca^{2+} handling [10–13] or mitochondrial Ca^{2+} dynamics [14–17] and only a handful of them have integrated the dynamics from both organelles [18,19]. Recently, there have been an increasing number of studies that combined both experimental and theoretical approaches to probe the cellular mechanisms underlying Ca^{2+} crosstalk between the ER and mitochondria. The model proposed by Szopa et al. [20] assumes that due to the minuscule volume of MAMs, the MCUs in MAMs sense Ca^{2+} concentration in the ER. Thus, the MCU Ca^{2+} flux in their model is essentially direct Ca^{2+} flow from the ER. Using numerical methods, they investigated the effects of this flow on the shape (bursting) and period of Ca^{2+} oscillations, and observed that mitochondrial Ca^{2+} concentrations tend to a high level in some regions of parameter space. Another recent model by Qi et al. [21] considers a range of possible distances between the IPRs and MCUs in MAMs, and expresses Ca^{2+} concentration in MAMs as a solution to a linearized reaction-diffusion equation. In this model, the concentration of Ca^{2+} that is sensed by the MCUs in MAMs depends on the distance of the MCUs from the point of source (a cluster of IPRs) and how fast Ca^{2+} diffuses in MAMs. The authors showed that Ca^{2+} signals can be significantly modulated by the distance, and determined an optimal distance between the IPRs and MCUs for effective Ca^{2+} exchange for the generation of Ca^{2+} oscillations. On the other hand, Wacquier et al. [22] published a model that associates Ca^{2+} oscillations with mitochondrial metabolism, and investigated the role of mitochondrial Ca^{2+} fluxes on the oscillation frequency. Their model modified one of the parameters that describes the Ca^{2+} concentration for the activation of the MCUs to a lower concentration than the one originally suggested by Magnus and Keizer [16]. By doing so, they implicitly included MAMs, with the following assumption: MCUs are activated at the average concentration of Ca^{2+} in the whole cytosol (including MAMs). They found that

mitochondrial Ca^{2+} fluxes can modulate the frequency of Ca^{2+} oscillations.   76

Here, we construct a mathematical model to investigate the cellular mechanisms   77
underlying the altered mitochondrial Ca^{2+} dynamics observed in obese mice. The model   78
extends the model of Wacquier et al. [22], and explicitly includes Ca^{2+} dynamics in   79
MAMs. We incorporated the model structure proposed by Penny et al. [23], wherein the   80
cytosol is compartmentalized to two separate domains: the bulk cytosol and membrane   81
contact sites between organelles. Rather than expressing the Ca^{2+} concentration in   82
MAMs as an algebraic function of that of the cytosol, we model it as a dynamic variable   83
that is determined by influxes and effluxes of the domain. We investigated how Ca^{2+} 84
signals are affected by the obesity related changes in Ca^{2+} channel expression levels.   85

Materials and Models   86

A better way of understanding the model structure is to consider the model as two   87
sub-models that are coupled by common factors, Ca^{2+} and ATP. One of the sub-models   88
describes intracellular Ca^{2+} dynamics, while the other models mitochondrial metabolic   89
pathways and voltage.   90

Ca^{2+} dynamics   91

Jones et al. [24] observed vasopressin-induced Ca^{2+} oscillations in hepatocytes in   92
Ca^{2+}-free environment. This experiment suggested that hepatocytes do not require   93
Ca^{2+} influx from the extracellular space to trigger Ca^{2+} oscillations, and that their 94
oscillations are primarily caused by the periodic release and re-uptake of Ca^{2+} by   95
intracellular stores. During the experiment, the oscillations were sustained for about 15   96
minutes after the stimulation, with decreasing frequencies, most likely due to the   97
continuous loss of Ca^{2+} across the plasma membrane as, eventually, cells became   98
completely depleted of Ca^{2+}. When modeling Ca^{2+} signals with these behaviors, it is   99
useful to consider a closed-cell model, where it is assumed that the total intracellular   100
Ca^{2+} is fixed (i.e., a cell does not lose or gain Ca^{2+} through the plasma membrane). 101
Since the main focus of this study is on Ca^{2+} dynamics in liver cells, we approached the   102
problem with a closed-cell model. We first compartmentalized the cellular domain into   103
four separate regions: the ER, the bulk cytosol, a mitochondrion, and MAM, and   104

Fig 1. Schematic diagram of Ca^{2+} dynamics described by the model.
The ER releases Ca^{2+} to the cytosol and MAM; the fluxes are denoted as J_{IPR} and J_{nIPR}, respectively. The cytosolic Ca^{2+} is pumped back into the ER lumen via SERCA (J_{SERCA}), and the Ca^{2+} in MAM are also pumped back into the ER (J_{nSERCA}). Mitochondria uptake Ca^{2+} from the cytosol (J_{MCU}) and MAM (J_{nMCU}), and mitochondrial Ca^{2+} is exchanged with Na$^+$ in the cytosol (J_{NCX}) and MAM (J_{nNCX}). Ca^{2+} can freely diffuse between the cytosol and MAM (J_{diff}).

assumed that Ca^{2+} concentration within each region, denoted by C_{ER}, C_{cyt}, C_{mito}, and C_{MAM}, respectively, is determined by Ca^{2+} influxes and effluxes going in and out of that region. Fig. 1 shows a schematic diagram of the compartments and Ca^{2+} fluxes in the model. Since the model does not include Ca^{2+} fluxes across the plasma membrane, the total intracellular Ca^{2+} concentration, C_{t}, is written as:

$$C_{\mathrm{t}} = C_{cyt} + \frac{1}{R_{\mathrm{V1}}} C_{MAM} + \frac{1}{R_{\mathrm{V2}}} C_{mito} + \frac{1}{R_{\mathrm{V3}}} C_{ER}.$$

The R_{V}'s account for compartment volume differences and are defined as:

$$R_{\mathrm{V1}} = \frac{\text{total cytosolic volume}}{\text{total MAM volume}},$$

$$R_{\mathrm{V2}} = \frac{\text{total cytosolic volume}}{\text{total ER volume}},$$

$$R_{\mathrm{V3}} = \frac{\text{total cytosolic volume}}{\text{total mitochondrial volume}}.$$

Given that the mitochondrial outer membrane is freely permeable to small molecules, such as Ca^{2+}, through the voltage dependent anion channel (VDAC), we assume that Ca^{2+} concentration in the inter-membrane space is equivalent to that of the cytosol. Thus, there is one effective layer of impermeable boundary, the mitochondrial inner membrane, that separates cytosolic Ca^{2+} from mitochondrial Ca^{2+}.

