# Estimating the time since admixture from phased and unphased molecular data 

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#### Abstract

After admixture, recombination breaks down genomic blocks of contiguous ancestry. The break down of these blocks forms a new 'molecular' clock, that ticks at a much faster rate than the mutation clock, enabling accurate dating of admixture events in the recent past. However, existing theory on the break down of these blocks, or the accumulation of delineations between blocks, so called 'junctions', has been limited to using regularly spaced markers on phased data. Here, we present an extension to the theory of junctions using the Ancestral Recombination Graph that describes the expected number of junctions for any distribution of markers along the genome. Furthermore, we provide a new framework to infer the time since admixture using unphased data. We demonstrate both the phased and unphased methods on simulated data and show that our new extensions perform much better than previous methods, especially for more ancient admixture times. Lastly, we demonstrate the applicability of our method on an empirical dataset of labcrosses of yeast (Saccharomyces cerevisae) and on two case studies of hybridization in swordtail fish and Populus trees.


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## Keywords

Admixture, hybridization, recombination, junctions, phasing

## 1 Introduction

 by Fisher (1949, 1954), and inheritance of these junctions is similar to that of point-mutations. Further work on the theory of junctions has shown how they accumulate over time for sib-sib mating (Fisher, 1954), self-fertilization (Bennett, 1953), alternate parent-offspring mating (Fisher, 1959; Gale, cichlid fishes (Koblmüller et al., 2007, Keller et al. 2013), warblers (Brelsford et al., 2011), fruit flies (Schwarz et al., 2005) and sculpins (Nolte et al., 2005).Understanding the timeline of these hybridization events is paramount in obtaining a full understanding of the process and its impact. Often, hybridization processes occur fast, on a timescale that is too rapid to accumulate enough mutations, which prevents the use of traditional molecular clocks to infer the onset of hybridization. Instead, recombination processes are sufficiently rapid so as to be used to study the recent evolutionary dynamics of a population. For example, they have been used to infer selective sweeps (Sabeti et al., 2007) or recent demography (Ralph and Coop, 2013, Ringbauer et al., 2017) in human populations. Recombination also leaves a footprint in genomes undergoing hybridization. After admixture of two lineages, contiguous genomic blocks are broken

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1964), a randomly mating population (Stam, 1980, Baird, 1995), and for sub-structured populations (Chapman and Thompson, 2002, 2003).

So far, applying the theory of junctions has shown to be difficult, as it requires extensive genotyping of the admixed lineage, but also of the parental lineages. With the current decrease in genotyping costs (Muir et al., 2016), such analyses are coming within reach, and frameworks are being developed that assist in inferring local ancestry and detecting junctions, given molecular data of parental and admixed lineages (Paşaniuc et al., 2009, Maples et al., 2013, Guan, 2014 Corbett-Detig and Nielsen, 2017). Nevertheless, molecular data always paints an imperfect image of ancestry along the genome, and inferring the number of junctions in a chromosome remains limited by the number of diagnostic markers available (see Fig 1. first panel). Previous work on the theory of junctions does not take into account the effect of a limited number of genetic markers, and so far this effect had to be corrected using simulations (MacLeod et al., 2005, Buerkle and Rieseberg, 2008). Recent work by Janzen et al. (2018) resolves this issue by extending the theory of junctions with the effect of using a limited number of markers, but they had to assume an evenly spacing of markers. However, molecular markers are rarely evenly spaced. The first result we present here is an extension of the theory of junctions which includes the effect of marker spacing on inferring the number of junctions in a genome.

Furthermore, existing theory on the accumulation of junctions is only developed for the case where ancestry can be determined within a single chromosome. For diploid species, sequencing data presents itself as the pileup of ancestry across both chromosomes, requiring an additional step to separate the contributions of both chromosomes, called 'phasing' (see Fig 1, second and third panel). Phasing methods can be classified into three main categories. Firstly, direct methods are based on haplotype-resolved genome sequencing (reviewed in Snyder et al. (2015)). These methods yield accurate results, but are expensive and require large amounts of DNA. Recently Lutgen et al. (2020) have shown that linked-read sequencing is efficient enough to provide haplotype resolved sequencing at a population scale at reasonable cost. However, linked-read sequencing is still a fairly new technology, and not yet widespread. Secondly, phasing can be performed using methods based on the analysis of genotypes of closely related individuals. These methods often

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yield good results but their application has been limited to humans, where large pedigree datasets are available (Browning and Browning, 2011, Loh et al., 2016a; Kong et al., 2008). Lastly, phasing can be performed using statistical methods, based on estimating the recombination rates and allele frequencies in a population. While some algorithms make use of a reference genome (for example Eagle (Loh et al., 2016b), Beagle (Browning and Browning, 2007) or ShapeIt (O’Connell et al., 2016) ), others allow de novo number of individuals in the sample is small, accuracy is low and only local haplotypes can reliably be inferred (Browning and Browning, 2011; Choi et al., 2018). More recently, statistical methods developed for third generation sequencing data (sometimes combined with Hi-C), do allow to infer long-range haplotypes with good accuracy (Tourdot and Zhang, 2019 Kronenberg et al., 2019; Ebler et al., 2019; Tangherloni et al., 2019). However, data from hybrid populations are not often available in this form. Across these three groups of methods, phasing is often costly and accuracy can be left wanting. Yet, inclusion of information from both chromosomes is expected to improve inference of the onset of admixture considerably and hence expansion of the theory of junctions towards a framework that takes into account data from both chromosomes is warranted.

Here we provide a full framework to estimate the time since admixture using phased or unphased data from two homologous chromosomes, taking into account marker spacing along the chromosome.

Our framework is based on modelling the joint genealogy of loci that are located in the same chromosome or in two homologous chromosomes, using the Ancestral Recombination Graph (ARG) (Hudson, 1983; Griffiths, 1991; Griffiths and Marjoram, 1997). It has the advantage of being fast since it relies on mathematical computations and does not require simulations. It has been implemented in the R package 'junctions'.

Our paper is organised as follows. In section 2.1, we introduce our model, which is a simplified version of the ARG. In 2.2, we present three maximum-likelihood methods to infer the time since admixture in hybrid populations: the first one uses information from a single chromosome and the others use phased or unphased data from two homologous chromosomes. In section 2.3, we validate our methods using simulations. In section 3 we apply them to a dataset from yeast experimental evolution and to two case studies of hybridization in swordtail fish and Populus tree.

## 2 Materials and methods

### 2.1 Mathematical model

We assume a diploid population that evolves according to a Wright-Fisher dynamics, i.e. generations are non-overlapping, mating is random and all individuals are hermaphrodites. We only keep track of one chromosome (or one pair of chromosomes), assuming that the accumulation of junctions on different pairs of chromosomes is independent on each other. We assume that hybridization occurred at time 0 between two populations, $\mathcal{P}$ and $\mathcal{Q}$. The proportion of individuals from population $\mathcal{P}$ at time 0 is $p$ and the proportion of individuals of type $\mathcal{Q}$ is $q=1-p$.

