

# Re-evaluation of SNP heritability in complex human traits

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**SNP heritability, the proportion of phenotypic variance explained by SNPs, has been reported for many hundreds of traits. Its estimation requires strong prior assumptions about the distribution of heritability across the genome, but the assumptions in current use have not been thoroughly tested. By analyzing imputed data for a large number of human traits, we empirically derive a model that more accurately describes how heritability varies with minor allele frequency, linkage disequilibrium and genotype certainty. Across 19 traits, our improved model leads to estimates of common SNP heritability on average 43% (SD 3) higher than those obtained from the widely-used software GCTA, and 25% (SD 2) higher than those from the recently-proposed extension GCTA-LDMS. Previously, DNaseI hypersensitivity sites were reported to explain 79% of SNP heritability; using our improved heritability model their estimated contribution is only 24%.**

The SNP heritability ( $h_{\text{SNP}}^2$ ) of a trait is the fraction of phenotypic variance explained by additive contributions from SNPs.<sup>1</sup> Accurate estimates of  $h_{\text{SNP}}^2$  are central to resolving the missing heritability debate, indicate the potential utility of SNP-based prediction and help design future genome-wide association studies (GWAS).<sup>2,3</sup> Whereas techniques for estimating (total) heritability have existed for decades,<sup>4,5</sup> the first method for estimating  $h_{\text{SNP}}^2$  was proposed only in 2010,<sup>1</sup> but has since been applied to many hundreds of traits. Extensions of this method are now being used to partition heritability across chromosomes, biological pathways and by SNP function, and to calculate the genetic correlation between pairs of traits.<sup>6-8</sup>

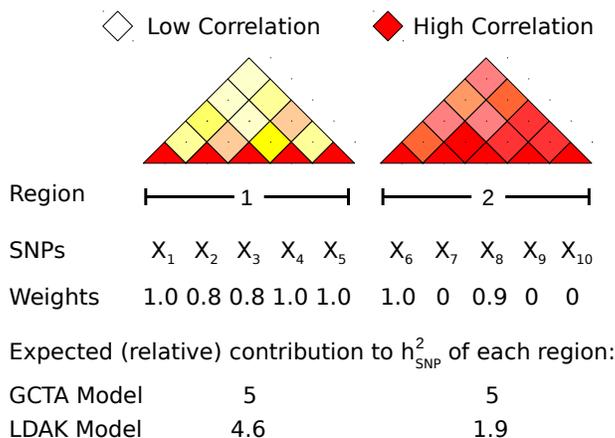
As the number of SNPs in a GWAS is usually much larger than the number of individuals, estimation of  $h_{\text{SNP}}^2$  requires steps to avoid over-fitting. Most reported estimates of  $h_{\text{SNP}}^2$  are based on assigning the same Gaussian prior distribution to each SNP effect size, in a way which implies that all SNPs are expected to contribute equal heritability.<sup>1,9</sup> By examining a large collection of real datasets, we derive approximate relationships between the expected heritability of a SNP and minor allele frequency (MAF), levels of linkage disequilibrium (LD) with other SNPs and genotype certainty. This provides us with an improved model for heritability estimation and a better understanding of the genetic architecture of complex traits.

## Results

When estimating  $h_{\text{SNP}}^2$ , the “LDAK Model” assumes

$$\mathbb{E}[h_j^2] \propto [f_j(1 - f_j)]^{1+\alpha} \times w_j \times r_j, \quad (1)$$

where  $\mathbb{E}[h_j^2]$  is the expected heritability contribution of SNP  $j$  and  $f_j$  is its (observed) MAF. The parameter  $\alpha$  determines the assumed relationship between heritability and MAF. In human genetics it is commonly assumed that heritability does not depend on MAF, which is achieved by setting  $\alpha = -1$ , however, we consider alternative relationships. The SNP weights  $w_1, \dots, w_m$  are computed based on local levels of LD;<sup>9</sup>  $w_j$  tends to be higher for SNPs in regions of low LD, and thus the LDAK Model assumes that these SNPs contribute



**Figure 1: Comparison of the GCTA and LDAK Models.** Region 1 contains five SNPs in low LD (lighter colors indicate weaker pairwise correlations). Each SNP contributes unique genetic variation, reflected by SNP weights close to one. Region 2 contains five SNPs in high LD (strong correlations). The total genetic variation tagged by the region is effectively captured by two of the SNPs, and so the others receive zero weight. Under the GCTA Model, the regions are expected to contribute heritability proportional to their numbers of SNPs, here equal. Under the LDAK Model, they are expected to contribute proportional to their sums of SNP weights, here in the ratio 4.6:1.9. Note that the expected heritability can also depend on the allele frequencies and genotype certainty of the SNPs, but for simplicity, these factors are ignored here.

38 more than those in high-LD regions. Finally,  $r_j \in [0, 1]$  is an information score measuring genotype certainty; the LDAK Model expects  
 39 that higher-quality SNPs contribute more than lower-quality ones.  $r_j$  is defined in Online Methods, where we also explain how (1) arises  
 40 by assuming a genome-wide random regression in which SNP effect sizes are assigned Gaussian distributions.

41 The “GCTA Model” is obtained from (1) by setting  $w_j = 1$  and  $r_j = 1$ , and thus assumes that expected heritability does not  
 42 vary with either LD or genotype certainty. To date, most reported estimates of  $h_{\text{SNP}}^2$  have used the GCTA Model with  $\alpha = -1$ , which  
 43 corresponds to the assumption that  $\mathbb{E}[h_j^2]$  is constant, and so the expected contribution of a SNP set depends only on the number of  
 44 SNPs it contains.<sup>1</sup> To appreciate the major difference between the GCTA and LDAK Models, consider a region containing two SNPs:  
 45 under the GCTA Model, the expected heritability of these two SNPs is the same irrespective of the LD between them, whereas under the  
 46 LDAK Model, two SNPs in perfect LD are expected to contribute only half the heritability of two SNPs showing no LD. See Figure 1  
 47 for a more detailed example.

48 An alternative method for estimating  $h_{\text{SNP}}^2$  is LDSC (LD Score Regression).<sup>10</sup> The LDSC Model expects that each SNP con-  
 49 tributes equal heritability,<sup>10,11</sup> and therefore closely resembles the GCTA Model with  $\alpha = -1$ . When applied to the same dataset,  
 50 estimates from LDSC will typically have standard error 25-100% higher than those from GCTA;<sup>11</sup> this is partly because the LDSC  
 51 Model includes an extra parameter, designed to capture confounding biases, and partly because LDSC estimates are moment-based,  
 52 whereas GCTA (like LDAK) uses restricted maximum likelihood (REML).<sup>12,13</sup> However, as LDSC requires only summary statistics  
 53 (i.e.,  $p$ -values from single-SNP analysis), it can be used on much larger datasets than GCTA and LDAK, which need raw genotype data,  
 54 and can be applied to results from large-scale meta-analyses.<sup>10</sup>