The Ca^{2+} dynamics part of the model consists of the following ordinary differential

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equations (ODEs):

$$\frac{d}{dt}C_{cyt} = (1 - R_{S1})(J_{IPR} - J_{SERCA} + J_{leak}) + \frac{1 - R_{S2}}{R_{V3}}(J_{NCX} - J_{MCU}) + J_{diff}$$

$$\frac{d}{dt}C_{MAM} = R_{V1}R_{S1}(J_{nIPR} - J_{nSERCA} + J_{nleak}) + \frac{R_{V1}R_{S2}}{R_{V3}}(J_{nNCX} - J_{nMCU}) - R_{V1}J_{diff}$$

$$\frac{d}{dt}C_{mito} = R_{S2}(J_{nMCU} - J_{nNCX}) + (1 - R_{S2})J_{MCU} - J_{NCX}$$

$$\frac{d}{dt}P = \tau_p(P_s - P) + \text{pulse} \tag{1}$$

$$\frac{d}{dt}P_{MAM} = D_p(P - P_{MAM})$$

$$\frac{d}{dt}h_{42} = \lambda_{h_{42}}(h_{42}^\infty - h_{42})$$

$$\frac{d}{dt}h_{n42} = \lambda_{h_{n42}}(h_{n42}^\infty - h_{n42})$$

P and P_{MAM} represent the concentrations of IP$_3$ in the bulk cytosol and the MAM, 116
respectively. h_{42} and h_{n42} denote the activation variables of the IPRs in the bulk 117
cytosol and the MAM, respectively. The R_S's are surface ratios, defined as 118

$$R_{S1} = \frac{\text{surface area of the ER that adjoins the MAM}}{\text{the total ER surface area}},$$

$$R_{S2} = \frac{\text{surface area of mitochondrion that adjoins the MAM}}{\text{the total mitochondrion surface area}};$$

see Table for their values. Short descriptions for the J_* Ca^{2+} fluxes are given below. 119

- J_{IPR} and J_{nIPR}: IPR Ca^{2+} flux into the bulk cytosol and MAMs from the ER 120

- J_{SERCA} and J_{nSERCA}: SERCA pump Ca^{2+} flux into the ER from the bulk cytosol 121
  and MAMs 122

- J_{leak} and J_{nleak}: a small Ca^{2+} leak from the ER into the bulk cytosol and MAMs 123

- J_{NCX} and J_{nNCX}: NCX Ca^{2+} flux from mitochondria into the bulk cytosol and 124
  MAMs 125

- J_{MCU} and J_{nMCU}: MCU Ca^{2+} flux into mitochondria from the bulk cytosol and 126
  MAMs 127

- J_{diff}: Ca^{2+} diffusion between the bulk cytosol and MAMs 128

IPR model

We incorporated the IPR model proposed in Cao et al. [25], which assumes that the receptors are either in drive mode when they are mostly open, or in park mode when they are mostly closed. The drive mode has one open state (O_6) and one closed state (C_2), while there is one closed state (C_4) in the park mode. The transition rates between the modes are denoted by q_{24} and q_{42}, and the rates between the states within the drive mode are q_{26} and q_{62}. The open probability of the drive mode is $q_{26}/(q_{26} + q_{62})$ ($\approx 70\%$). q_{24} and q_{42} are given by

$$q_{24} = q_{24} + V_{24}(1 - m_{24}^{\infty} h_{24}^{\infty}),$$

$$q_{42} = a_{42} + V_{42} m_{42}^{\infty} h_{42}.$$

The m's and h's are gating variables that govern the opening and closing kinetics of the receptors, with the following quasi-equilibria:

$$m_{24}^{\infty} = \frac{C_p^3}{C_p^3 + k_{24}^3},$$

$$h_{24}^{\infty} = \frac{k_{-24}^2}{C_p^2 + k_{-24}^2},$$

$$m_{42}^{\infty} = \frac{C_{cyt}^3}{C_{cyt}^3 + k_{24}^3},$$

$$h_{42}^{\infty} = \frac{k_{-42}^3}{C_{cyt}^3 + k_{-42}^3}.$$

While m_{24}, m_{42}, and h_{24} are assumed to have reached their quasi-equilibria instantaneously, there is the rate at which h_{42} approaches its equilibrium, $\lambda_{h_{42}}$,

$$\lambda_{h_{42}} = (1 - D)L_{\text{IPR}} + DH_{\text{IPR}}.$$

$C_p = C_{p0}(C_{ER}/680)$ denotes the concentration of Ca^{2+} at the pore of a receptor. The expressions for the V's, a's, and k's are

$$V_{24} = 62 + 880/(P^2 + 4) \qquad\qquad a_{24} = 1 + 5/(P^2 + 0.25)$$

$$k_{24} = 0.35 \qquad\qquad k_{-24} = 80$$

$$V_{42} = 110P^2/(P^2 + 0.01) \qquad\qquad a_{42} = 1.8P^2/(P^2 + 0.34)$$

$$k_{42} = 0.49 + 0.543P^3/(P^3 + 64) \qquad k_{-42} = 0.41 + 25P^3/(P^3 + 274.6)$$

The open probability of the IPRs in the cytosol, O_{IPR} is defined as   141

$$O_{\text{IPR}} = \frac{q_{42}}{q_{62} + q_{26}}D,$$

where D,   142

$$D = \frac{q_{42}(q_{62} + q_{26})}{q_{42}q_{62} + q_{42}q_{26} + q_{24}q_{62}},$$

represents the proportion of the IPRs that are in the drive mode. Then the IPR fluxes   143
are modeled as: 144

$$J_{\text{IPR}} = k_{\text{IPR}}O_{\text{IPR}}(C_{ER} - C_{cyt}),$$

$$J_{\text{nIPR}} = k_{\text{nIPR}}O_{\text{nIPR}}(C_{ER} - C_{MAM}),$$

where O_{nIPR} is in the same functional form as O_{IPR}, except that the composing   145
functions are now expressed with C_{MAM} and P_{MAM}, instead of C_{cyt} and P.   146

The diffusion flux between the MAM and the cytosol is a linear function of the   147
concentration difference,   148

$$J_{\text{diff}} = D_c(C_{MAM} - C_{cyt}).$$

Small leak fluxes across the ER membrane are given by

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\begin{aligned}
J_{\text{leak}} &= k_{\text{leak}}(C_{ER} - C_{cyt}), \\
J_{\text{nleak}} &= k_{\text{leak}}(C_{ER} - C_{MAM}).
\end{aligned}
$$