We assume that the length of the chromosome is $C$ Morgan and that there are $n$ molecular markers whose positions are given by $\left(z_{1}, \ldots, z_{n}\right) \in[0, C]$. For two consecutive markers at sites $z_{i}$ and $z_{i+1}$, we define $d_{i}=z_{i+1}-z_{i}$, the distance between them in Morgan. The genealogy of these $n$ (or $2 n$ ) loci is given by the Ancestral Recombination Graph (ARG), defined in Hudson (1983); Griffiths (1991); Griffiths and Marjoram (1997). This process is a branching-coalescence process in which loci that belong to the same block at time $t$ are those which were carried by the same ancestor $t$ units of time ago. Although the ARG for many loci has complicated transition rates and is a computationally intensive model, here we consider only two loci (or two pairs of loci) at a time.

We assume that $N \gg 1$ so that we can neglect some transitions (double coalescences and simultaneous coalescence and recombination), $d_{i} \ll 1$ so that there is no more than one crossover per generation between two molecular markers and the mutation rates are small enough so that we can neglect mutations that happened between the admixture time and the present.

### 2.1.1 Two sites, one chromosome

The aim of this section is to derive a formula for the expected number of observed junctions on one chromosome given $N$, the distances between the markers $\left(d_{1}, \ldots, d_{n}\right)$ and the initial heterozygosity $H_{0}:=2 p q$. We start by considering two consecutive loci $z_{i}$ and $z_{i+1}$. The ARG for these two sites has two possible states $\left(z_{i} \sim z_{i+1}\right)$ (where both loci are carried by the same lineage) and state $\left(z_{i} \nsim z_{i+1}\right)$ (where each locus is carried by a different lineage). The dynamics of this process are controlled by two types of events:

- Recombination $\left(z_{i} \sim z_{i+1}\right) \rightarrow\left(z_{i} \nsim z_{i+1}\right)$ with probability $d_{i}$,
- Coalescence $\left(z_{i} \nsim z_{i+1}\right) \rightarrow\left(z_{i} \sim z_{i+1}\right)$ with probability $\frac{1}{2 N}$.

Other events (such as simultaneous coalescence and recombination events) have probabilities that are negligible when $N$ is large. This yields the following transition matrix:

$$
\bar{M}=\left(\begin{array}{cc}
1-d_{i} & d_{i} \\
\frac{1}{2 N} & 1-\frac{1}{2 N}
\end{array}\right) .
$$

Let $\bar{P}_{t}$ be the probability vector at time $t$ for this Markov chain with two states. $\left(\bar{P}_{t}\right)_{1}$ is the 110 probability of $\left(z_{i} \sim z_{i+1}\right)$ at time $t$ and $\left(\bar{P}_{t}\right)_{2}$ the probability of $\left(z_{i} \nsim z_{i+1}\right)$ at time $t$. We have ${ }_{111}$ $\bar{P}_{0}=(1,0)$ (in the present we sample the two loci in the same individual) and $\bar{P}_{t}=\bar{P}_{0} \bar{M}^{t}$. We denote $\quad{ }_{112}$ by $\mathbb{P}\left(J_{t}\left(z_{i}, z_{i+1}\right)\right)$ the probability that a junction is observed between $z_{i}$ and $z_{i+1}$, if the hybridization ${ }_{113}$ event happened $t$ generations ago. We have

$$
\begin{equation*}
\mathbb{P}\left(J_{t}\left(z_{i}, z_{i+1}\right)\right)=H_{0}\left(\bar{P}_{t}\right)_{2}, \tag{1}
\end{equation*}
$$

which corresponds to the probability that the two loci were carried by different lineages $t$ generations 115 ago and the two lineages correspond to individuals from different ancestral subpopulations (see Fig ${ }_{116}$ 2. left panel).

Solving equation (1) gives

$$
\mathbb{P}\left(J_{t}\left(z_{i}, z_{i+1}\right)\right)=H_{0} \frac{2 N}{2 N+1 / d_{i}}\left(1-\left(1-d_{i}-\frac{1}{2 N}\right)^{t}\right) .
$$

Let $\mathbb{E}\left(J_{t}\right)$ be the expected number of observed junctions, we have

$$
\begin{equation*}
\mathbb{E}\left(J_{t}\right)=\sum_{i=1}^{n-1} \mathbb{P}\left(J_{t}\left(z_{i}, z_{i+1}\right)\right)=\sum_{i=1}^{n-1} \frac{H_{0} 2 N d_{i}}{2 N d_{i}+1}\left(1-\left(1-d_{i}-\frac{1}{2 N}\right)^{t}\right) . \tag{2}
\end{equation*}
$$

| State |  | $n_{i}$ | $n_{i+1}$ | $n_{\text {tot }}$ |
| :---: | :---: | :---: | :---: | :---: |
| $S^{1}$ | $\left(z_{i} z_{i+1}\right),\left(z_{i} z_{i+1}\right)$ | 2 | 2 | 2 |
| $S^{2}$ | $\left(z_{i} z_{i+1}\right)\left(z_{i}\right)\left(z_{i+1}\right)$ | 2 | 2 | 3 |
| $S^{3}$ | $\left(z_{i}\right)\left(z_{i}\right)\left(z_{i+1}\right)\left(z_{i+1}\right)$ | 2 | 2 | 4 |
| $S^{4}$ | $\left(z_{i} z_{i+1}\right)\left(z_{i}\right)$ or $\left(z_{i} z_{i+1}\right)\left(z_{i+1}\right)$ | 2 (or 1) | 1 (or 2) | 2 |
| $S^{5}$ | $\left(z_{i}\right)\left(z_{i+1}\right)\left(z_{i+1}\right)$ or $\left(z_{i+1}\right)\left(z_{i}\right)\left(z_{i}\right)$ | 1 (or 2) | 2 (or 1) | 3 |
| $S^{6}$ | $\left(z_{i} z_{i+1}\right)$ | 1 | 1 | 1 |
| $S^{7}$ | $\left(z_{i}\right),\left(z_{i+1}\right)$ | 1 | 1 | 2 |

Table 1. States of the reduced ARG. $n_{i}$ (resp. $n_{i+1}$ ) denotes the number of ancestors of site $z_{i}$ (resp. $z_{i+1}$ ) and $n_{\text {tot }}$ the total number of ancestors to the sample.

### 2.1.2 Two sites, two chromosomes

We consider two consecutive loci $z_{i}$ and $z_{i+1}$, which are at distance $d_{i}$ (in Morgan), that we sample ${ }_{119}$ in two homologous chromosomes. The ARG for these 2 sites in 2 chromosomes has 7 states (see ${ }_{120}$ Durrett (2008), Chapter 3). To describe them, we borrow the notation from Durrett (2008) and we ${ }^{121}$ write $\left(z_{i} z_{i+1}\right)$ to indicate an ancestor that is ancestor to site $z_{i}$ and $z_{i+1}$, and notation $\left(z_{i}\right)$ or $\left(z_{i+1}\right){ }_{122}$ for an ancestor that is only ancestor to one of the two sites. The resulting 7 states are summarized ${ }^{123}$ in Fig 1. An example of realization of this process is shown in Fig 2 (right panel).