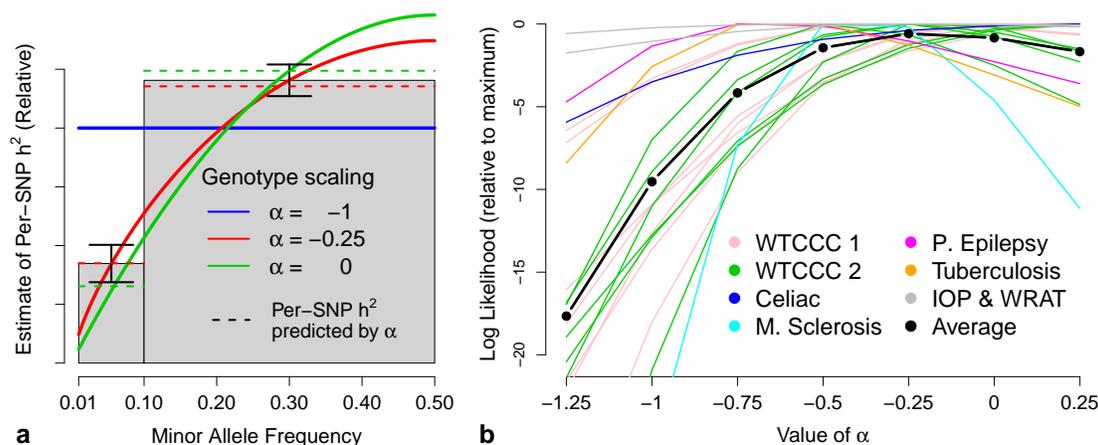
55 **SNP partitioning:** (1) can be generalized by dividing SNPs into tranches across which the constant of proportionality is allowed to  
 56 vary (so  $\mathbb{E}[h_j^2] = c_k \times [f_j(1 - f_j)]^{1+\alpha} \times w_j \times r_j$  for SNPs in Tranche  $k$ ). This is known as SNP partitioning.<sup>6</sup> Two examples are  
 57 GCTA-MS<sup>14</sup> and GCTA-LDMS:<sup>15</sup> when applied to common SNPs ( $\text{MAF} > 0.01$ ), GCTA-MS divides the genome into five tranches  
 58 based on MAF, using the boundaries 0.1, 0.2, 0.3 and 0.4, while GCTA-LDMS first divides SNPs into four tranches based on local  
 59 average LD Score,<sup>10</sup> then divides each of these into five based on MAF, resulting in a total of 20 tranches. In general, we prefer to avoid  
 60 SNP partitioning when estimating  $h_{\text{SNP}}^2$ , because it introduces (often arbitrary) discontinuities in the model assumptions and can cause

| Collection   | Trait (Disease Prevalence, %) | $n$           | $m$     | $\sum_{j=1}^m w_j$ | $h_{\text{GWAS}}^2$       | Estimates of $h_{\text{SNP}}^2$ (SD) |             |
|--|-------------------------------|---------------|---------|--------------------|---------------------------|--------------------------------------|-------------|
|  |                               |               |         |                    |                           | Previous                             | LDAK        |
| <b>Welcome Trust<br/>Case Control<br/>Consortium 1<br/>(WTCCC 1)</b> | Bipolar Disorder (0.5)        | 1 840 + 2 913 | 3 729 K | 79 K               | 0.02                      | 0.24 (0.04) <sup>7</sup>             | 0.35 (0.03) |
|  | Coronary Artery Disease (6)   | 1 907 + 2 918 | 3 738 K | 80 K               | 0.03                      | 0.25 (0.06) <sup>7</sup>             | 0.40 (0.06) |
|  | Crohn's Disease (0.5)         | 1 691 + 2 905 | 3 723 K | 79 K               | 0.21                      | 0.26 (0.01) <sup>16</sup>            | 0.32 (0.03) |
|  | Hypertension (5)              | 1 918 + 2 916 | 3 739 K | 80 K               | <0.01                     | 0.33 (0.06) <sup>7</sup>             | 0.46 (0.06) |
|  | Rheumatoid Arthritis (0.5)    | 1 846 + 2 918 | 3 735 K | 80 K               | 0.19                      | 0.09 (0.03) <sup>7</sup>             | 0.21 (0.03) |
|  | Type 1 Diabetes (0.5)         | 1 941 + 2 907 | 3 731 K | 80 K               | 0.27                      | 0.13 (0.03) <sup>7</sup>             | 0.31 (0.02) |
|  | Type 2 Diabetes (8)           | 1 896 + 2 917 | 3 735 K | 80 K               | 0.08                      | 0.42 (0.07) <sup>7</sup>             | 0.54 (0.07) |
| <b>Welcome Trust<br/>Case Control<br/>Consortium 2<br/>(WTCCC 2)</b> | Barrett's Oesophagus (1.6)    | 1 861 + 5 138 | 4 830 K | 116 K              | <0.01                     | 0.25 (0.05) <sup>17</sup>            | 0.32 (0.04) |
|  | Ischaemic Stroke (2)          | 3 769 + 5 139 | 4 797 K | 115 K              | <0.01                     | 0.25 (0.03) <sup>18</sup>            | 0.34 (0.03) |
|  | Parkinson's Disease (0.2)     | 1 687 + 5 136 | 4 819 K | 116 K              | 0.03                      | 0.27 (0.05) <sup>19</sup>            | 0.20 (0.03) |
|  | Psoriasis (0.5)               | 2 267 + 5 143 | 4 814 K | 116 K              | 0.21                      | 0.35 (0.06) <sup>20</sup>            | 0.34 (0.02) |
|  | Schizophrenia (1)             | 2 068 + 2 615 | 3 481 K | 111 K              | 0.07                      | 0.23 (0.01) <sup>21</sup>            | 0.30 (0.04) |
| Ulcerative Colitis (0.2)   | 2 614 + 5 327                 | 4 061 K       | 115 K   | 0.12               | 0.19 (0.01) <sup>16</sup> | 0.28 (0.02)                          |             |
| <b>WTCCC 2+</b>  | Celiac Disease (1)            | 2 492 + 7 376 | 3 681 K | 88 K               | 0.29                      | 0.33 (0.04) <sup>22</sup>            | 0.35 (0.02) |
|  | Multiple Sclerosis (0.1)      | 8 553 + 5 667 | 4 702 K | 113 K              | 0.17                      | 0.17 (0.01) <sup>7</sup>             | 0.24 (0.01) |
|  | Partial Epilepsy (0.3)        | 1 217 + 5 152 | 3 399 K | 108 K              | <0.01                     | 0.33 (0.05) <sup>3</sup>             | 0.27 (0.04) |
| <b>RPTB<br/>Blue Mountain<br/>CHOP</b>                               | Pulmonary Tuberculosis (4)    | 5 142 + 5 283 | 3 987 K | 102 K              | <0.01                     | None Found                           | 0.26 (0.03) |
|  | Intraocular Pressure          | 2 235         | 4 149 K | 125 K              | 0.02                      | None Found                           | 0.38 (0.17) |
|  | Wide-Range Achievement Test   | 3 747         | 3 593 K | 88 K               | <0.01                     | 0.43 (0.10) <sup>23</sup>            | 0.21 (0.09) |

**Table 1: Properties of datasets and estimates of  $h_{\text{SNP}}^2$ .**  $n$  = sample size (cases + controls),  $m$  = number of SNPs,  $\sum_{j=1}^m w_j$  = sum of SNP weights which can be interpreted as an effective number of independent SNPs. All values are post quality control; values for  $m$  and  $\sum w_j$  are rounded to the nearest K (thousand). For UCLEB,  $m$  and  $\sum w_j$  refer to our main analysis, which considers only high-quality, common SNPs. The final column provides our best estimates of  $h_{\text{SNP}}^2$  from common SNPs, computed using LDAK with  $\alpha = -0.25$  (see main text for explanation of  $\alpha$ ). For comparison, we include previously published estimates of  $h_{\text{SNP}}^2$  (note that the previous analyses for rheumatoid arthritis, type 1 diabetes and multiple sclerosis excluded major histocompatibility SNPs, which we estimate contribute 0.07, 0.20 and 0.05, respectively), as well as  $h_{\text{GWAS}}^2$ , the proportion of phenotypic variance explained by SNPs reported as GWAS significant ( $P < 5 \times 10^{-8}$ ). For disease traits, estimates of  $h_{\text{SNP}}^2$  and  $h_{\text{GWAS}}^2$  have been converted to the liability scale assuming the stated prevalence.

61 convergence problems. However, we show below that partitioning based on MAF enables reliable estimation of  $h_{\text{SNP}}^2$  when rare SNPs  
62 (MAF < 0.01) are included. Additionally, SNP partitioning provides a way to visually assess the fit of different heritability models;  
63 it allows us to estimate average  $h_j^2$  for different SNP tranches, which can then be compared to the values predicted under different  
64 assumptions.