We follow Wacquier et al. [22] in modeling the SERCA, the MCU, and the NCX fluxes,

$$
\begin{aligned}
J_{\text{SERCA}} &= V_{\text{SERCA}} \frac{C_{cyt}^2}{K_{\text{SERCA}}^2 + C_{cyt}^2} \frac{ATP_c}{K_{\text{e}} + ATP_c}, \\[2mm]
J_{\text{nSERCA}} &= V_{\text{nSERCA}} \frac{C_{MAM}^2}{K_{\text{SERCA}}^2 + C_{MAM}^2} \frac{ATP_c}{K_{\text{e}} + ATP_c}, \\[2mm]
J_{\text{MCU}} &= V_{\text{MCU}} \frac{\frac{C_{cyt}}{K_1}\left(1 + \frac{C_{cyt}}{K_1}\right)^3 e^{p_1 V_m}}{\left(1 + \frac{C_{cyt}}{K_1}\right)^4 + \dfrac{L_{\text{MCU}}}{\left(1 + \frac{C_{cyt}}{K_2}\right)^{2.8}}}, \\[2mm]
J_{\text{nMCU}} &= V_{\text{nMCU}} \frac{\frac{C_{MAM}}{K_1}\left(1 + \frac{C_{MAM}}{K_1}\right)^3 e^{p_1 V_m}}{\left(1 + \frac{C_{MAM}}{K_1}\right)^4 + \dfrac{L_{\text{MCU}}}{\left(1 + \frac{C_{MAM}}{K_2}\right)^{2.8}}}, \\[2mm]
J_{\text{NCX}} &= V_{\text{NCX}} \left(\frac{C_{mito}}{C_{cyt}}\right) e^{p_2 V_m}, \\[2mm]
J_{\text{nNCX}} &= V_{\text{nNCX}} \left(\frac{C_{mito}}{C_{MAM}}\right) e^{p_2 V_m}.
\end{aligned}
$$

They set the parameter K_1 to 6 μM, but this represents the average level of Ca^{2+} in the whole cytosol, including MAMs, when the concentration reaches a physiologically reasonable level in MAMs. Since we consider MAMs explicitly in the model, we set K_1 to 19 μM, as originally proposed by Magnus et al. [16].

IP$_3$ metabolism

A number of studies suggest Ca^{2+} oscillations in hepatocytes can occur at a constant level of IP$_3$. In particular, an experimental study reported oscillating Ca^{2+}

concentrations in the ER lumen in permeabilized hepatocytes, while the concentration   158
of IP$_3$ was clamped at a submaximal concentration [26]. Thus, we assumed that Ca^{2+} 159
oscillations are primarily generated by Ca^{2+} feedback on the opening and closing   160
kinetics of the IPR. For simplicity, we did not consider cellular formation or breakdown   161
of IP$_3$ that involves Ca^{2+} or protein kinase C (PKC). Instead, we model IP$_3$ dynamics   162
as a gradual increase from 0 μM to its steady-state concentration, P_s, at a rate of τ_p.   163
However, if hepatocytes were to exhibit IP$_3$ oscillations, it is possible to include positive   164
and/or negative Ca^{2+} feedback on IP$_3$ metabolism in the model to introduce IP$_3$ 165
oscillations as passive reflections of Ca^{2+} oscillations.   166

Apart from continuous stimulation, the model can be perturbed with a single   167
stimulation in a pulsatile manner. In such a case, cells would be exposed to a certain   168
amount of agonist for a short period of time. While continuous stimulation saturates   169
the concentration of IP$_3$ to its steady state, a pulse stimulation produces a sudden   170
increase in IP$_3$ concentration, followed by a natural decay at a rate of degradation. In   171
the model, a pulse of stimulation is described by   172

$$\text{pulse} = MH(t - t_0)H(t_0 + \triangle - t), \qquad (2)$$

where H is the Heaviside function   173

$$H(t - t_0) = \begin{cases} 0 & \text{if } t \leq t_0, \\ 1 & \text{if } t > t_0, \end{cases}$$

M is the pulse magnitude, t_0 is the time at which the pulse is given, and $\triangle$ is the pulse   174
duration.   175

Mitochondrial metabolic pathway model   176

The main function of mitochondria is to create ATP by oxidative phosphorylation. Due   177
to this particular role, mitochondria are the powerhouse of the cell. The mitochondrial   178
metabolic pathway is initiated by the uptake of pyruvate, which is the end product of   179
cytosolic glycolysis. Pyruvate in the mitochondrial matrix then enters the tricarboxylic   180
acid (TCA) cycle, also known as the citric acid cycle or the Krebs cycle, to generate the   181

reducing agent NADH that has electrons with a high transfer potential. The $\qquad$ 182

concentration of mitochondrial NADH can also be increased by the activity of the $\qquad$ 183

malate-aspartate shuttle (MAS). NADH then goes through an electron transport chain $\qquad$ 184

(ETC), where the electrons are separated and used to drive protons (H^+) across the $\qquad$ 185

inner membrane and generate a proton gradient between the intermembrane space and $\qquad$ 186

mitochondrial matrix. As protons accumulate in the intermembrane space, the gradient $\qquad$ 187

across the inner membrane is used by the F1FO-ATPase to convert mitochondrial $\qquad$ 188

adenosine diphosphate (ADP) to ATP via phosphorylation. The produced ATP is then $\qquad$ 189

transported to the cytosol by adenine nucleotide translocases (ANT), which carry out $\qquad$ 190

the exchange of cytosolic ADP and mitochondrial ATP across the inner mitochondrial $\qquad$ 191

membrane. $\qquad$ 192

Ca^{2+} is an important component in mitochondrial metabolism, as it promotes the production of NADH. An increase in mitochondrial Ca^{2+} concentration upregulates the TCA cycle, and an increase in cytosolic Ca^{2+} concentration stimulates the aspartate-glutamate carrier (AGC), a protein involved in the MAS. We incorporated a part of the model in Wacquier et al. [22] to describe Ca^{2+}-stimulated mitochondrial metabolism. We combined the calcium model with a model of mitochondrial metabolic pathways proposed by Wacquier et al. [22]:

$$
\begin{aligned}
\frac{d}{dt}ADP_c &= I_{\mathrm{hyd}} - \frac{I_{\mathrm{ant}}}{R_{\mathrm{V2}}} \\
\frac{d}{dt}ADP_m &= I_{\mathrm{ant}} - I_{\mathrm{F1FO}} \\
\frac{d}{dt}N &= I_{\mathrm{pdh}} - I_{\mathrm{o}} + I_{\mathrm{agc}} \\
\frac{d}{dt}V_m &= \frac{1}{C_p}(a_1 I_{\mathrm{o}} - a_2 I_{\mathrm{F1FO}} - I_{\mathrm{ant}} - I_{\mathrm{Hleak}} \\
&\qquad - (1 - R_{\mathrm{S2}})(J_{\mathrm{NCX}} + 2J_{\mathrm{MCU}}) - R_{\mathrm{S2}}(J_{\mathrm{nNCX}} + 2J_{\mathrm{nMCU}}) - I_{\mathrm{agc}})
\end{aligned}
\tag{3}
$$

The variables ADP_c and ADP_m measure ADP concentrations in the cytosol and $\qquad$ 193

mitochondrion, while N is the concentration of mitochondrial NADH. V_m models the $\qquad$ 194

voltage difference across the inner mitochondrial membrane. The I_* rates are: $\qquad$ 195