The initial state is $S^{1}$ because in the present time we sample two different loci in two different chromosomes. The transition matrix of the ARG with 2 loci and a sample size 2 can be approximated, when $N \gg 1$ by

$$
M^{(i)}=\left(\begin{array}{ccccccc}
1-\frac{1}{2 N}-2 d_{i} & 2 d_{i} & 0 & 0 & 0 & \frac{1}{2 N} & 0 \\
\frac{1}{2 N} & 1-3 \frac{1}{2 N}-d_{i} & d_{i} & 2 \frac{1}{2 N} & 0 & 0 & 0 \\
0 & 2 \frac{1}{2 N} & 1-4 \frac{1}{2 N} & 0 & 2 \frac{1}{2 N} & 0 & 0 \\
0 & 0 & 0 & 1-\frac{1}{2 N}-d_{i} & d_{i} & \frac{1}{2 N} & 0 \\
0 & 0 & 0 & 2 \frac{1}{2 N} & 1-3 \frac{1}{2 N} & 0 & \frac{1}{2 N} \\
0 & 0 & 0 & 0 & 0 & 1-d_{i} & d_{i} \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2 N} & 1-\frac{1}{2 N}
\end{array}\right) .
$$

All other potential events (e.g. double crossovers or simultaneous crossover and coalescence events) ${ }^{125}$ have probabilities that are negligible compared to $1 / 2 N$.
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Let $P_{t}^{(i)}$ be the vector containing the probabilities of observing each of the states $\left(S^{1}, \ldots, S^{7}\right)$ at time $t . P_{t}^{(i)}$ satisfies

$$
P_{t}^{(i)}=P_{0}\left(M^{(i)}\right)^{t},
$$

where $P_{0}=(1,0,0,0,0,0,0)$, since at time 0 we sample all loci in two homologous chromosomes. ${ }^{127}$ This equation can only be solved numerically.

Recall that the stationary distribution of this process $P^{(i)}$ satisfies

$$
P^{(i)}=P^{(i)} M^{(i)}
$$

and has the analytical expression

$$
P^{(i)}=\left(0,0,0,0,0, \frac{1}{2 d_{i} N+1}, \frac{2 d_{i} N}{2 d_{i} N+1}\right) .
$$

Thus, for large values of $t$ the system reduces to states $S^{6}$ and $S^{7}$, which means that each locus has only one ancestor i.e. forwards in time the process has reached fixation (at each locus). Recall that ${ }_{130}$ state $S^{6}$ is the state where there is one ancestor for the sample thus we observe no junctions on ${ }^{131}$ either chromosome and that state $S^{7}$ is the state where there are two ancestors, one for the first $\quad{ }_{132}$ locus and one for the second locus, and with probability $2 p q$ each one of them comes from a different ancestral subpopulation. This is exactly the probability of observing a junction when $t \rightarrow \infty$ for one chromosome. In other words, when $t$ is very large, fixation is reached and the two sampled chromosomes are homozygous so the problem reduces to the single chromosome case.

### 2.2 Maximum likelihood estimation

### 2.2.1 One chromosome case

To infer the admixture time given an observed number of junctions $J_{o b s}$, we have to numerically solve equation (2). The solution of this equation is the maximum likelihood estimator of the time
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### 2.2.2 Two chromosomes, phased data

We first consider the case of phased data. Each pair of homologous markers can be in one of four states:

- $P P$ i.e. both homologous markers carry the allele from parent $\mathcal{P}$,
- $Q Q$ i.e. both homologous markers carry the allele from parent $\mathcal{Q}$,
- $P Q$ i.e. the marker on the first chromosome carries the allele from parent $\mathcal{P}$ and the marker on the second chromosome carries the allele from parent $\mathcal{Q}$,
- $Q P$ i.e. the marker on the first chromosome carries the allele from parent $\mathcal{Q}$ and the marker on the second chromosome carries the allele from parent $\mathcal{P}$.

The data can then be represented as a sequence $\left(O_{i}, 1 \leq i \leq n\right)$ that takes values in $\{P P, Q Q, P Q, Q P\}$ such that $O_{i}$ is the state of the i-th marker. To derive a maximum likelihood formula for the time since admixture $T$, we compute the probability of each sequence in $\{P P, Q Q, P Q, Q P\}^{n}$ given $T$, $N, C$, the distances between the $n$ loci and the initial heterozygosity $H_{0}$.

We want to compute the probability of our observations $\left(O_{1}, \ldots, O_{n}\right)$. These $n$ observations are not independent, as there are non-trivial correlations between loci along the chromosome. However, we can neglect long-range dependencies and assume that $O_{i}$ only depends on $O_{i-1}$, i.e. that the probability of observing $\left(O_{1}, \ldots, O_{n}\right), t$ units of time after hybridization is

$$
\mathbb{P}_{t}\left(\left(O_{1}, \ldots, O_{n}\right)\right)=\mathbb{P}_{t}\left(O_{1}, O_{2}\right) \prod_{i=2}^{n-1} \mathbb{P}_{t}\left(O_{i+1} \mid O_{i}\right)
$$

Recall that ignoring long-range dependencies is a natural approximation and it has been used for example by McVean and Cardin (2005) to define the sequentially Markov coalescent. To compute $\mathbb{P}_{t}\left(O_{i+1} \mid O_{i}\right)$, we use the ARG for markers at $z_{i}$ and $z_{i+1}$ denoted by $\left(\Gamma_{t}^{i}\right)$ (and to compute $\mathbb{P}\left(O_{1}, O_{2}\right), \quad 157$ we use $\left.\left(\Gamma_{t}^{1}\right)\right)$. For example, we can observe $O_{1}=P P$ and $O_{2}=Q Q$ if:

- $\Gamma_{t}^{1}=S^{3}$ and the two ancestors for locus 1 are from subpopulation $\mathcal{P}$ and the two ancestors for locus 2 from $\mathcal{Q}$, which happens with probability $p^{2} q^{2}$ or,
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- $\Gamma_{t}^{1}=S^{5}$, with probability $1 / 2$ there are two ancestors for locus 1 and one for locus 2 and they are of from desired subpopulations with probability $p^{2} q$. With probability $1 / 2$ there is one ancestor for locus 1 and two for locus 2 and they are from the desired subpopulations with probability $p q^{2}$ or,
- $\Gamma_{t}^{1}=S^{7}$ and the ancestor to 1 is from subpopulation $\mathcal{P}$ and the ancestor to 2 from $\mathcal{Q}$, which happens with probability $p q$.

To sum up, when $O_{1}=P P$ and $O_{2}=Q Q$,

$$
\mathbb{P}_{t}\left(O_{1}, O_{2}\right)=p^{2} q^{2}\left(P_{t}^{(1)}\right)_{3}+\frac{1}{2}\left(p q^{2}+q p^{2}\right)\left(P_{t}^{(1)}\right)_{5}+p q\left(P_{t}^{(1)}\right)_{7} .
$$

The probabilities for all combinations of $O_{1}$ and $O_{2}$ are listed in Fig 3. To compute $\mathbb{P}_{t}\left(O_{i+1} \mid O_{i}\right)$ we use Bayes' formula:

$$
\mathbb{P}_{t}\left(O_{i+1} \mid O_{i}\right)=\frac{\mathbb{P}_{t}\left(O_{i}, O_{i+1}\right)}{\mathbb{P}_{t}\left(O_{i}\right)}
$$

where, using the total probability theorem, $\mathbb{P}_{t}\left(O_{i}\right)$ can be obtained by summing over the appropriate row in Fig 3. Then, the total probability of observing the data, given $N$ and $t$, i.e.

$$
\begin{equation*}
\mathbb{P}_{t}\left(\left(O_{1}, \ldots, O_{n}\right)\right)=\mathbb{P}_{t}\left(O_{1}, O_{2}\right) \prod_{i=2}^{n-1} \frac{\mathbb{P}_{t}\left(O_{i}, O_{i+1}\right)}{\mathbb{P}_{t}\left(O_{i}\right)} \tag{3}
\end{equation*}
$$

can be maximized in order to find the maximum likelihood estimator of $t$ and $N$.