65 **Datasets:** In total, we analyze data for 42 traits. Table 1 describes the 19 “GWAS traits” (17 case-control, 2 quantitative). For these, in-  
66 dividuals were genotyped using either genome-wide Illumina or Affymetrix arrays (typically 500 K to 1.2 M SNPs). We additionally ex-  
67 amine data from eight cohorts of the UCLEB consortium,<sup>24</sup> which comprise about 14 000 individuals genotyped using the Metabochip;<sup>25</sup>  
68 (a relatively sparse array of 200 K SNPs selected based on previous GWAS) and recorded for a wide range of clinical phenotypes. From  
69 these, we consider 23 quantitative phenotypes (average sample size 8 200), which can loosely be divided into anthropomorphic (height,  
70 weight, BMI and waist circumference), physiological (lung capacity and blood pressure), cardiac (e.g., PR and QT intervals), metabolic  
71 (glucose, insulin and lipid levels) and blood chemistry (e.g., fibrinogen, Interleukin 6 and haemoglobin levels). In general, our quality  
72 control is extremely strict; after imputing using IMPUTE2<sup>26</sup> and the 1000 Genome Phase 3 (2014) reference panel,<sup>27</sup> we retain only  
73 autosomal SNPs with MAF > 0.01 and information score  $r_j > 0.99$ . We only relax quality control when, using the UCLEB data, we  
74 explicitly examine the consequences of including lower-quality and rare SNPs.

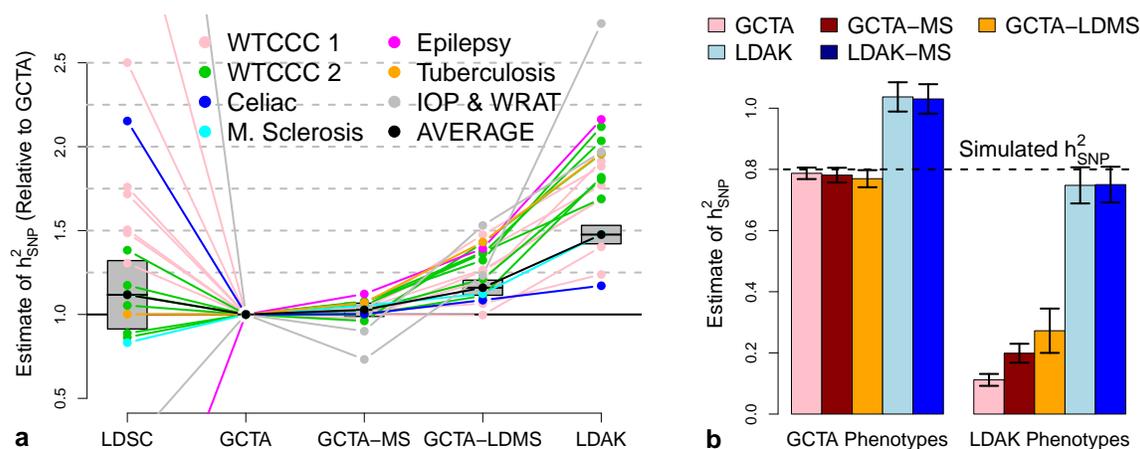


**Figure 2: (a) Relationship between heritability and MAF.** The parameter  $\alpha$  specifies the assumed relationship between heritability and MAF: in human genetics,  $\alpha = -1$  is typically used (solid blue line), while in animal and plant genetics,  $\alpha = 0$  is more common (green); we instead found  $\alpha = -0.25$  (red) provides a better fit to real data. The gray bars report (relative) estimates of the per-SNP heritability for  $\text{MAF} < 0.1$  and  $\text{MAF} > 0.1$  SNPs, averaged across the 19 GWAS traits (vertical lines provide 95% confidence intervals); the dashed lines indicate the per-SNP heritability predicted by each  $\alpha$ . **(b) Determining best-fitting  $\alpha$  for the GWAS traits.** We compare  $\alpha$  based on likelihood; higher likelihood indicates better-fitting  $\alpha$ . Lines report log likelihoods from LDAK for seven values of  $\alpha$ , relative to the highest observed. Line colors indicate the seven trait categories, while the black line reports averages.

75 Further details of our methods and datasets are provided in Online Methods. In particular, we explain how when estimating  $h_{\text{SNP}}^2$   
 76 we give special consideration to highly-associated SNPs, which we define as those with  $P < 10^{-20}$  from single-SNP analysis, and how  
 77 for the UCLEB data, we confirm that genotyping errors do not correlate with phenotype (which is important for the analyses where we  
 78 include lower-quality SNPs).

79 **Relationship between heritability and MAF:** Varying the value of  $\alpha$  in (1) changes the assumed relationship between heritability  
 80 and MAF; three example relationships are shown in Figure 2a. To determine suitable  $\alpha$ , we analyze each of the 42 traits using seven  
 81 values:  $-1.25, -1, -0.75, -0.5, -0.25, 0$  and  $0.25$ , seeing which lead to best model fit (highest likelihood). Full results are provided in  
 82 Supplementary Figure 1 and Supplementary Table 2. First, to remove any confounding due to LD, we use only a pruned subset of SNPs  
 83 (with  $w_j = 1$ ); next, we repeat without LD pruning (the results for the GWAS traits are shown in Figure 2b); finally, for the UCLEB  
 84 traits, we repeat including lower-quality and rare SNPs. We find that model fit is typically highest for  $-0.5 \leq \alpha \leq 0$ , whereas the most  
 85 widely-used value,  $\alpha = -1$ , results in sub-optimal fit. On the basis that it performs consistently well across different traits and SNP  
 86 filterings, we recommend that  $\alpha = -0.25$  becomes the default. This value implies that expected heritability declines with MAF; this is  
 87 seen in Figure 2a which reports, averaged across the 19 GWAS traits, the (weight-adjusted) per-SNP heritability for low- and high-MAF  
 88 SNPs (see Supplementary Figure 2 for further details).

89 While  $\alpha = -0.25$  provides the best fit overall, for individual traits, optimal  $\alpha$  may differ, and therefore we investigate sensitivity  
 90 of  $h_{\text{SNP}}^2$  estimates to the value of  $\alpha$ . Full results are provided in Supplementary Figures 3, 4 & 5, while Figure 6a provides a summary for  
 91 the UCLEB traits. When analyzing only common SNPs, we find that changes in  $\alpha$  have little impact on  $h_{\text{SNP}}^2$ . For example, across the  
 92 23 UCLEB traits, estimates from high-quality common SNPs using  $\alpha = -0.25$  are on average only 5% (SD 4) lower than those using  
 93  $\alpha = -1$ , and 4% (SD 4) higher than those using  $\alpha = 0$ . However, this is no longer the case when rare SNPs are included in the analysis:  
 94 for example, when the MAF threshold is reduced to 0.0005, estimates using  $\alpha = -0.25$  are on average 18% (SD 4) lower than those  
 95 using  $\alpha = -1$  and 30% (SD 6) higher than those from  $\alpha = 0$ . Therefore, when including rare SNPs, we guard against misspecification  
 96 of  $\alpha$  by partitioning based on MAF (with boundaries at 0.001, 0.0025, 0.01 and 0.1); we find that this provides stable estimates of  $h_{\text{SNP}}^2$   
 97 and also allows estimation of the relative contributions of rare and common variants (Figure 6a and Supplementary Figure 6).