- I_{hyd}: rate of ATP hydrolysis $\qquad$ 196

- I_{ant}: rate of the ADP/ATP translocator $\qquad$ 197

- I_{F1FO}: rate of ADP phosphorylation 198

- I_{pdh}: the production rate of NADH by the pyruvate dehydrogenase 199

- I_{o}: rate of NADH oxidation 200

- I_{agc}: the production rate of NADH from the MAS 201

- I_{Hleak}: the ohmic mitochondrial proton leak 202

Along with the conservation of the total intracellular Ca^{2+} concentration, C_{t}, the model 203
suggests the conservation of the following ion concentrations: total NADH (oxidized and 204
reduced), mitochondrial di- and trisphosphorylated adenine nucleotides, and cytosolic 205
di- and trisphosphorylated adenine nucleotides. Mathematically speaking, 206

$$N_{mito}^{\text{tot}} = N + NAD,$$

$$A_{mito}^{\text{tot}} = ADP_m + ATP_m,$$

$$A_{cyt}^{\text{tot}} = ADP_c + ATP_c.$$

Other functions of the mitochondrial model are reproduced below for convenience. The 207
model parameters are in Table . For modeling purposes, some of the parameters are 208
modified from their original values as in Wacquier et al. [22]. We find these 209
modifications justifiable, as the original values were chosen by the authors to reproduce 210
their experimental data, and hence were not based on any physiological evidence. 211

$$I_{\text{hyd}} = \frac{J_{\text{SERCA}} + J_{\text{nSERCA}}}{2} + V_{\text{hyd}} \frac{ATP_c}{ATP_c + K_{\text{hyd}}}$$

$$I_{\text{ant}} = V_{\text{ant}} \frac{1 - \frac{\alpha_c ATP_c ADP_m}{\alpha_m ADP_c ATP_m} e^{\frac{FV_m}{RT}}}{\left(1 + \alpha_c \frac{ATP_c}{ADP_m} e^{-0.5 \frac{FV_m}{RT}}\right) \left(1 + \frac{ADP_m}{\alpha_m ATP_m}\right)}$$

$$I_{\text{F1FO}} = V_{\text{F1FO}} \left(\frac{q_6}{q_6 + ATP_m}\right) \left(1 + e^{\frac{q_7 - V_m}{q_8}}\right)^{-1}$$

$$I_{\text{pdh}} = k_{\text{gly}} \frac{1}{q_1 + \frac{N}{NAD}} \frac{C_{mito}}{q_2 + C_{mito}}$$

$$I_{\text{o}} = k_{\text{o}} \frac{N}{q_3 + N} \left(1 + e^{\frac{V_m - q_4}{q_5}}\right)^{-1}$$

$$I_{\text{agc}} = V_{\text{agc}} \frac{C_{cyt}}{K_{\text{agc}} + C_{cyt}} \frac{q_2}{q_2 + C_{mito}} e^{p_4 V_m}$$

$$I_{\text{Hleak}} = q_9 V_m + q_{10}$$

Table 1. Parameter values of the model. The parameters with * are modified from their suggested values as in Wacquier et al. [22].

Parameter	Value (Unit)	Description
Parameters for the Ca^{2+} and IP$_3$ concentration equations		
R_{V1}	2000	the volume ratio between the cytosol and MAM
R_{V2}	10	the volume ratio between the cytosol and the ER
R_{V3}	15	the volume ratio between the cytosol and mitochondria
R_{S1}	0.2	the proportion of the ER membrane surface that adjoins the MAM
R_{S2}	0.1	the proportion of mitochondrial membrane surface that adjoins the MAM
C_t	50 (μM)	the total intracellular Ca^{2+} concentration
D_c	0.1	intracellular Ca^{2+} diffusion rate
D_p	1	intracellular IP$_3$ diffusion rate
k_{leak}	0.001 (s^{-1})	leak Ca^{2+} flux coefficient
k_{IPR}	0.3 (s^{-1})	coefficient of the IPR flux entering the bulk cytosol
k_{nIPR}	0.3 (s^{-1})	coefficient of the IPR flux entering the MAM

V_{SERCA}	30 (μM s^{-1})	coefficient of the SERCA flux from the bulk cytosol
V_{nSERCA}	20 (μM s^{-1})	coefficient of the SERCA flux from the MAM
K_{SERCA}	0.35 (μM)	half-maximal activating cytosolic Ca^{2+} concentration of SERCA
K_{e}	0.05 (μM)	dissociation constant of ATP from SERCA
V_{MCU}	0.00005 (μM s^{-1})	coefficient of the MCU flux from the bulk cytosol
V_{nMCU}	0.00005 (μM s^{-1})	coefficient of the MCU flux from the MAM
V_{NCX}	0.5 (μM s^{-1})	coefficient of the NCX flux entering the bulk cytosol
V_{nNCX}	0.5 (μM s^{-1})	coefficient of the NCX flux entering the MAM
K_1	19 (μM)	dissociation constant for Ca^{2+} translocation by the MCU
K_2	0.38 (μM)	dissociation constant for MCU activation by Ca^{2+}
p_1	0.1 (mV^{-1})	coefficient of MCU activity dependence on voltage
p_2	0.016 (mV^{-1})	coefficient of NCX activity dependence on voltage
Parameters for the IPR model equations		
q_{26}	10500 (s^{-1})	transition rate from C_2 to O_6
q_{62}	4010 (s^{-1})	transition rate from O_6 to C_2
L_{IPR}	0.2 (s^{-1})	the rate at which h_{42} and $h_{\text{n}42}$ reach their equilibria, if the channel is in C_4
H_{IPR}	10 (s^{-1})	the rate at which h_{42} and $h_{\text{n}42}$ reach their equilibria, if the channel is in the drive mode
C_{p0}	700 (μM)	Ca^{2+} concentration at the pore of an open IPR
Parameters for the mitochondrial metabolic pathway equations		
N_{mito}^{tot}	250 (μM)	the total mitochondrial pyridine nucleotide concentration
A_{mito}^{tot}	15000 (μM)	the total mitochondrial adenine nucleotide concentration
A_{cyt}^{tot}	2500 (μM)	the total cytosolic adenine nucleotide concentration
L_{MCU}	50	allosteric equilibrium constant for uniporter conformations