### 2.2.3 Two chromosomes, unphased data

If the data is unphased, we cannot distinguish which allele is in which of the two homologous chromosomes. We can observe one of these three states at each marker:

- $P$ i.e. we only observe the allele from parent $\mathcal{P}$, i.e. both chromosomes carry the allele from parent $\mathcal{P}$,
- $Q$ i.e. we only observe the allele from parent $\mathcal{Q}$.
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- $x$ i.e. we observe both alleles, i.e. each one of the two homologous chromosomes carries a 176 different allele.

The data can then be represented as a sequence $\left(O_{i}\right)$ of length $n$ that takes values in $\{P, Q, x\}$ such ${ }_{178}$ that $O_{i}$ is the state of the i-th marker. We can perform exactly the same method, as in the previous section, except that now the probabilities of each state are given by Fig 4 .

### 2.3 Individual based simulations

To test the validity of our maximum likelihood approach, we use individual based simulations, 182 as described in (Janzen et al., 2018), i.e. Wright-Fisher type simulations of randomly mating 183 populations of constant size $N$, with non-overlapping generations. We then recover local ancestry by 184 analyzing ancestry at $n$ markers whose positions are chosen uniformly at random along the genome. 185

As a proof of concept, we show how time can be accurately inferred for a population of $10,000 \quad 186$ individuals, for time points between the first generation and 1,000 generations. We use $n=10,000{ }_{187}$ markers, which should be sufficient to detect the majority of accumulated junctions. We report our 188 findings across 100 replicates, where in each replicate 10 individuals were randomly selected from 189 the population and used to infer the time since the onset of hybridization. We have simulated with 190 three different values of the initial proportion of subpopulation $\mathcal{P},(p \in\{0.053,0.184,0.5\})$, to vary ${ }_{191}$ the initial heterozygosity $H_{0}$ in $\{0.1,0.3,0.5\}$.

In Fig 5 we compare the methods we have developed here to previous methods based on the theory of junctions. We observe that, when the number of markers is low, previous methods, that 194 do not take into account marker spacing, tend to underestimate the time since admixture, which is not the case for our methods.

In Fig 6 we compare the estimations of the time since admixture, using the method for one ${ }^{197}$ chromosome and the method for two chromosomes (phased). We observe that using data from the two homologous chromosomes allows to infer the time since admixture more accurately, since it reduces uncertainty.

In Fig 7 we compare the methods that use phased or unphased information of two homologous 201 chromosomes. We observe that both methods yield very similar results in terms of the relative error. 202

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This can be due to the fact that homozygous sites have an important contribution to the likelihood ${ }_{203}$ and the uncertainty that comes from sites that are of type $x$ (in the unphased case) is well managed 204 by our method.

Finally, we explore error in phasing assignment (switching error). We simulate the effect of 206 error in phasing assignment by randomly swapping a fraction of the markers between chromosomes. 207 We explore phasing error in $\{0.0025,0.005,0.0075,0.01,0.02\}$ These errors are comparable to the 208 switching error rates reported in the literature. For example, (Choi et al. 2018) compared different ${ }^{209}$ phasing methods and reported switching error rates between $0.1 \%$ and $2 \%$. (Notice that these error ${ }^{210}$ rates are for human data where there are good quality references and sample sizes are large). More ${ }_{211}$ recent reference-free methods (based on third generation sequencing techniques) report switching $\quad 212$ error rates of 1-2\% (see for example (Tourdot and Zhang, 2019; Ebler et al., 2019, Kronenberg et al., 213 2019). Switching error rate error has strong effects on the inferred time since admixture, as shown 214 in the bottom panel in Fig 8 . Generally, imposed errors increase the inferred age, by introducing 215 novel junctions due to mis-phased markers. ${ }_{216}$

Another important source of error is the lack of coverage, which would have the effect of reducing ${ }^{217}$ the number of markers. An analysis of the sensitivity of our method to reducing the number of markers can be found in S1 Appendix.

## 3 Results

### 3.1 Saccharomyces cerevisiae

Experimental evolution provides an important reference point to verify our findings. Here, we 222 re-analyze data from an Advanced Intercross Line (AIL) experiment, where two highly differentiated ${ }_{223}$ yeast (Saccharomyces cerevisiae) lines were crossed, and the resulting hybrid offspring was outbred $\quad 224$ for 12 generations in order to obtain maximum genetic diversity (Parts et al., 2011, Illingworth et al., 225 2013). The data consists of sequencing data for 171 individuals, for all 16 chromosomes. There are ${ }^{226}$ on average 3271 ancestry informative markers per chromosome ( $95 \%$ CI: [929, 6284]). Local ancestry ${ }^{227}$ was inferred using a custom procedure, making use of the high levels of homozygosity in the parental 228
lineages. $H_{0}$ was 0.5 , reflecting a $50 / 50$ contribution of both strains to the first generation. We used three different recombination rate estimates: firstly, we used the linkage map of (Cherry et al. 1997) where the average recombination rate is $1 c M / 2.7 k b(1$ centimorgan per 2.7 kilobases), secondly, we used the average recombination rate of $1 c M / 2.2 k b$ as inferred in (Mancera et al., 2008) and lastly, we used the average recombination rate of $1 c M / 5.8 k b$ as inferred for the two-way cross in (Illingworth et al., 2013). In the absence of a detailed recombination map, we assume that recombination is constant across the chromosome, ignoring hotspots and coldspots. We assume a large population (N $=100,000$ ), which makes inbreeding effects negligible.

We find that when using the older recombination rate estimates, we consistently underestimate the age of the hybrids (median age using (Mancera et al. 2008): 6.69 generations, median age ${ }^{238}$ using (Cherry et al., 1997): 8.45 generations), suggesting that the true recombination rate is slightly ${ }_{239}$ lower than assumed. When using the most recent recombination rate estimate (i.e. $1 c M / 5.8 k b$ ), we slightly overestimate the age (median age estimate: 17.7 generations). Alternatively, we could be overestimating population size, suggesting that perhaps the rate of inbreeding in the experimental design was higher than anticipated. However, this only applies when assuming extremely high degrees of inbreeding, which seems unrealistic.