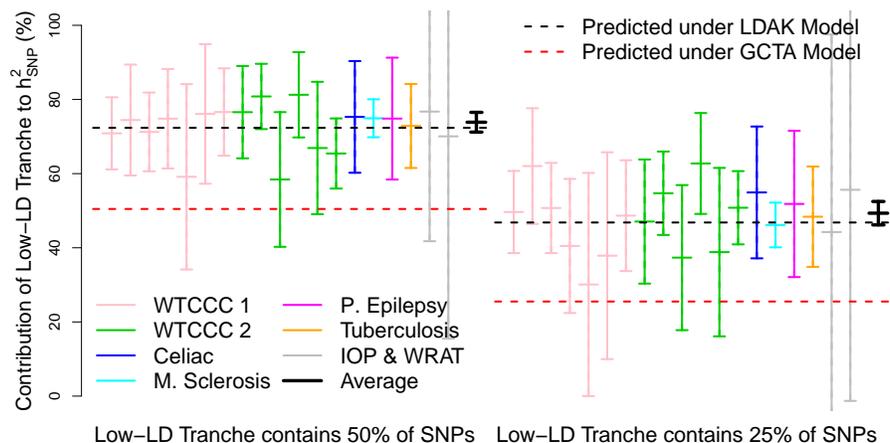


**Figure 3: (a) Relative estimates of  $h^2_{\text{SNP}}$  for the GWAS traits.**  $h^2_{\text{SNP}}$  estimates from LDSC, GCTA-MS (SNPs partitioned by MAF), GCTA-LDMS (SNPs partitioned by LD and MAF) and LDAK are reported relative to those from GCTA. For versions of GCTA and LDAK, we use  $\alpha = -0.25$  (see main text for explanation of  $\alpha$ ). Line colors indicate the seven trait categories; the black line reports the (inverse variance weighted) averages, with gray boxes providing 95% confidence intervals for these averages. Numerical values are provided in Supplementary Table 3. **(b) Simulation studies can be misleading.** Phenotypes are simulated with 1000 causal SNPs and  $h^2_{\text{SNP}} = 0.8$  (black horizontal line), then analyzed using GCTA, GCTA-MS, GCTA-LDMS, LDAK and LDAK-MS (LDAK with SNPs partitioned by MAF). Bars report average  $h^2_{\text{SNP}}$  across 200 simulated phenotypes (vertical lines provide 95% confidence intervals). **Left:** copying the study of Yang *et al.*,<sup>1</sup> causal SNP effect sizes are sampled from  $\mathbb{N}(0, 1)$ , similar to the GCTA Model. **Right:** causal SNP effect sizes are sampled from  $\mathbb{N}(0, w_j)$ , similar to the LDAK Model.

98 **Relationship between heritability and LD:** The LDAK Model assumes that heritability varies according to local levels of LD, whereas  
 99 the GCTA Model assumes that heritability is independent of LD. First we demonstrate that choice of model matters when estimating  
 100  $h^2_{\text{SNP}}$ . For the GWAS traits, Figure 3a reports relative estimates of  $h^2_{\text{SNP}}$  from GCTA, GCTA-MS, GCTA-LDMS and LDAK (all using  
 101  $\alpha = -0.25$ ); see Supplementary Figure 7 for an extended version. We find that estimates based on the LDAK Model are on average 48%  
 102 (SD 3) higher than estimates based on the GCTA Model. For the UCLEB traits, estimates from LDAK are on average 88% (SD 7)  
 103 higher than those from GCTA (Supplementary Fig. 8). Figure 3a also includes results from LDSC, run as described in the original publication<sup>10</sup>  
 104 (see Supplementary Table 3 for numerical values). Estimates from LDSC are not significantly different to those from GCTA, which is to  
 105 be expected considering that GCTA and LDSC assume the same relationship between heritability and LD. In Supplementary Figure 9 we  
 106 consider alternative versions of LDSC (e.g., varying how LD Scores are computed, forcing the intercept term to be zero and excluding  
 107 highly-associated SNPs). While changing settings can have a large impact, in all cases the average estimate of  $h^2_{\text{SNP}}$  from LDSC remains  
 108 substantially below that from LDAK.

109 A recent article which asserted that GCTA estimates  $h^2_{\text{SNP}}$  more accurately than LDAK, based this claim on a simulation study in  
 110 which causal SNPs were assigned effect sizes from the same Gaussian distribution, irrespective of LD.<sup>6</sup> This resembles the GCTA Model  
 111 but not the LDAK Model, and so it is no surprise that GCTA performed better. Figure 3b shows that if instead effect size variances had  
 112 been scaled by SNP weights, and so vary with LD similar to the LDAK Model, then the study would have found LDAK to be superior  
 113 to GCTA. Thus using simulations to compare different heritability models is problematic, because the conclusions will depend on the  
 114 assumptions used when generating phenotypes. See Supplementary Figure 10 for a full reanalysis of the reported simulation study and  
 115 Supplementary Figure 11 for further simulations.

116 Rather than using simulations, we compare LDAK and GCTA empirically. Supplementary Table 4 shows that when  $\alpha = -0.25$ ,  
 117 assuming the LDAK Model leads to higher likelihood than assuming the GCTA Model for all 19 GWAS traits and for 17 of the 23  
 118 UCLEB traits (if we instead use  $\alpha = -1$ , likelihood is higher under the LDAK Model for 31 of the 42 traits). To visually demonstrate  
 119 the superior fit of the LDAK Model, we partition SNPs into low- and high-LD (for this, we rank SNPs according to the average LD  
 120 Score<sup>10</sup> of non-overlapping 100 kb segments, the metric used by GCTA-LDMS<sup>15</sup>). First, we partition so that the two tranches contain an

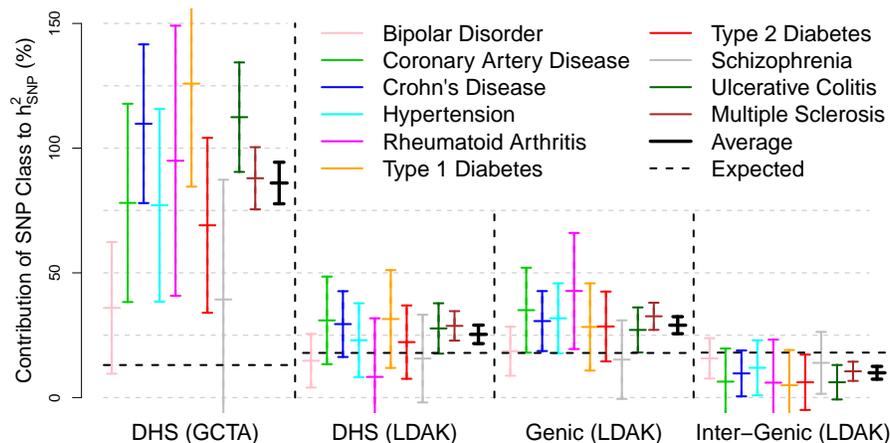


**Figure 4: Comparing the GCTA and LDAK Models for the GWAS traits:** We partition SNPs into low- or high-LD, with the low-LD tranche containing either 50% (left) or 25% (right) of SNPs. For each partition, the horizontal red and black lines indicate the predicted contribution of the low-LD tranche to  $h^2_{\text{SNP}}$  under the GCTA and LDAK Models, respectively. Vertical lines provide point estimates and 95% confidence intervals for the contribution of the low-LD tranche to  $h^2_{\text{SNP}}$ , estimated assuming the GCTA Model. Line colors indicate the seven trait categories, while the black lines provide the (inverse variance weighted) averages.