$a_1{}^*$	10	scaling factor between NADH oxidation and change in mitochondrial membrane voltage
a_2	3.43	scaling factor between ADP phosphorylation and change in mitochondrial membrane voltage
$V_{\mathrm{hyd}}{}^*$	150 (μM s^{-1})	maximum rate of ATP hydrolysis
K_{hyd}	1000 (μM)	half-maximal activating cytosolic ATP concentration of ATP hydrolysis
V_{ant}	5000 (μM s^{-1})	rate coefficient of the adenine nucleotide translocator
α_c	0.111	cytosolic ADP and ATP buffering coefficient
α_m	0.139	mitochondrial ADP and ATP buffering coefficient
F	96480 (C mol^{-1})	Faraday constant
R	8315 (mJ mol^{-1} K^{-1})	perfect gas constant
T	310.16 (K)	temperature
V_{F1FO}	35000 (μM s^{-1})	rate coefficient of the F$_1$F$_\mathrm{O}$-ATPase
k_{gly}	450 (μM s^{-1})	rate coefficient of glycolysis
k_{o}	600 (μM s^{-1})	rate coefficient of NADH oxidation by ETC
$V_{\mathrm{agc}}{}^*$	180 (μM s^{-1})	rate coefficient of NADH production
K_{agc}	0.14 (μM)	dissociation constant of cytosolic Ca^{2+} from AGC
p_4	0.01 (mV^{-1})	coefficient of AGC activity dependence on voltage
q_1	1	Michaelis-Menten-like constant for NAD$^+$ consumption by the TCA
q_2	0.1 (μM)	half-maximal activating mitochondrial Ca^{2+} concentration of the TCA
q_3	100 (mV)	Michaelis-Menten constant for NADH consumption by the ETC
q_4	177 (mV)	coefficient of ETC activity dependence on voltage
q_5	5 (mV)	coefficient of ETC activity dependence on voltage
q_6	10000 (μM)	inhibition constant of ATPase activity by ATP
q_7	190 (mV)	coefficient of ATPase activity dependence on voltage
q_8	8.5 (mV)	coefficient of ATPase activity dependence on voltage

q_9	2 (μM s^{-1} mV^{-1})	proton leak dependence on voltage
q_{10}	-30 (μM s^{-1})	rate coefficient of voltage-independent proton leak

Numerical simulations   212

All the numerical simulations presented in this paper were computed with   213
XPPAUT [27].   214

Results   215

Up-regulation of MAMs and mitochondrial Ca^{2+} activities   216

Due to the morphology of MAMs, where the ER and mitochondria are in close contact,   217
it is generally accepted that there is direct Ca^{2+} exchange between the organelles at   218
such sites. Based on this notion, it is reasonable to expect that an increase in the degree   219
of MAM formation up-regulates mitochondrial Ca^{2+} intake via MAMs, and   220
consequently induces larger amplitudes of mitochondrial Ca^{2+} signals. In fact, Arruda et   221
al. [9] used synthetic linkers to mechanically induced more MAMs in hepatocyte-derived   222
mouse Hepa1-6 cells, and observed higher ATP-stimulated mitochondrial Ca^{2+} peaks,   223
compared to the cells without the linkers. This experiment revealed a correlation   224
between mitochondrial Ca^{2+} activity and ER–mitochondrial interactions. Through the   225
following model simulations, we confirmed that the model behaves in this manner.   226

To determine whether the model can reproduce the experimental observations of   227
Arruda et al. [9], we performed model simulations using two different sets of values for   228
the R_S parameters, the portions of the ER and mitochondrial membranes that face each   229
other. The model with $(R_{S1}, R_{S2}) = (0.2, 0.1)$ was stimulated with a pulse of IP$_3$,   230
generated from Eq. 2 with $M = 0.5$, $t_0 = 4$, and $\triangle = 0.5$. Then the same stimulation   231
was applied to the model with $(R_{S1}, R_{S2}) = (0.3, 0.15)$, which mimics the up-regulation   232
of MAMs induced by the synthetic linkers. The cytosolic Ca^{2+} transient was not   233
affected by the parameter increases, however, the mitochondrial Ca^{2+} transient   234
exhibited a larger amplitude (Fig. 2A and Fig. 2C). These results are compatible with   235
the experimental observation. Given that the MCU has a low Ca^{2+} affinity, i.e., it   236

requires high concentrations of Ca^{2+} for channel activation, increasing the $\qquad$ 237

mitochondrial surface portion that faces MAMs, and thus exposing more MCUs to $\qquad$ 238

higher concentrations of Ca^{2+}, would lead to augmented MCU Ca^{2+} flux, and $\qquad$ 239

consequently, induce larger amplitudes of mitochondrial Ca^{2+} activities. $\qquad$ 240

We also studied model behavior under continuous stimulation. Instead of producing $\qquad$ 241

IP_3 in a pulsatile manner, we modeled the concentration of IP_3 to gradually increase $\qquad$ 242

until it reaches a steady state, P_s. The model exhibited stable oscillations in cytosolic $\qquad$ 243

and mitochondrial Ca^{2+} concentrations (Fig. 2B and Fig. 2D). The increases of the R_S $\qquad$ 244

ratios were followed by an increased amplitude in mitochondrial oscillations, while the $\qquad$ 245

amplitude change in the cytosolic Ca^{2+} oscillations was negligible. $\qquad$ 246

Fig 2. Effects of increased MAMs on amplitudes of cytosolic (top) and mitochondrial (bottom) Ca^{2+} activities.
(A and C) We stimulated the model with a pulse of IP_3, Eq. 2 with $M = 0.5$, $t_0 = 4$, and $\Delta = 0.5$, shown by the inset graph in (A). The blue solid spikes were simulated with $(R_{S1}, R_{S2}) = (0.2, 0.1)$, while the red dashed spikes were generated with the increased MAM surface ratios, $(R_{S1}, R_{S2}) = (0.3, 0.15)$. The green dashed lines indicate the onset of the pulse. (B and D) The model was given continuous stimulation from IP_3, which had a saturating concentration at $P_s = 0.3$, shown by the inset graph in (B). The blue oscillations were generated with the control parameters. Again, when we generated the red oscillations, R_{S1} and R_{S2} were increased by 50%.

Ca^{2+} activities in MAMs $\qquad$ 247

Experimental observations suggest that Ca^{2+} concentration in MAMs is about 10-fold $\qquad$ 248

higher than that of the cytosol, and about 10-fold lower than that in the ER $\qquad$ 249

lumen [28, 29]. We examined whether the model reproduces these phenomena. The $\qquad$ 250

model was continuously stimulated with a constant IP_3 concentration at its steady state $\qquad$ 251

$P_s = 0.3$ μM to generate stable oscillations in all three compartments (Fig. 3A). The $\qquad$ 252

model simulated the expected order differences in the Ca^{2+} concentrations between the $\qquad$ 253

domains. We note that this model behavior is solely induced from the model $\qquad$ 254

assumption of a significantly large volume difference between the cytosol and MAMs. $\qquad$ 255

Since the portion of the ER membrane that faces the cytosol is larger than the other $\qquad$ 256

section juxtaposing MAMs, it is not surprising to see the larger cytosolic IPR Ca^{2+} flux $\qquad$ 257

(Fig. 3B and Fig. 3C). Nonetheless, the model simulated MAM Ca^{2+} oscillations with $\qquad$ 258

higher peaks, despite the relatively smaller MAM IPR Ca^{2+} fluxes. $\qquad$ 259

Fig 3. Ca^{2+} oscillations generated from the model exhibit varying orders of magnitude in different compartments.
(A) The model was given continuous stimulation of IP$_3$ with $P_s = 0.3$ μM. From the top, the panels show Ca^{2+} oscillations in the ER, the MAM, and the bulk cytosol.
(B and C) The magnitudes of IPR Ca^{2+} fluxes from the ER to the bulk cytosol and the MAM, respectively, during the oscillations shown in (A).