### 3.2 Swordtail Fish

Here, we re-analyze data of hybridizing swordtail fish published in (Schumer et al., 2018). Swordtail fish have received considerable attention in the past years, as they have been shown to hybridize readily in nature. We focus here on a hybrid population located in Tlatemaco, Mexico (Schumer et al., 2018, 2014a). The population is the result of a hybridization event between Xiphophorus birchmanni and X. malinche, approximately 100-200 generations ago (Pers. Comm. M. Schumer and (Schumer et al., 2018)). Currently, the hybrid genome consists for $75 \%$ of $X$. malinche, suggesting that the initial hybrid swarm was strongly biased towards $X$. malinche, or that strong selection after hybridization has favored genomic material from X. malinche. We use ancestry information provided in the data supplement of (Schumer et al. 2018), which contains unphased local ancestry estimates based on multiplexed shotgun genotyping (MSG) results (Andolfatto et al. 2011), with on
average 38,462 markers per chromosome ( $95 \%$ CI: $[18605,50242]$ ). The MSG pipeline provides a 256 posterior probability of observing local ancestry. Following (Schumer et al., 2018), we converted ${ }^{257}$ local ancestry probabilities of $>95 \%$ to hard ancestry calls. To obtain age estimates, we use the ${ }^{258}$ estimated population size in (Schumer et al. 2014a): 1830 individuals. We infer the age for each 259 of the 24 linkage groups separately, and analyze 187 individuals from the Tlatemaco population. 260 As a recombination map, we use three approaches. Firstly, we use the average recombination ${ }^{261}$ rate of $1 c M / 378 k b$ as used in (Schumer et al., 2014a), which is based on the average genome-wide 262 recombination rate in Xiphophorus (Walter et al. 2004). Secondly, we use the average recombination 263 rate of $1 c M / 500 \mathrm{~kb}$ as reported in (Powell et al. 2020). Lastly, we use the high density recombination 264 map reconstructed from Linkage Disequilibrium patterns as presented in (Schumer et al. 2018), 265 which represents an average recombination rate of $1 c M / 485 \mathrm{~kb}$.

When we compare age estimates across chromosomes (see Fig 10 A), we find that chromosomes ${ }^{267}$ 17 and 24 are inferred to be much younger, in line with the notion that these chromosomes include 268 large inversions (Schumer et al. 2018), making them unsuitable for admixture analysis. In any 269 subsequent analysis, we have removed these two chromosomes from the dataset. We find that the ${ }_{270}$ distribution of ages inferred for individuals from the Tlatemaco population is overall higher than the 271 previously inferred age but still consistent with those estimates (see Fig 10 B). We recover a median ${ }^{272}$ age of 167 generations (mean: 165, $95 \%$ CI: [75, 242]). when using the recombination rate reported $\quad 273$ in (Schumer et al., 2014b). Using the high density recombination map from (Schumer et al., 2018) 274 we obtain an age estimate of 194 generations ( $95 \%$ CI: [84, 349]), due to the shorter map length. 275 Alternatively, using the most recent recombination rate estimate of $1 c M / 500 \mathrm{~kb}$ reported in (Powell 276 et al. 2020), we recover a median age of 221 generations ( $95 \%$ CI: [100, 320]).

### 3.3 Populus trees

Here, we re-analyze a dataset of Populus trees, published in (Suarez-Gonzalez et al., 2016). The 279 dataset focuses on two species of trees, P. trichocarpa, found mainly in West-America, in humid, 280 moist conditions, and P. balsamifera, which is found in Northern America (e.g. Alaska, Canada) 281 and is more frost tolerant. The two species are thought to have diverged relatively recently, around 282

760k years ago. Where their ranges meet (around the southern tip of Alaska), the two species hybridize, and a hybrid population has been established. The dataset consists of 32 individuals 284 which are mainly $P$. balsamifera, admixed with $P$. trichocarpa and 36 individuals that are mainly $P .285$ trichocarpa, admixed with P. balsamifera. Three chromosomes of interest were Illumina sequenced, 286 being chromosomes 6,12 and 15. 68 admixed individuals were included, and unphased data was 287 available for on average 60071 ancestry informative markers per chromosome $(95 \% \mathrm{CI}:[28745,288$ 101425]). We use three different population level recombination rates recovered from the literature, 289 being $\rho=0.00219$ (Wang et al., 2016), $\rho=0.0092$ (Olson et al., 2010) and $\rho=0.0197$ (Slavov et al., 290 2012). We converted these population level recombination rates to individual rates using an effective 291 population size of 5106 individuals, as estimated using phylogenetic methods in (Slavov et al., 2012). 292 ${ }_{\square}$ This yielded three local recombination rates of $1 c M / 10.4 k b$ (Slavov et al., 2012), $1 c M / 22.2 k b$ (Olson 293 et al., 2010) and $1 c M / 93.3 k b$ (Wang et al. 2016). Local ancestry was determined using ANCESTRY 294 HMm (Corbett-Detig and Nielsen, 2017), assuming equal admixture of both parental species. Because 295 admixture differed strongly across samples, we used the average local ancestry per sample as input 296 for a second run of ANCESTRY HMM in order to obtain accurate local ancestry calls. Local ancestry 297 was translated into hard ancestry calls based on fixed thresholds. These thresholds are presented as 298 Phred ancestry scores, which are $-10 \log _{10}(p)$, where $p$ indicates the ancestry uncertainty. 299

Across all Phred Ancestry scores, we find that the time since admixture strongly correlates with 300 the recombination rate used (See Figure 11A), with a median number of generations since admixture 301 of $6(95 \% \mathrm{CI}:[3,15])$ when using the highest estimate of recombination $(1 c M / 10.4 k b$ (Slavov et al., 302 2012) , an intermediate estimate of 12 generations ( $95 \% \mathrm{CI}$ : $[6,30]$ ) when using a recombination ${ }_{303}$ rate of $1 c M / 22.2 k b($ Olson et al. 2010) and a much higher age estimate of 48 generations (95\% CI: 304 $[22,122]$ ) when using the lowest recombination estimate (Wang et al. 2016). When we correlate ${ }_{305}$ the age estimate for a Phred Ancestry score of 30 with the fraction of local ancestry in the sample ${ }_{306}$ attributable to $P$. trichocarpa, we find that individuals with intermediate ancestry tend to have a 307 higher estimated age, and that individuals with a genomic ancestry more similar to either of the 308 parental species tend to be younger.

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## 4 Discussion

The aim of this article was to improve the estimation of the time since admixture in hybrid populations. ${ }^{311}$ To do so, we have extended the theory of junctions in two directions. First, we have derived a 312 formula for the expected number of observed junctions in one chromosome that takes into account ${ }_{313}$ the number of markers and their positions (equation (2)). Second, we have considered the case in 314 which there is sequencing data from two homologous chromosomes. We have developed a maximum 315 likelihood approach that allows to infer the time since admixture, whether the data is phased or ${ }_{316}$ unphased. We have used a powerful mathematical model which is the ARG (Hudson, 1983; Griffiths, ${ }_{317}$ 1991. Griffiths and Marjoram, 1997). In the one chromosome case, we get an explicit formula for the number of junctions (equation (22) and in the two chromosomes case, we get a semi-explicit formula 319 for the likelihood of the observations (equation (3)) that can be solved numerically. 320

We have validated our method using simulations. We have shown that our formula for the 321 number of observed junctions in one chromosome performs better than previous methods that ignore 322 the effect of having a limited number of markers or asume that they are even-spaced (see Fig 5). We ${ }^{323}$ expected that using information from two chromosomes would improve accuracy of the estimation ${ }^{324}$ considerably, and this is also what we find when using phased data, especially for small population 325 sizes (see Fig 6 and Fig 2 in S2 Appendix. Surprisingly, a similar performance is achieved by the method that uses unphased data (see Fig 7 and Fig 3 in S2 Appendix. The phased and unphased ${ }^{327}$ approaches differ mainly in their treatment of markers that are heterozygous for ancestry, and hence ${ }_{328}$ we expected mainly differences between these methods to manifest themselves during the initial ${ }_{329}$ stages of admixture, when heterozygosity is still high. We did find that there were slight differences during these stages (Fig 7 and Fig 3 in S2 Appendix), but these were neglibile compared to the

When we take into account additional errors in ancestry inference due to incorrect phasing, we ${ }_{333}$ have shown that our unphased method outperforms the phased method (see Fig 8). Our findings here are conservative, as we show that the unphased method performs better even for small error rates, 335 comparable to error rates for human data (for example in (Choi et al., 2018)). Human data sets are ${ }^{336}$ typically of very high quality, and these error rates represent an extremely favourable scenario. In ${ }^{337}$
addition, not all data can be phased, for example if no reference haplotypes are available or if the sample sizes are small, which is often the case of data from hybrid species. This makes the unphased method particularly interesting.