121 equal number of SNPs. The left half of Figure 4b reports, for each of the GWAS traits, the contribution of the low-LD tranche, estimated  
 122 using the GCTA Model (with  $\alpha = -0.25$ ). Under the GCTA Model, the low-LD tranche is expected to contribute 50% of  $h^2_{\text{SNP}}$ ; under  
 123 the LDAK Model, it is expected to contribute 72% of  $h^2_{\text{SNP}}$ . We see that the estimated contribution of the low-LD tranche is consistent  
 124 with the GCTA Model (95% confidence interval includes 50%) for only 5 of the 19 traits, whereas it is consistent with the LDAK Model  
 125 (confidence interval includes 72%) for 18. Next we partition so that the low-LD tranche contains a quarter of the SNPs; now the low-LD  
 126 tranche is predicted to contribute 26% of  $h^2_{\text{SNP}}$  under the GCTA Model, but 47% of  $h^2_{\text{SNP}}$  under the LDAK Model. The right half of  
 127 Figure 4b shows that its estimated contribution is consistent with the GCTA Model for only 7 of the 19 traits, but again consistent with  
 128 the LDAK Model for 18. Additional results are provided in Supplementary Figure 12; these show that regardless of whether we estimate  
 129 heritabilities using LDAK (rather than GCTA), whether we use  $\alpha = -1$  (instead of  $\alpha = -0.25$ ) or whether we analyze the UCLEB  
 130 traits, it remains the case that the LDAK Model better predicts the heritability contribution of each tranche than the GCTA Model.

131 **Relationship between heritability and genotype certainty:** The LDAK Model assumes that SNP heritability contributions vary with  
 132 genotype certainty (measured by the information score  $r_j$ ). So far, our analyses have used only very high-quality SNPs ( $r_j > 0.99$ ), so  
 133 this assumption has been redundant. Now we also include lower-quality common SNPs; we focus on the UCLEB traits, as for these we  
 134 were able to test for correlation between genotyping errors and phenotype (Supplementary Fig. 13). Supplementary Table 5 compares  
 135 model fit with and without allowance for genotype certainty; it shows that including  $r_j$  in the heritability model tends to provide a modest  
 136 improvement in model fit, resulting in a higher likelihood for 18 out of 23 traits.

137 **Estimates of  $h^2_{\text{SNP}}$  for the GWAS traits:** Table 1 presents our final estimates of  $h^2_{\text{SNP}}$  for the 19 GWAS traits, obtained using the LDAK  
 138 Model (with  $\alpha = -0.25$ ). For comparison, we include previously-reported estimates of  $h^2_{\text{SNP}}$ , as well as the proportion of phenotypic  
 139 variance explained by SNPs reported as genome-wide significant (see Supplementary Table 6). For the disease traits, estimates are on  
 140 the liability scale, obtained by scaling according to the observed case-control ratio and (assumed) trait prevalence.<sup>28,29</sup> We are unable to  
 141 find previous estimates of  $h^2_{\text{SNP}}$  for tuberculosis or intraocular pressure, indicating that for these two traits, we are the first to establish  
 142 that common SNPs contribute sizable heritability. Extended results are provided in Supplementary Table 7. These show that our final  
 143 estimates of  $h^2_{\text{SNP}}$  are on average 43% (SD 3) and 25% (SD 2) higher than, respectively, those obtained using the original versions (i.e.,  
 144 with  $\alpha = -1$ ) of GCTA<sup>30</sup> and GCTA-LDMS.<sup>15</sup>

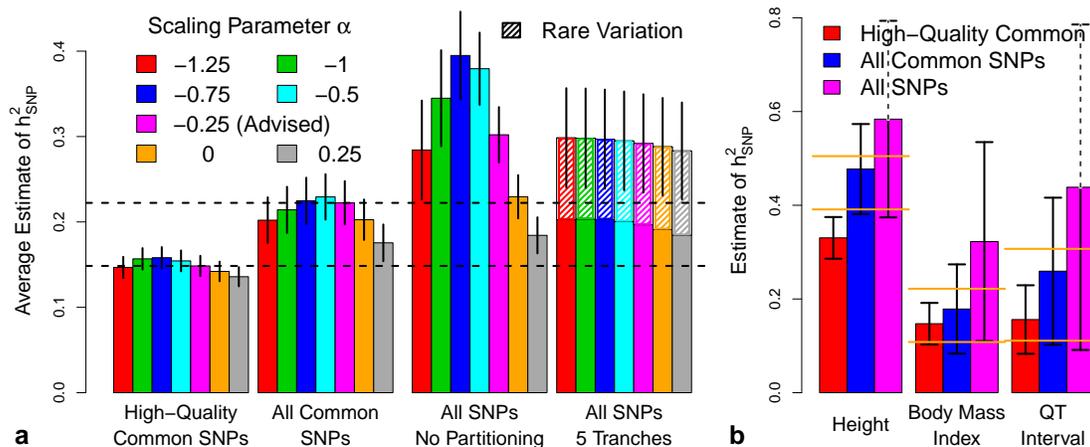


**Figure 5: Enrichment of SNP Classes.** Block 1 reports the contributions to  $h^2_{\text{SNP}}$  of DNaseI hypersensitivity sites (DHS), estimated under the GCTA Model with  $\alpha = -1$  (see main text for explanation of  $\alpha$ ). The vertical lines provide point estimates and 95% confidence intervals for each trait, and for the (inverse variance weighted) average; for 3 of the traits, the point estimate is above 100%, as was also the case for Gusev *et al.*<sup>7</sup> Block 2 repeats this analysis, but now assuming the LDAK Model with  $\alpha = -0.25$ . Blocks 3 & 4 estimate the contribution of “genic SNPs” (those inside or within 2 kb of an exon) and “inter-genic SNPs” (further than 125 kb from an exon), again assuming the LDAK Model with  $\alpha = -0.25$ . To assess enrichment, estimated contributions are compared to those expected under the GCTA or LDAK Model, as appropriate (horizontal lines).

145 **Role of DNaseI hypersensitivity sites (DHS):** Gusev *et al.*<sup>7</sup> used SNP partitioning to assess the contributions of SNP classes defined by  
 146 functional annotations. Across 11 diseases they concluded that the majority of  $h^2_{\text{SNP}}$  was explained by DHS, despite these containing less  
 147 than 20% of all SNPs. For Figure 5, we perform a similar analysis using the 10 traits we have in common with their study (for 9 of these,  
 148 we are using the same data). When we copy Gusev *et al.* and assume the GCTA Model with  $\alpha = -1$ , we estimate that on average DHS  
 149 contribute 86% (SD 4) of  $h^2_{\text{SNP}}$ , close to the value they reported (79%). When instead we assume the LDAK Model (with  $\alpha = -0.25$ ),  
 150 the estimated contribution of DHS reduces to 25% (SD 2). Under the LDAK Model, DHS are predicted to contribute 18% of  $h^2_{\text{SNP}}$  so  
 151 25% represents 1.4-fold enrichment. To add context, we also consider “genic” SNPs, which we define as SNPs inside or within 2 kb of  
 152 an exon (using RefSeq annotations<sup>31</sup>), and “inter-genic,” SNPs further than 125 kb from an exon; these definitions ensure that these two  
 153 SNP classes are also predicted to contribute 18% of  $h^2_{\text{SNP}}$  under the LDAK Model. We estimate that genic SNPs contribute 29% (SD  
 154 2), while inter-genic SNPs contribute 10% (SD 2), representing 1.6-fold and 0.6-fold enrichment, respectively. When we extend this  
 155 analysis to all 42 traits, DHS on average contribute 24% (SD 2) of  $h^2_{\text{SNP}}$ , and in contrast to Gusev *et al.*, enrichment remains constant  
 156 when we reduce SNP density (Supplementary Fig. 14 & 15 and Supplementary Table 8).

157 Finucane *et al.*<sup>32</sup> performed a similar analysis, but considered 52 SNP classes and estimated enrichment using LDSC; across nine  
 158 traits, they identified five classes with >4-fold enrichment, the highest of which, “conserved SNPs,” had 13-fold enrichment. When we  
 159 use LDAK to estimate enrichment for our 19 GWAS traits, the results are more modest; the highest enrichment is 2.5-fold, with only  
 160 1.3-fold enrichment for conserved SNPs (Supplementary Fig. 16).