Comparing Ca^{2+} oscillations: Control vs. obesity $\quad$ 260

Arruda et al. [9] reported the effects of obesity on the morphology of hepatic ER and $\quad$ 261
mitochondria. Their study involved lean mice and two different groups of obese mice, $\quad$ 262
one that had been on high-fat diet (HFD) for 16 weeks, and the other, genetically obese $\quad$ 263
(*ob/ob*) mice. One of their main findings was that both groups of obese mice had a $\quad$ 264
greater proportion of MAMs in liver cells. The authors also examined the expression $\quad$ 265
levels of ER and mitochondrial proteins in liver lysates from the mice. Their western $\quad$ 266
blot analysis showed that the obese mice had a higher expression of the IPR and $\quad$ 267
PACS-2, an ER-mitochondrial tether protein. Interestingly, the expression level of MCU $\quad$ 268
was higher in the *ob/ob* mice, while the difference between the lean and the HFD mice $\quad$ 269
was negligible. This suggests that the change in the expression of MCU may not occur $\quad$ 270
at the early stages of obesity, although it is likely to be associated with obesity in the $\quad$ 271
long-term. $\quad$ 272

Another part of their study traced intracellular Ca^{2+} activities, both in the cytosol $\quad$ 273
and mitochondrial lumen. Liver cells from lean and obese mice were stimulated with $\quad$ 274
ATP to induce subsequent Ca^{2+} releases from the ER. The results showed higher peaks $\quad$ 275
of mitochondrial Ca^{2+} concentration in cells from obese mice, and similar increases in $\quad$ 276
cytosolic Ca^{2+} concentrations. From this, the authors speculated that the higher $\quad$ 277
mitochondrial Ca^{2+} peaks observed under obesity are related to having more $\quad$ 278
ER-mitochondrial interactions in the cells, and consequently, increased direct Ca^{2+} $\quad$ 279
transport through MAMs. $\quad$ 280

We do not know the exact population of Ca^{2+} channels in mouse hepatocytes. $\quad$ 281
However, the experimental data provide some guidelines for the relative expression $\quad$ 282
levels between the control (lean) group and the obese group. The parameters shown in $\quad$ 283
Table are control values, and when we simulated the model with this parameter set, $\quad$ 284
which will be referred to as the *control model* from here on, model outcomes were $\quad$ 285
regarded as the baseline behaviors for later comparison. Based on the experimental $\quad$ 286

results discussed above, we modified some of the parameters to reflect cellular changes $_{287}$

that are associated with obesity. Table 2 shows the parameter modifications. The model $_{288}$

with the modified parameters will be referred to as the *obesity model*. We compared $_{289}$

behaviors of the obesity model to those of the control model to confirm that the models $_{290}$

correctly capture the gap between Ca^{2+} activities observed in hepatocytes from lean $_{291}$

and obese mice. $_{292}$

Table 2. Modified parameters for the obesity model simulations.

Parameter	control model	obesity model
R_{S1}	0.2	0.3
R_{S2}	0.1	0.15
k_{IPR}	0.3 s^{-1}	0.36 s^{-1}
k_{nIPR}	0.3 s^{-1}	0.36 s^{-1}
V_{MCU}	0.00005 μM s^{-1}	0.0001 μM s^{-1}
V_{nMCU}	0.00005 μM s^{-1}	0.0001 μM s^{-1}

The panels in Fig. 4 show Ca^{2+} oscillations generated from the control and obesity $_{293}$

models with the same magnitude of stimulation. The simulations suggest that $_{294}$

hepatocytes from obese mice may exhibit faster Ca^{2+} oscillations with a higher $_{295}$

amplitude of mitochondrial Ca^{2+}, compared to cells from lean mice. In fact, the $_{296}$

amplitude in the cytosol showed a relatively small change ($\approx 5\%$), compared to the $_{297}$

mitochondrial Ca^{2+} amplitude, which was increased by 126%. From the experimental $_{298}$

data, we cannot distinguish which cellular changes are responsible for which effects, and $_{299}$

to what degree. Using our Ca^{2+} model, we investigated how Ca^{2+} oscillations are $_{300}$

modulated by each cellular change associated with obesity. Model results are discussed $_{301}$

in the following subsections. $_{302}$

Fig 4. Effects of the cellular changes associated with obesity on Ca^{2+} oscillations.
Oscillations of (A) cytosolic and (B) mitochondrial Ca^{2+} concentrations. The blue oscillations were generated from the model with the control parameters. When simulating the red oscillations, some of the parameters were modified as in Table 2. Throughout the simulation, IP$_3$ concentration was held constant at 0.3 μM.

Increased MAM formation   303

We have previously discussed that a possible mechanism underlying the amplitude   304
change in mitochondrial Ca^{2+} activities associated with the enhancement of MAM   305
formation is a synergistic process that comes from the MCU's low Ca^{2+} affinity and the   306
small volume of MAMs. The increase in the amplitude of mitochondrial Ca^{2+} 307
oscillations induced from the increased degree of MAM formation is already evident in   308
Fig. 2. Interestingly, expressing more MAMs in the model slightly increased the   309
oscillation frequency. Table 3 shows the quantified effects of 20%, 50% and 100%   310
increase in MAM formation on the amplitudes and frequency of cytosolic and   311
mitochondrial Ca^{2+} oscillations. The parameter adjustments induced a trend of   312
negative effects on the amplitude of cytosolic Ca^{2+} oscillations, while having positive   313
effects on that of mitochondrial oscillations. As stated in the introduction, cells tightly   314
regulate frequencies and amplitudes of Ca^{2+} oscillations to orchestrate many cellular   315
activities. Thus, although the magnitude of the frequency change is small, its effect on   316
cellular homeostasis may be significant.   317

Table 3. Effects of increasing the degree of MAM formation, IPR activity, and MCU activity on the amplitudes of cytosolic and mitochondrial Ca^{2+} oscillations and the oscillation frequency.

$\triangle$ in MAMs	$\triangle$ in C_{cyt} amplitude	$\triangle$ in C_{mito} amplitude	$\triangle$ in oscillation freq.
20%	−0.56%	10%	0.5%
50%	−1.3%	26%	0.8%
100%	−2.4%	55%	0.8%
$\triangle$ in k_{IPR}, k_{nIPR}			
20%	17%	42%	30%
50%	39%	99%	61%
100%	68%	181%	84%
$\triangle$ in V_{MCU}, V_{nMCU}			
20%	−1.4%	14%	−0.7%
50%	−3.2%	33%	−1.6%
100%	−6%	61%	−2.9%
200%	−11%	104%	−4.4%

Increased IPR activity   318

Secondly, we looked at the effects of increased IPR activity on Ca^{2+} oscillations, by   319

increasing k_{IPR} and k_{nIPR}, the rate constants for the IPR Ca^{2+} fluxes. These   320

adjustments lead to an increase in the magnitude of total Ca^{2+} flux through the IPR,   321

either from an increased number of channels or an increase in the maximum strength of   322

each channel. Since the model is a deterministic model that incorporates all-or-none   323

activation of IPRs, we intuitively expected the increased IPR parameters to generate   324