In addition, we have tested the sensitivity of our method to different parameters such as the ${ }_{341}$ number of markers $n$, the population size $N$, the initial heterozygosity $H_{0}$ and the total recombination rate $C$ (see S1 Appendix). We have found that our method is quite sensitive to $H_{0}$ but this parameter ${ }^{343}$ can easily be estimated from the proportion of markers that come from each parental population. 344 One advantage of our approach is that age inference is not very sensitive to population size (see Fig ${ }_{345}$ 1 in S1 Appendix), which was not true for previous methods that rely on a good estimation of $N$ (see (Janzen et al., 2018)). Our method is not very sensitive either to the number of markers (see Fig 4 in S1 Appendix), provided that it is above a certain threshold. Janzen et al. (2018) inferred ${ }_{348}$ that when using regularly spaced markers and information for a single chromosome, the number ${ }_{349}$ of markers typically needs to be an order of magnitude larger than $\frac{1}{2} C t$, where $t$ is the admixture ${ }_{350}$ time and $C$ the total amount of recombination. We find similar results when using information ${ }^{351}$ from a single chromosome with arbitrarily spaced markers or information from both chromosomes $\quad 352$ (see S1 Appendix). When analyzing empirical data, it is often impossible to know a priori whether ${ }_{353}$ the number of ancestry informative markers is much larger than the admixture time. However, 354 our simulation results indicate that when the number of markers is too small, variation in the age 355 estimate across different chromosomes tends to increase. Thus, large variation in the estimate of 356 admixture time, or inferred admixture times that tend to extremely large values, are indicative of an insufficient marker number.

The main issue with our method is its sensitivity to the recombination rate. This is shown in Fig 2 of S1 Appendix but also exemplified by the varying results in the empirical datasets, dependent on our assumptions about recombination rates. However, it should be noted that this issue is not 361 novel to our approach, but is a general issue with the theory of junctions. Apart from sensitivity 362 to the average recombination rate, local hot-spots or cold-spots of recombination could potentially also influence admixture time estimates. Hence, we advocate for extending research on inferring recombination landscapes. At the same time we realize that inferring local recombination rates is

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labour intensive, and restrictive for organisms with large generation times (where crossing schemes would take very long to realize). The recombination rate does not only factor in during admixture ${ }_{367}$ time inference, but is also typically used to infer local ancestry. Methods such as AncestryHMM 368 (Corbett-Detig and Nielsen, 2017), ELAI (Guan, 2014) and MSG (Andolfatto et al., 2011) use the 369 local recombination landscape to assess the probability of an ancestry switch between neighboring 370 nucleotides. Thus, any variation introduced at the start of the analysis in the recombination ${ }_{371}$ landscape, echoes down the analysis pipeline both through impact on local ancestry and on the time ${ }^{372}$ since admixture. This further stresses the need for improved methods of inferring the recombination ${ }^{373}$ landscape. 374

To validate our approach we have re-analysed three datasets. The first dataset is from a crossing ${ }^{375}$ experiment on S. cerevisiae. We found that equation (2) provides a slightly better estimation of the ${ }^{376}$ time since admixture than previous methods. However, since the number of markers is very large, we ${ }^{377}$ did not expect a major improvement (see Fig 5). In addition, taking into account the marker positions 378 is particularly interesting when a detailed recombination map is available and the recombination ${ }^{379}$ rates between each pair of markers are known (here they are assumed to be proportional to the 380 distance in base pairs, which is not necessarily true). Nevertheless, our estimates of the time since 381 the onset of admixture line up well with the experimental design, although assumptions regarding $\quad 382$ the recombination rate remain of strong influence on the admixture time estimates. 383

The second dataset we re-analyzed is of Swordtail fish (Xiphophorus). We infer an admixture ${ }^{384}$ time that is older than previous estimates (Schumer et al., 2014b) but that is in line with more 385 recent estimates done by the same authors (M. Schumer, personal communication) using more recent ${ }_{386}$ recombination rate estimates (Powell et al. 2020) . The advantage of our method is that it is faster, ${ }^{387}$ since it does not rely on simulations. In the original dataset, the authors removed chromosomes 388 17 and 24 from their analysis because these chromosomes contain large inversions. We also find 389 strongly differing age estimates for these chromosomes, indicating that indeed these chromosomes 390 have not been subject to the same evolutionary history as the others. Again, we find that the results are sensitive to assumptions made regarding the recombination rate.

Finally, we have re-analyzed a dataset on Populus trees (Suarez-Gonzalez et al. 2016). We infer ${ }^{393}$
an admixture time that is in line with previous findings, but would like to stress that the original 394 analysis did not focus on admixture time, and only used admixture time to infer local ancestry. 395 Furthermore, we find that the time since admixture correlates strongly with the genetic distance ${ }^{396}$ to either of the parents, with individuals more closely related to either of the parents inferred to ${ }^{397}$ be younger. In the case of incidental hybridization and subsequent backcrossing, we would expect 398 the exact opposite, with individuals more related to the parents to be relatively older. In contrast, 399 the pattern we recover here suggests a hybrid zone between the two parents. However, admixture 400 mapping analyses have shown that perhaps late generation backcrosses have contributed as well 401 to the hybrid population (Suarez-Gonzalez et al., 2018), suggesting perhaps an intermediate form 402 between on the one hand some initial adaptive introgression and back-crossing, and on the other 403 hand the ongoing hybridization across a spatial gradient. 404

Here we have presented a full framework to estimate the time since admixture using phased or ${ }^{405}$ unphased data from two homologous chromosomes, taking into account marker spacing along the 400 chromosome. We have shown that using data from two chromosomes improves the estimations of the ${ }^{407}$ admixture time compared to the method that uses only one chromosome. This is true whether the 408 data is phased or unphased. In addition we have shown, using simulations, that applying the phased 409 or the unphased method yields very similar results. However, given that even small (unavoidable) 410 phasing errors produce overestimates in the time since admixture, we suggest that, in most cases, 411 using unphased data is the best strategy. With our new framework, we hope to have opened new ${ }_{412}$ avenues towards inferring the time since admixture in admixed populations, and primarily hope $\quad{ }^{413}$ to have brought this analysis within reach also for datasets where phased data is unavailable or ${ }_{414}$ impossible to acquire. We have included the derivations and the numerical solution framework in 415 the R package 'junctions'. By providing the code in an easy to use package, we hope to lower the threshold for other users to apply the theory of junctions to their model system.