161 **Relaxing quality control:** For the UCLEB data, we consider nine alternative SNP filterings. Supplementary Figure 17 reports estimates  
 162 of  $h^2_{\text{SNP}}$  for each trait / filtering, while Figure 6a provides a summary. First we vary the information score threshold:  $r_j > 0.99$ ,  $> 0.95$ ,  
 163  $> 0.9$ ,  $> 0.6$ ,  $> 0.3$  and  $> 0$  (each time continuing to require  $\text{MAF} > 0.01$ ). Simulations suggest that by including all 8.8 M common  
 164 SNPs ( $r_j > 0$ ), instead of using just the 353 K high-quality ones ( $r_j > 0.99$ ), we can expect estimates of  $h^2_{\text{SNP}}$  to increase by 50-60%  
 165 (Supplementary Fig. 18). This is similar to what we observe in practice, as across the 23 traits, estimates of  $h^2_{\text{SNP}}$  (using  $\alpha = -0.25$ )  
 166 are on average 45% (SD 8) higher. The simulations further predict that, even though the MetaboChip provides relatively low coverage  
 167 of the genome (after quality control, it contains only 60 K SNPs, predominately within genes), we can expect estimates of  $h^2_{\text{SNP}}$  to be  
 168 approximately 80% as high as those obtained starting from genome-wide genotyping arrays. While we are unable to test this claim  
 169 directly, it is consistent with our results for height, body mass index and QT Interval, the three traits for which reasonably precise



**Figure 6: Varying quality control for the UCLEB traits.** We consider three SNP filterings: 353 K high-quality common SNPs (information score  $> 0.99$ ,  $MAF > 0.01$ ), 8.8 M common SNPs ( $MAF > 0.01$ ) and all 17.3 M SNPs ( $MAF > 0.0005$ ). **(a)** Blocks indicate SNP filtering; bars report (inverse variance weighted) average estimates of  $h^2_{SNP}$  using LDAK (vertical lines provide 95% confidence intervals). Bar color indicates the value of  $\alpha$  used. For Blocks 1, 2 & 3,  $h^2_{SNP}$  is estimated using the non-partitioned model. For Block 4, SNPs are partitioned by MAF; we find this is necessary when rare SNPs are included, and also allows estimation of the contribution of  $MAF < 0.01$  SNPs (hatched areas). **(b)** bars report our final estimates of  $h^2_{SNP}$  for height, body mass index and QT interval, the three traits for which common SNP heritability has been previously estimated with reasonable precision<sup>6</sup> (orange lines mark the 95% confidence intervals from these previous studies). Bar colors now indicate SNP filtering; all estimates are based on  $\alpha = -0.25$ , using either a non-partitioned model (red and blue bars) or with SNPs partitioned by MAF (purple bars).

170 estimates of common SNP  $h^2_{SNP}$  are available<sup>6</sup> (Figure 6b). For the final three SNP filterings, we vary the MAF threshold: MAF  
 171  $> 0.0025$ ,  $MAF > 0.001$  and  $MAF > 0.0005$  (all with  $r_j > 0$ ). Across the 23 traits, we find that rare SNPs contribute substantially to  
 172  $h^2_{SNP}$ : for example, when we use the 17.3 M SNPs with  $MAF > 0.0005$ , estimates of  $h^2_{SNP}$  (using  $\alpha = -0.25$  and MAF partitioning) are  
 173 on average 29% (SD 12) higher than those based on the 8.8 M common SNPs (median increase 22%), with rare SNPs contributing on  
 174 average 33% (SD 5) of  $h^2_{SNP}$  (Figure 6a).

## 175 Discussion

176 With estimates of  $h^2_{SNP}$  so widely reported, it is easy to forget that calculating the variance explained by large numbers of SNPs is  
 177 a challenging problem. To avoid over-fitting, it is necessary to make strong prior assumptions about SNP effect sizes, but different  
 178 assumptions can lead to substantially different estimates of  $h^2_{SNP}$ . Previous attempts to assess the validity of assumptions have used  
 179 simulation studies,<sup>14,15</sup> but this approach will tend to favor assumptions similar to those used to generate the phenotypes. Instead, we  
 180 have compared different heritability models empirically, by examining how well they fit real datasets.

181 We began by investigating the relationship between heritability and MAF. Across 42 traits, we found that best fit was achieved  
 182 by setting  $\alpha = -0.25$  in (1), which implies that average heritability varies with  $[MAF(1-MAF)]^{0.75}$ . As explained in Online Methods,  
 183 the value of  $\alpha$  corresponds to the scaling of genotypes. Therefore, our result indicates that the performance (i.e., detection power and/or  
 184 prediction accuracy) of many penalized and Bayesian regression methods, for example, the Lasso, ridge regression and Bayes A,<sup>33-35</sup>  
 185 could be improved simply by changing how genotypes are scaled. Although we recommend  $\alpha = -0.25$  as the default value, with  
 186 sufficient data available, it should be possible to estimate  $\alpha$  on a trait-by-trait basis, or to investigate more complex relationships between  
 187 heritability and MAF. In particular, with a better understanding of the relationship between heritability and MAF for low frequencies, it  
 188 may no longer be necessary to partition by MAF when rare SNPs are included.

189 We also examined the relationship between heritability and LD. To date, most estimates of  $h^2_{SNP}$  have been based on the GCTA

190 Model; this model can be motivated by a belief that each SNP is expected to have the same effect on the phenotype, from which it  
191 follows that the expected heritability of a region should depend on the number of SNPs it contains. By contrast, the LDK Model views  
192 highly-correlated SNPs as tagging the same underlying variant, and therefore believes that the expected heritability of a region should  
193 vary according to the total amount of distinct genetic variation it contains. Across our traits, we found that the relationship between  
194 heritability and LD specified by the LDK Model consistently provides a better description of reality.

195 This finding has important consequences for complex trait genetics. Firstly, it implies that for many traits, common SNPs  
196 explain considerably more phenotypic variance than previously reported, which represents a significant advance in the search for missing  
197 heritability.<sup>2</sup> It also impacts on a large number of closely-related methods. For example, LDSC,<sup>10</sup> like GCTA, assumes that heritability  
198 contributions are independent of LD and therefore it also tends to under-estimate  $h_{\text{SNP}}^2$ . Similarly, we have shown that estimates of  
199 the relative importance of SNP classes via SNP partitioning can be misleading when the GCTA Model is assumed.<sup>7,32</sup> Further afield,  
200 most software for mixed model association analyses (e.g., FAST-LMM, GEMMA and MLM-LOCO) use an extension of the GCTA  
201 Model,<sup>36–38</sup> and likewise most bivariate analyses, including those performed by LDSC.<sup>8,39,40</sup> It remains to be seen how much these  
202 methods would be affected if they employed more realistic heritability models.

203 Attempts have been made to improve the accuracy of heritability models via SNP partitioning.<sup>14,15,41</sup> We find that partitioning by  
204 MAF can be advantageous, as it guards against misspecification of the relationship between heritability and MAF when rare variants are  
205 included. Figure 3a and Supplementary Figure 7 indicate that the realism of the GCTA Model can be improved by partitioning based on  
206 LD; for example, across the GWAS traits, estimates from GCTA-LDMS are on average 16% (SD 2) higher than those from GCTA, and  
207 now only 23% (SD 2) lower than those from LDK. The improvement arises because model misspecification is reduced by allowing  
208 SNPs in lower-LD tranches to have higher average heritability. However, Supplementary Table 9 illustrates why we consider such an  
209 approach sub-optimal; in particular, SNP partitioning can be computationally expensive, and even with LD-partitioning, model fit tends  
210 to be worse than that from LDK.