Ca^{2+} oscillations with larger amplitudes in both the cytosol and mitochondria. The   325

model was stimulated with $P_s = 0.3$ μM to produce the stable oscillations in Fig. 5. At   326

$t = 25$ s, only k_{IPR} and k_{nIPR} were increased by 20%, while other parameters were kept   327

the same. The parameter adjustments induced increases in the amplitudes of cytosolic   328

and mitochondrial Ca^{2+} oscillations by 17% and 42%, respectively, as expected. The   329

model simulations also showed a 30% increase in the oscillation frequency. With the   330

increased IPR channel activity, it takes less time for cytosolic Ca^{2+} concentration to   331

reach the threshold concentration for spiking. However, this also means that the   332

channels have a shorter refractory time, the period of time that it takes for the IPR to   333

recover its state between spikes, and hence, at the time of spiking, the receptors would   334

have a lower open probability. Fig. 5C and Fig. 5D confirm this conjecture.   335

**Fig 5. Effects of increased IPR activity on Ca^{2+} oscillations and IPR open
probabilities.**
(A and B) Oscillations in cytosolic and mitochondrial Ca^{2+} concentrations, respectively.
The blue oscillations are generated from the control model, while the red oscillations are
generated with increased k_{IPR} and k_{nIPR}. The model was given continuous stimulation
of IP_3 with $P_s = 0.3$ μM. (C and D) Corresponding open probabilities of the bulk
cytosol IPR and the MAM IPR, respectively.

We further increased k_{IPR} and k_{nIPR} by 50%, and then by 100%, to affirm the   336

positive correlation between the parameters and the oscillation amplitudes and   337

frequency (Table 3). It is evident that the corresponding parameter modifications had   338

strong positive effects on all three properties of oscillations that were of interest.   339

Increased MCU activity   340

Lastly, we simulated the effects of an increase in MCU activity level on cytosolic and   341

mitochondrial Ca^{2+} oscillations. Following the same steps as the previous simulations,   342

Fig 6. Effects of increased MCU activity on Ca²⁺ oscillations.
Oscillations of (A) cytosolic and (B) mitochondrial Ca^{2+} concentrations generated from the control model, in blue, and with increased V_{MCU} and V_{nMCU}, in red. The model was given continuous stimulation of IP_3 with $P_s = 0.3$ μM.

the model was given continuous stimulation with $P_s = 0.3$ μM, which gave rise to the   343

stable oscillations shown in the panels of Fig. 6. At $t = 25$ s, V_{MCU} and V_{nMCU} were   344

increased by 100%, and the model continued to exhibit oscillations shown in the figures.   345

The increased parameters had a negative effect on the amplitude of cytosolic Ca^{2+} 346

oscillations and the oscillation frequency, whereas the amplitude of mitochondrial Ca^{2+} 347

oscillations was increased. The relative magnitudes of these changes are quantified in   348

Table 3. The most obvious explanation for the amplitude changes is that the enhanced   349

activity of MCUs transports a larger amount of Ca^{2+} from the cytosol to mitochondria,   350

and the magnitude of change is more substantial in mitochondria than in the cytosol   351

due to the compartments' volume difference. We also examined the changes under 20%,   352

50%, and 200% increase of the parameters, to verify the overall trend of the effects.   353

As mentioned above, no significant difference was reported between the expression   354

levels of MCU in liver cells from the lean and the HFD mice. However, cells from ob/ob 355

mice had a marked increase in MCU expression level. These experimental data suggest   356

that the increase in the MCU expression level may not be a direct response to the onset   357

of obesity. We have shown that both increases of MAM formation and IPR channel   358

density, the cellular changes that were observed in the HFD mice, and thus are early   359

events associated with obesity, induce higher peaks in mitochondrial Ca^{2+}.   360

Model prediction   361

We postulate that liver cells from different health conditions have varying thresholds for   362

the breaking point of their Ca^{2+} oscillations. In order to test this, we studied model   363

behaviors under three different sets of parameters: one is without any modification,   364

which represents the healthy case, another one is with increased R_S's, k_{IPR} and k_{nIPR},   365

which corresponds to the cell condition associated with HFD, and the last set is with   366

additional parameter increases of V_{MCU} and V_{nMCU}, which refers to cells from ob/ob 367

animals. For each parameter set, the model was given continuous stimulation with five   368

distinct regimes where P_s is incrementally adjusted, as shown in the panels of Fig. 7. It   369

is clearly shown that the model representing the *ob/ob* condition reached the breaking $\qquad$ 370

point for Ca^{2+} oscillations at a lower concentration of IP_3, compared to the HFD and $\qquad$ 371

healthy conditions. The control model exhibited Ca^{2+} oscillations that are robust to $\qquad$ 372

higher concentrations of IP_3. $\qquad$ 373

To verify these predictions, we suggest measuring Ca^{2+} responses in liver cells with $\qquad$ 374

each of the three conditions to a wide range of IP_3 concentration, and see if the average $\qquad$ 375

IP_3 oscillatory range was decreased in the *ob/ob* condition. However, one of the $\qquad$ 376

challenges of this experiment is that, due to cell-to-cell variability, it is critical to $\qquad$ 377

measure responses from a myriad of cells before drawing a conclusion. $\qquad$ 378

Fig 7. Robustness of Ca^{2+} oscillations under different model conditions.
We perturbed the model with gradually increasing stimulation, under three model
conditions: (A) control, (B) HFD, and (C) *ob/ob*. Initially, P_s was at 0.6 μM, then was
increased to 0.75 μM, 0.9 μM, 1 μM, and then to 1.2 μM at $t = 20$ s, 40 s, 60 s, and 80
s, respectively. The cytosolic Ca^{2+} concentrations are shown in blue, with the scale on
the left y-axis. The green timeseries represent the cytosolic IP_3 concentration, with the
scale on the right y-axis.