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## Data Accessibility

We have included the derivations and the numerical solution framework in the R package 'junctions', ${ }^{426}$ which can be found on CRAN on https://CRAN.R-project.org/package=junctions. A develop- ${ }^{427}$ ment version of the package can be found at https://www.github.com/thijsjanzen/junctions. All code used in data analysis and visualization for this manuscript has been included in the Supporting Information.

## Author contributions

TJ and VMP jointly designed the research. VMP inferred the ARG based mathematics, TJ verified findings using individual based simulations and analyzed the empirical data. TJ and VMP jointly wrote the paper.

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## Supporting information

S1 Appendix. Sensitivity analysis. Using individual based simulations, we test how sensitive 575 our new framework is to variation of the different parameters. 576

S2 Appendix. Small population size. We test the validity of our method using a smaller value ${ }^{577}$
of the population size $(N=1000)$

S3 Appendix. Phasing error. We extend the analysis done in the main text to the case of a 579 smaller number of markers.

S4 Appendix. Simulation code. A collection of simulation code, simulation data and visual- 581 ization scripts, for all figures in the main text and in appendices S1, S2 and S3.


Fig 1. Visual depiction of the observed data. We show the differences between the type of data generated by the three methods we present in this paper. On each panel, the chromosome in the center is colored according to ancestry (blue represents parental population $\mathcal{P}$ and red represents parental population $\mathcal{Q}$ ). Above the chromosome are indicated the locations of ancestry informative markers $z_{i}$. Resulting inferred ancestry on these markers is shown below, where grey indicates heterozygous ancestry. The first panel represents the one chromosome method. There are 7 junctions in the chromosome, but only 3 are observed in the data due to a limited marker coverage. The second and third panels represent the methods that use information from two chromosomes. In the second panel data is phased whereas in the second panel data is unphased.


Fig 2. The ARG with two markers. Each color represents one parental population ( $\mathcal{P}$ and $\mathcal{Q}$ ). The black and grey lines (or dotted lines) represent the ancestral lineage of each marker. In the left panel, we show the ARG for two markers in one chromosome. In the present, there is an observed junction between the two markers. In the past ( $t$ generations ago, when hybridization took place), each lineage is carried by a different individual and these two individuals are from different subpopulations. The right panel shows the ARG for two markers in two homologous chromosomes.

|  | $O_{i+1}=P P$ | $O_{i+1}=Q Q$ | $O_{i+1}=P Q$ | $O_{i+1}=Q P$ |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & O_{i}= \\ & P P \end{aligned}$ | $\begin{aligned} & p^{2}\left(\left(P_{t}^{i}\right)_{1}+\left(P_{t}^{i}\right)_{4}+\right. \\ & \left.\left(P_{t}^{i}\right)_{7}\right)+p^{3}\left(\left(P_{t}^{i}\right)_{2}+\right. \\ & \left.\left(P_{t}^{i}\right)_{5}\right)+p^{4}\left(P_{t}^{i}\right)_{3}+ \\ & p\left(P_{t}^{i}\right)_{6} \end{aligned}$ | $\begin{aligned} & p q\left(p q\left(P_{t}^{i}\right)_{3}\right. \\ & \left.\frac{1}{2}\left(P_{t}^{i}\right)_{5}+\left(P_{t}^{i}\right)_{7}\right) \end{aligned}+$ | $\begin{array}{ll} \frac{p q}{2}\left(p\left(P_{t}^{i}\right)_{2}\right. & + \\ 2 p^{2}\left(P_{t}^{i}\right)_{3} \\ \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+p\left(P_{t}^{i}\right)_{5}\right) \end{array}+$ | $\begin{array}{ll} \frac{p q}{2}\left(p\left(P_{t}^{i}\right)_{2}\right. & + \\ 2 p^{2}\left(P_{t}^{i}\right)_{3} \\ \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+p\left(P_{t}^{i}\right)_{5}\right) \end{array}+$ |
| $\begin{aligned} & O_{i}= \\ & Q Q \end{aligned}$ | $\begin{aligned} & p q\left(p q\left(P_{t}^{i}\right)_{3}\right. \\ & \left.\frac{1}{2}\left(P_{t}^{i}\right)_{5}+\left(P_{t}^{i}\right)_{7}\right) \end{aligned}+$ | $\begin{aligned} & q^{2}\left(\left(P_{t}^{i}\right)_{1}+\left(P_{t}^{i}\right)_{4}+\right. \\ & \left.\left(P_{t}^{i}\right)_{7}\right)+q^{3}\left(\left(P_{t}^{i}\right)_{2}+\right. \\ & \left.\left(P_{t}^{i}\right)_{5}\right)+q^{4}\left(P_{t}^{i}\right)_{3}+ \\ & q\left(P_{t}^{i}\right)_{6} \end{aligned}$ | $\begin{array}{ll} \frac{p q}{2}\left(q\left(P_{t}^{i}\right)_{2}\right. \\ 2 q^{2}\left(P_{t}^{i}\right)_{3} \\ \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+q\left(P_{t}^{i}\right)_{5}\right) \end{array}+$ | $\begin{aligned} & \frac{p q}{2}\left(q\left(P_{t}^{i}\right)_{2}+\right. \\ & 2 q^{2}\left(P_{t}^{i}\right)_{3}+ \\ & \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+q\left(P_{t}^{i}\right)_{5}\right) \end{aligned}+$ |
| $\begin{aligned} & O_{i}= \\ & P Q \end{aligned}$ | $\begin{array}{ll} \frac{p q}{2}\left(p\left(P_{t}^{i}\right)_{2}\right. & + \\ 2 p^{2}\left(P_{t}^{i}\right)_{3} & + \\ \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+p\left(P_{t}^{i}\right)_{5}\right) \end{array}+$ | $\begin{array}{ll} \frac{p q}{2}\left(q\left(P_{t}^{i}\right)_{2}\right. & + \\ 2 q^{2}\left(P_{t}^{i}\right)_{3} \\ \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+q\left(P_{t}^{i}\right)_{5}\right) \end{array}+$ | $\begin{aligned} & p q\left(\left(P_{t}^{i}\right)_{1}+\right. \\ & \left.\frac{1}{2}\left(P_{t}^{i}\right)_{2}+p q\left(P_{t}^{i}\right)_{3}\right) \end{aligned}$ | $p^{2} q^{2}\left(P_{t}^{i}\right)_{3}$ |
| $\begin{aligned} & O_{i}= \\ & Q P \end{aligned}$ | $\begin{array}{ll} \frac{p q}{2}\left(p\left(P_{t}^{i}\right)_{2}\right. \\ 2 p^{2}\left(P_{t}^{i}\right)_{3} \\ \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+p\left(P_{t}^{i}\right)_{5}\right) \end{array}+$ | $\begin{array}{ll} \frac{p q}{2}\left(q\left(P_{t}^{i}\right)_{2}\right. \\ 2 q^{2}\left(P_{t}^{i}\right)_{3} \\ \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+q\left(P_{t}^{i}\right)_{5}\right) \end{array}+$ | $p^{2} q^{2}\left(P_{t}^{i}\right)_{3}$ | $\begin{aligned} & p q\left(\left(P_{t}^{i}\right)_{1}\right. \\ & \left.\frac{1}{2}\left(P_{t}^{i}\right)_{2}+p q\left(P_{t}^{i}\right)_{3}\right) \end{aligned}+$ |