211 While we have investigated the role of MAF, LD and genotype certainty, there remain other factors on which heritability could  
212 depend, in particular the available functional annotations of genomes.<sup>42</sup> For example, our comparison of genic and inter-genic SNPs  
213 indicates that the effect-size prior distribution could be improved by taking into account proximity to coding regions. By way of demon-  
214 stration, Supplementary Table 10 shows that model fit is improved by assuming  $\mathbb{E}[h_j^2] \propto [f_j(1 - f_j)]^{1+\alpha} \times w_j \times r_j \times \exp(-\frac{(D_j+50)}{500})$ ,  
215 where  $D_j$  is the distance (in kb) between SNP  $j$  and the nearest exon (under this model, genic SNPs are expected to have about twice  
216 the heritability of inter-genic SNPs). In general, we believe that modifications of this type will have a relatively small impact; we note  
217 that across the 19 GWAS traits, scaling by  $\exp(-\frac{(D_j+50)}{500})$  increases model log likelihood by on average only 1.5, much less than the  
218 average increase obtained by using  $\alpha = -0.25$  instead of  $\alpha = -1$  (8.9), or by choosing the LD-model specified by LDK instead of  
219 GCTA (17.7), and does not significantly change estimates of  $h_{\text{SNP}}^2$ . However, with sufficient data, it may be possible to obtain more  
220 substantial improvement by tailoring model assumptions to individual traits.

221 When estimating  $h_{\text{SNP}}^2$ , care should be taken to avoid possible sources of confounding. Previously, we advocated a test for  
222 inflation of  $h_{\text{SNP}}^2$  due to population structure and familial relatedness.<sup>3</sup> The conclusions of a recent paper claiming that  $h_{\text{SNP}}^2$  estimates  
223 are unreliable,<sup>43</sup> would have changed substantially had this test been applied (Supplementary Fig. 19). We also recommend testing for  
224 inflation due to genotyping errors, particularly before including lower-quality and/or rare SNPs. For the 23 UCLEB traits, we showed  
225 that including poorly-imputed SNPs resulted in significantly higher estimates of  $h_{\text{SNP}}^2$ , and made it possible to capture the majority  
226 of genome-wide heritability despite the very sparse genotyping provided by the Metachip. We found that including rare SNPs also  
227 led to significantly higher  $h_{\text{SNP}}^2$ . Although sample size prevented us from obtaining precise estimates of  $h_{\text{SNP}}^2$  for individual traits, our  
228 analyses indicated that for larger datasets, including rare SNPs will be both practical and fruitful in the search for the remaining missing  
229 heritability.<sup>2</sup>

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## 242 Author Contributions

243 DS and NC performed the analyses. DS and DJB wrote the manuscript with assistance from NC, MRJ, SN and members of the UCLEB  
 244 Consortium.

## 245 Online Methods

246 Supplementary Note 1 summarizes the different analyses we performed, and the conclusions we drew from each, while Supplementary  
 247 Protocol 1 provides step-by-step instructions for estimating  $h_{\text{SNP}}^2$  starting from raw genotype data. In general, we assume there are  $n$   
 248 individuals, recorded for  $p$  covariates and genotyped (either directly or via imputation) for  $m$  SNPs: the length- $n$  vector  $\mathbf{Y}$  contains  
 249 phenotypic values, the  $n \times p$  matrix  $\mathbf{Z}$  contains covariates, while the  $n \times m$  matrix  $\mathbf{S}$  contains (expected) allele counts.

**Information score  $r_j$ :** Let the vector  $\mathbf{S}_j = (S_{1,j}, \dots, S_{n,j})^T \in [0, 2]^n$ , denote the allele counts for SNP  $j$  (i.e.,  $S_j$  is Column  $j$  of  $\mathbf{S}$ ).  
 Our information score  $r_j$  estimates the squared correlation between  $\mathbf{S}_j$  and  $\mathbf{G}_j = (G_{1,j}, \dots, G_{n,j})^T \in \{0, 1, 2\}^n$ , the true genotypes  
 for SNP  $j$ . When using imputed data,  $\mathbf{G}_j$  is typically not known; instead for each individual we have a triplet of state probabilities  
 $(p_{i,j,0}, p_{i,j,1}, p_{i,j,2})$ , where  $p_{i,j,g} = \mathbb{P}(G_{i,j} = g)$  and  $p_{i,j,0} + p_{i,j,1} + p_{i,j,2} = 1$ . Therefore, we define  $r_j$  by taking expectations over the  
 $3^n$  possible realizations of  $\mathbf{G}_j$ .

$$r_j = \frac{\mathbb{E}[\sum_{i=1}^n (S_{i,j} - \bar{S}_j)(G_{i,j} - \bar{G}_j)]^2}{(\sum_{i=1}^n (S_{i,j} - \bar{S}_j)^2) \mathbb{E}[\sum_{i=1}^n (G_{i,j} - \bar{G}_j)^2]}, \quad \text{where } \bar{S}_j = \frac{1}{n} \sum_{i=1}^n S_{i,j} \quad \text{and} \quad \bar{G}_j = \frac{1}{n} \sum_{i=1}^n G_{i,j}.$$

250  $S_j$  is known, so computing  $\sum_{i=1}^n (S_{i,j} - \bar{S}_j)^2$  is straightforward. The two expectations can also be calculated explicitly:

$$\begin{aligned} 251 \quad \mathbb{E}[\sum_{i=1}^n (S_{i,j} - \bar{S}_j)(G_{i,j} - \bar{G}_j)] &= \sum_i (S_j - \bar{S}_j) \mathbb{E}[G_j - \mu] = \sum_i (S_j - \bar{S}_j) (p_{i,1} + 2p_{i,2} - \mu) \\ 252 \quad \mathbb{E}[\sum_i (G_j - \bar{G}_j)^2] &= \sum_i \mathbb{E}[(G_j - \mu)^2] = \sum_i [p_{i,j,0}(-\mu)^2 + p_{i,j,1}(1 - \mu)^2 + p_{i,j,2}(2 - \mu)^2], \end{aligned}$$

253 where  $\mu = \mathbb{E}[\bar{G}_j] = \frac{1}{n} \sum_i (p_{i,j,1} + 2p_{i,j,2})$ . For our analyses, we use expected allele counts (dosages), so  $S_{i,j} = p_{i,j,1} + 2p_{i,j,2}$ . In this  
 254 case,  $\mathbb{E}[\sum_{i=1}^n (S_{i,j} - \bar{S}_j)(G_{i,j} - \bar{G}_j)] = \sum_{i=1}^n (S_{i,j} - \bar{S}_j)^2$  and so the score reduces to  $r_j = \sum_{i=1}^n (S_{i,j} - \bar{S}_j)^2 / \sum_{i=1}^n (G_{i,j} - \bar{G}_j)^2$ .  
 255 For a directly genotyped SNP, each triplet of state probabilities will be (1,0,0), (0,1,0) or (0,0,1), which will result in  $S_{i,j} = G_{i,j}$  for all  
 256  $i$  and  $r_j = 1$ ; so for these, in place of  $r_j$ , we use the metric `r2_type0` reported by IMPUTE2.<sup>26</sup> Additional details on our information  
 257 score are provided in Supplementary Figure 20.