Discussion $\qquad$ 379

We have presented a mathematical model for Ca^{2+} dynamics in mouse hepatocytes. $\qquad$ 380

Based on experimental data that showed oscillatory Ca^{2+} signals in hepatocytes in $\qquad$ 381

Ca^{2+}-free media, the model is the closed-cell type, wherein the total intracellular Ca^{2+} $\qquad$ 382

concentration is assumed to be conserved (i.e., there is no Ca^{2+} flux across the plasma $\qquad$ 383

membrane). To our knowledge, this model is the first mathematical model to explicitly $\qquad$ 384

express Ca^{2+} concentration in MAMs as a dynamical variable and also show MAM $\qquad$ 385

Ca^{2+} levels to be within reasonable proportions of the Ca^{2+} levels in the other domains. $\qquad$ 386

The first aim of the model was to reproduced the data reported by Arruda et al. where $\qquad$ 387

hepatocytes with more MAMs exhibited ATP-induced Ca^{2+} transient with higher peaks $\qquad$ 388

in mitochondrial Ca^{2+} concentration [9]. We assumed that the IPRs, MCUs and NCXs $\qquad$ 389

are uniformly expressed on the residing membranes, while the SERCA pumps are $\qquad$ 390

predominant on the cytosolic side of the ER membrane. $\qquad$ 391

Arruda et al. [9] also compared hepatic cellular characteristics between different $\qquad$ 392

groups of mice. They had a group of lean mice as their control, and two groups of mice, $\qquad$ 393

one that had been under high fat diet (HFD) and the other genetically obese (*ob/ob*) $\qquad$ 394

mice, for mouse models of obesity. The *ob/ob* mouse cells showed higher expression levels of IPR and MCU, as well as a higher degree of MAM formation. The HFD mouse cells also had more MAMs and a higher IPR channel density than the lean group, but the groups' MCU expression levels did not differ. We interpreted the morphological change of MAMs and and the increase in IPR channel density as the traits of obesity in early stages, while considering the increase in MCU channel density as an effect that appears in later stages.

We used the model to study how Ca^{2+} signals are altered by each cellular change associated with obesity. According to model simulations, an increase of MAM proportion speeds up Ca^{2+} oscillations and increases mitochondrial Ca^{2+} amplitudes, while decreasing the amplitude of cytosolic Ca^{2+}. An increase in IPR channel activity level induces faster Ca^{2+} oscillations with higher amplitudes in both cytosol and mitochondria. Furthermore, an increase in the level of MCU activity increases mitochondrial Ca^{2+} amplitude and oscillation period. We found that the cellular changes observed in the HFD mouse cells have a positive effect on oscillation frequency, while the additional change that only appeared in the *ob/ob* has the opposite effect. Thus, we postulate that the increase of MCU channel density observed in the *ob/ob* mouse cells is a secondary effect of obesity that counteracts other cellular changes that occur in the HFD mouse cells.

Metabolic flexibility refers to the ability of the organism to adapt its fuel source, depending on availability and need [30], and emerging evidence suggests the involvement of MAMs in metabolic flexibility [31]. Interestingly, Rieusset et al. [32] reported a link between the disruption of ER-mitochondrial Ca^{2+} exchange and hepatic insulin resistance in their mouse model. As we have shown in this paper, hepatic cellular changes associated with HFD and obesity affect Ca^{2+} oscillation frequencies and amplitudes. However, the question of whether the altered Ca^{2+} dynamics plays a causal role in the development of hepatic insulin resistance and metabolic diseases remains to be explored. We eagerly anticipate that our model can be one of stepping stones addressing such conundrum.

Mitochondrial Ca^{2+} oscillations and ROS   424

There are many studies that associate mitochondrial Ca^{2+} dynamics with the   425
generation of ROS [8,33,34]. It has been suggested that mitochondrial Ca^{2+} 426
up-regulates ETC, which in turn increases the generation of mitochondrial superoxide,   427
one of the main forms of ROS. As a counter mechanism, Ca^{2+} also activates enzymes   428
that scavenge superoxide. The optimal feedback mechanisms between mitochondrial   429
Ca^{2+} and ROS would keep ROS at a tolerable level to avoid ROS-induced apoptosis.   430
Achieving this state requires a balance between the production and degradation of ROS,   431
and most likely in a time-delay manner. We have shown that the cellular changes   432
associated with HFD induce faster Ca^{2+} oscillations in mitochondria, which potentially   433
leads to a faster rate of ROS production. If the process of ROS degradation cannot keep   434
up with the increased level of ROS, it would cause the accumulation of ROS in   435
mitochondria and consequently, apoptosis. However, according to the model simulations,   436
oscillation frequencies can be decreased by an increase of MCU channel density, another   437
cellular change associated with obesity. Unfortunately, this reaction potentially leads to   438
mitochondrial Ca^{2+} overload, which is a deleterious phenomenon for cells. There are   439
many studies that discuss links between mitochondrial Ca^{2+} overload and oxidative   440
stress and ER stress, which are the hallmark of apoptosis [3,35–38]. Nonetheless, liver   441
cells from the *ob/ob* mice have an increased expression level of MCU, which aggravates   442
mitochondrial Ca^{2+} overload. The fact that the cells increase their MCU channels on   443
top of the other cellular changes is an interesting and physiologically counterintuitive   444
behavior. Why would they push themselves in an adverse direction and worsen their   445
condition? Based on our model simulations, we conjecture that the MCU expression   446
level change is a counter response to the other cellular changes. The increases in MAM   447
formation and IPR channel density both accelerated the oscillations, while the increase   448
in MCU channel density had an opposite effect on the oscillations. Thus, we suggest   449
that cells increase their MCU expression level in an attempt to adjust their oscillation   450
frequency to a sustainable level. Of course, this mechanism has to be finely controlled,   451
as there are other problems, such as mitochondrial Ca^{2+} overload, that come with it.   452

Other Ca^{2+} fluxes

As discussed before, we presented a closed-cell model, and did not include Ca^{2+} fluxes across the plasma membrane, such as the ones via store-operated Ca^{2+} channels (SOCCs), receptor-operated Ca^{2+} channels (ROCCs), and plasma membrane Ca^{2+}-ATPase (PMCA) pumps. SOCCs draw Ca^{2+} from the extracellular space when Ca^{2+} stores (ER/SR) get depleted. ROCCs, which also carry Ca^{2+} into the cytosol, are activated by agonists binding on G-protein-coupled receptors. PMCA pumps remove Ca^{2+} from the cytosol. The fluxes through these channels also modulate intracellular Ca^{2+} dynamic, and thus could be added in the model. In particular, experimental evidence suggests that mitochondrial Ca^{2+} regulate SOCCs [39–41]. When mitochondria sequester a substantial amount of Ca^{2+} from the cytosol and the ER, the level of Ca^{2+} concentration in the ER could get sufficiently low and activate SOCCs.

The model proposed by Wacquier et al. [22] includes a bidirectional Ca^{2+} flux between the cytosol and mitochondria. They suggested that this flux could represent the mitochondrial permeability transition pore (PTP) in the low conductance mode. It would be interesting to see how our model behaves in the presence of the bidirectional flux, and compare the models.

Model reduction

Due to the high dimensionality of the model, which is a system of 11 ODEs, numerical integration can be computationally expensive. In an attempt to simplify the model, we applied quasi-steady state reduction and assumed that the activation variables of the IPRs, h_{42} and h_{n42}, instantaneously reached their quasi-equilibria. However, this reduction destroyed the desired order differences between the magnitudes of Ca^{2+} transients in the cytosol, the ER, and MAMs. This implies that the kinetics of IPR activation is an important factor for modeling the 10-fold difference in the amplitudes of cytosolic and MAMs Ca^{2+} activities, shown in Fig. 3.

Supporting information

S1 .ode file. The model's .ode file that runs on XPPAUT [27].

Acknowledgments

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fig2

fig3

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fig4

fig1