Fig 3. $\mathbb{P}_{t}\left(O_{i}, O_{i+1}\right)$ for phased data. The allele from parent $\mathcal{P}$ is represented in blue and the allele from parent $\mathcal{Q}$ is represented in red.

|  | $O_{i+1}=P$ | $O_{i+1}=Q$ | $O_{i+1}=x$ |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & O_{i}= \\ & P \end{aligned}$ | $\begin{aligned} & p^{2}\left(\left(P_{t}^{i}\right)_{1}+{ }^{\left(P_{t}^{i}\right)_{4}+}+\right. \\ & \left.\left(P_{t}^{i}\right)_{7}\right)+p^{3}\left(\left(P_{t}^{i}\right)_{2}+\right. \\ & \left.\left(P_{t}^{i}\right)_{5}\right)+p^{4}\left(P_{t}^{i}\right)_{3}+p\left(P_{t}^{i}\right)_{6} \end{aligned}$ | $\begin{aligned} & p q\left(p q\left(P_{t}^{i}\right)_{3}+\frac{1}{2}\left(P_{t}^{i}\right)_{5}+\right. \\ & \left.\left(P_{t}^{i}\right)_{7}\right) \end{aligned}$ | $\begin{aligned} & p q\left(p\left(P_{t}^{i}\right)_{2}+2 p^{2}\left(P_{t}^{i}\right)_{3}+\right. \\ & \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+p\left(P_{t}^{i}\right)_{5}\right) \end{aligned}$ |
| $\begin{aligned} & O_{i}= \\ & Q \end{aligned}$ | $\begin{aligned} & p q\left(p q\left(P_{t}^{i}\right)_{3}+\frac{1}{2}\left(P_{t}^{i}\right)_{5}+\right. \\ & \left.\left(P_{t}^{i}\right)_{7}\right) \end{aligned}$ | $\begin{aligned} & q^{2}\left(\left(P_{t}^{i}\right)_{1}+{ }^{2}+P_{t}^{i}\right)_{4}+ \\ & \left.\left(P_{t}^{i}\right)_{7}\right)+q^{3}\left(\left(P_{t}^{i}\right)_{2}+\right. \\ & \left.\left(P_{t}^{i}\right)_{5}\right)+q^{4}\left(P_{t}^{i}\right)_{3}+q\left(P_{t}^{i}\right)_{6} \end{aligned}$ | $\begin{aligned} & p q\left(q\left(P_{t}^{i}\right)_{2}+2 q^{2}\left(P_{t}^{i}\right)_{3}+\right. \\ & \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+q\left(P_{t}^{i}\right)_{5}\right) \end{aligned}$ |
| $\begin{aligned} & O_{i}= \\ & x \end{aligned}$ | $\begin{aligned} & p q\left(p\left(P_{t}^{i}\right)_{2}+2 p^{2}\left(P_{t}^{i}\right)_{3}+\right. \\ & \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+p\left(P_{t}^{i}\right)_{5}\right) \end{aligned}$ | $\begin{aligned} & p q\left(q\left(P_{t}^{i}\right)_{2}+2 q^{2}\left(P_{t}^{i}\right)_{3}+\right. \\ & \left.\frac{1}{2}\left(P_{t}^{i}\right)_{4}+q\left(P_{t}^{i}\right)_{5}\right) \end{aligned}$ | or $\begin{aligned} & p q\left(2\left(P_{t}^{i}\right)_{1}+\left(P_{t}^{i}\right)_{2}+\right. \\ & \left.4 p q\left(P_{t}^{i}\right)_{3}\right) \end{aligned}$ |

Fig 4. $\mathbb{P}_{t}\left(O_{i}, O_{i+1}\right)$ for unphased data. The allele from parent $\mathcal{P}$ is represented in blue and the allele from parent $\mathcal{Q}$ is represented in red.


Fig 5. Comparison to previous methods. Shown are the median estimates for the time since admixture (dots) for 100 replicates, where in each replicate 10 individuals were analyzed. The dashed line indicates the simulated time. 'Evenly spaced markers' corresponds to the method in (Janzen et al., 2018). 'Infinite markers' corresponds to an idealized scenario where ancestry is known for every locus in the chromosome and is there to quantify the amount of randomness in the process. The population size was 10,000 individuals, and 10,000 randomly spaced markers were used.


Fig 6. Accuracy in age estimate using information from one versus two chromosomes. Inferred time versus simulated time is represented. Shown are the median estimates (dots) for 100 replicates, where in each replicate 10 individuals were analyzed. The solid white line indicates the observed is equal to expected line and the shaded area indicates the $95 \%$ percentile. Shown are results using junction information from one chromosome (blue) and results using information from two chromosomes (gold). Numbers above the plots indicate the initial heterozygosity. The population size was 10,000 individuals, and 10,000 randomly spaced markers were used.


Fig 7. Accuracy in age estimate using the unphased framework versus the phased framework. Shown are the median difference across 100 replicates. We represent the results for three different initial heterozygozities, as indicated at the top of each plot. The population size was 10,000 individuals, and 10,000 randomly spaced markers were used. The inset plots show the same results, including the $95 \%$ confidence limits, which are far outside the boundaries of the main plot.


Fig 8. Effect of switching error on the estimated time since admixture. Data simulated with $N=10,000, p=0.5, C=1$ and $n=10,000$. The solid black line indicates the simulated $=$ estimated time. Dots indicate the median inferred age and the colored area indicates the $95 \%$ confidence interval (CI) envelope. Colors reflect different degrees of phasing error, where a phasing error of 0.01 represents a $1 \%$ probability of a SNP being phased incorrectly.


Fig 9. Inferred age for F12 Hybrid Yeast (Saccharomyces cerevisiae) individuals. Shown are estimates across all 16 chromosomes. The dotted line indicates the 12 generations used to generate the hybrid individuals. (A) Results using either equidistant markers ((Janzen et al., 2018) ) or using actual marker positions (this paper). Shown are results using average recombination rate of $1 c M / 5.8 \mathrm{~kb}$ as inferred in (Illingworth et al., 2013). (B) Inferred age for different recombination rates, assuming a population size of 100,000 individuals.


Fig 10. Inferred age for hybrid Xiphophorus fish from Tlatemaco (Mexico). (A):Results for each chromosome, where two chromosomes with large inversions are indicated in pink (these were excluded from the subsequent analysis). Shown are inferred ages using the recombination map from (Schumer et al., 2014a). (B) combined results (excluding chromosomes 17 and 24). The dashed line indicates the previously estimated age, based on the decay of linkage disequilibrium ( 56 generations). Shown are age inferences based on different recombination maps.


Fig 11. Inferred age for hybrid Populus trees. (A) .pdf Inferred time since admixture for all individuals, split out per Phred Ancestry score, where ancestry phred scores indicate the local ancestry uncertainty allowed for inclusion of markers. Colors indicate different recombination rates used: $1 c M / 10.4 k b$ (Slavov et al., 2012), $1 c M / 22.2 k b$ (Olson et al., 2010) and $1 c M / 93.3 k b$ (Wang et al. 2016). (B) Inferred time since admixture for a Phred Ancestry score of 30, split out across the average frequency of $P$. trichocarpa in the admixed individual.