258 **Estimating  $h_{\text{SNP}}^2$ :** We first construct the  $n \times m$  genotype matrix  $\mathbf{X}$ , by centering and scaling the allele counts for each SNP according  
 259 to  $X_{i,j} = (S_{i,j} - 2f_j) \times [2f_j(1 - f_j)]^{\alpha/2}$ , where  $f_j = \sum_i S_{i,j} / 2n$ . If  $w_j$  and  $r_j$  denote the LD weight<sup>9</sup> and information score for SNP  $j$ ,

260 then the LDAK Model for estimating SNP heritability  $h_{\text{SNP}}^2 = \sigma_g^2 / (\sigma_g^2 + \sigma_e^2)$  is:

$$Y_i = \sum_{k=1}^p \theta_k Z_{i,k} + \sum_{j=1}^m \beta_j X_{i,j} + e_i, \quad \text{with } \beta_j \sim \mathbb{N}(0, r_j w_j \sigma_g^2 / W), \quad e_i \sim \mathbb{N}(0, \sigma_e^2) \quad \text{and} \quad W = \sum_{j=1}^m r_j w_j [2f_j(1-f_j)]^{1-\alpha}. \quad (2)$$

261  $\theta_k$  denotes the fixed-effect coefficient for the  $k$ th covariate,  $\beta_j$  and  $e_i$  are random-effects indicating the effect size of SNP  $j$  and the  
 262 noise component for Individual  $i$ , while  $\sigma_g^2$  and  $\sigma_e^2$  are interpreted as genetic and environmental variances, respectively. Note that the  
 263 introduction of  $r_j$  is an addition to the model we proposed in 2012.<sup>9</sup> Model (2) is equivalent to assuming:<sup>45,46</sup>

$$Y \sim \mathbb{N}(Z\theta, K\sigma_g^2 + I\sigma_e^2), \quad \text{with } K = \frac{X\Omega X^T}{W}, \quad (3)$$

264 where  $I$  is an  $n \times n$  identity matrix and  $\Omega$  denotes a diagonal matrix with diagonal entries  $(r_1 w_1, \dots, r_m w_m)$ . The kinship matrix  $K$ ,  
 265 also referred to as a genetic relationship matrix (GRM)<sup>1</sup> or genomic similarity matrix (GSM),<sup>47</sup> consists of average allelic correlations  
 266 across the SNPs (adjusted for LD and genotype certainty). Model (3) is typically solved using REstricted Maximum Likelihood (REML),  
 267 which returns estimates of  $\theta_1, \dots, \theta_p, \sigma_g^2$  and  $\sigma_e^2$ .<sup>12</sup>

268 The heritability of SNP  $j$  can be estimated by  $h_j^2 = \beta_j^2 \text{Var}(X_j) / \text{Var}(Y)$ , which under Model (2), and assuming Hardy-Weinberg  
 269 Equilibrium,<sup>48,49</sup> has expectation

$$\mathbb{E}[h_j^2] = \frac{\mathbb{E}[\beta_j^2] \times \text{Var}(X_j)}{\text{Var}(Y)} = \frac{r_j w_j \sigma_g^2 / W \times [2f_j(1-f_j)]^{1+\alpha}}{\text{Var}(Y)}. \quad (4)$$

270 If  $P_1$  and  $P_2$  index two sets of SNPs of size  $|P_1|$  and  $|P_2|$ , then under the LDAK Model, they are expected to contribute heritability in  
 271 the ratio  $W_1 : W_2$ , where  $W_l = \sum_{j \in P_l} r_j w_j [2f_j(1-f_j)]^{1-\alpha}$ . The GCTA Model corresponds to setting  $w_j = r_j = 1$ , in which case  
 272  $W_l = \sum_{j \in P_l} [2f_j(1-f_j)]^{1-\alpha}$ . Most applications of GCTA have further assumed  $\alpha = -1$ , so that  $W_l = |P_l|$ , which corresponds to the  
 273 assumption that SNP sets are expected to contribute heritability proportional to the number of SNPs they contain.

274 Model (2) assumes that all effect-sizes can be described by a single prior distribution. This assumption is relaxed by SNP  
 275 partitioning. Suppose that the SNPs are divided into tranches  $P_1, \dots, P_L$  of sizes  $|P_1|, \dots, |P_L|$ ; typically these will partition the  
 276 genome, so that each SNP appears in exactly one tranche and  $\sum_l |P_l| = m$ , but this is not required. This correspond to generalizing  
 277 Model (2), so that SNPs in Tranche  $l$  have effect-size prior distribution  $\beta_j \sim \mathbb{N}(0, r_j w_j \sigma_l^2 / W_l)$ . Letting  $\Sigma = \sigma_1^2 + \dots + \sigma_L^2$ , then  
 278  $h_{\text{SNP}}^2 = \Sigma / (\Sigma + \sigma_e^2)$ , while  $\sigma_l^2 / \Sigma$  represents the contribution to  $h_{\text{SNP}}^2$  of SNPs in Tranche  $l$ . This model can equivalently be expressed  
 279 as  $Y \sim \mathbb{N}(Z\theta, K_1\sigma_1^2 + \dots + K_L\sigma_L^2 + I\sigma_e^2)$ , where  $K_l$  represents allele correlations across the SNPs in Tranche  $l$ .

280 For analyses under the LDAK Model, we used LDAK v.5; for analyses under the GCTA Model, we used GCTA v.1.26. For about  
 281 a third of GCTA-LDMS analyses, the GCTA REML solver failed with the error ‘‘information matrix is not invertible,’’ in which case  
 282 we rerun using LDAK (while the GCTA and LDAK solvers are both based on Average Information REML,<sup>30,50</sup> subtle differences mean  
 283 that when using a large number of tranches, one might complete while the other fails). For the few occasions when both solvers failed,  
 284 we instead used ‘‘GCTA-LD’’ (i.e., SNPs divided only by LD, rather than by LD and MAF), which we found gave very similar results  
 285 to GCTA-LDMS for traits where both completed (Supplementary Fig. 7). For diseases, we converted estimates of  $h_{\text{SNP}}^2$  to the liability  
 286 scale based on the observed case-control ratio and assumed prevalence.<sup>28,29</sup> In general, we copied the prevalences used by previous  
 287 studies; however for tuberculosis, where no previous estimate of  $h_{\text{SNP}}^2$  is available, we derived an estimate of prevalence from World  
 288 Health Organization data<sup>51</sup> (Supplementary Note 2).

289 **LDSC:** Originally designed as a way to quantify confounding in a GWAS, LDSC<sup>10</sup> also provides a method for estimating  $h_{\text{SNP}}^2$ , which  
 290 requires only summary statistics from single-SNP analysis (rather than raw genotype and phenotype data). LDSC is based on the  
 291 principal that in a single-SNP analysis, the  $\chi^2(1)$  test statistic for SNP  $j$  has expected value  $\mathbb{E}[\chi_j^2] = 1 + nh_j^2 + n \sum_{k \neq j} r_{j,k}^2 h_k^2 +$   
 292  $na_j$ , where  $r_{j,k}^2$  denotes the squared correlation between SNPs  $j$  and  $k$ , while  $a_j$  represents bias due to confounding factors (e.g.,  
 293 population structure and familial relatedness).<sup>10</sup> Under a polygenic model where every SNP is expected to contribute equally (i.e.,  
 294  $\mathbb{E}[h_j^2] = h_{\text{SNP}}^2 / m$ ), and the (widely-used) assumption that the bias is constant across SNPs ( $a_j = a$ ), we have  $\mathbb{E}[\chi_j^2] = 1 + nl_j h_{\text{SNP}}^2 / m +$

295  $na$ , where  $l_j = 1 + \sum_{k \neq j} r_{j,k}^2$  is referred to as the LD Score of SNP  $j$  (as it is not feasible to compute pairwise correlations across  
296 all SNPs, in practice these are approximated using a sliding window of, say, 1 centiMorgan). Therefore, LDSC estimates  $h_{\text{SNP}}^2$  and  $a$   
297 by regressing test statistics on LD Scores. In the absence of confounding ( $a = 0$ ), LDSC can be viewed as estimating  $h_{\text{SNP}}^2$  under  
298 the GCTA Model with  $\alpha = -1$  (as this satisfies the assumption that every SNP is expected to contribute equal heritability). As the  
299 authors of LDSC point out,<sup>10</sup> it is straightforward to accommodate alternative relationships between  $\mathbb{E}[h_j^2]$  and MAF (i.e.,  $\alpha \neq -1$ ) by  
300 changing how genotypes are scaled when computing LD Scores, and potentially genotype certainty could be accommodated. However,  
301 the similarity with the GCTA Model appears intrinsic to LDSC; while the assumption that heritability is independent of LD can be  
302 relaxed via SNP partitioning,<sup>41</sup> we can not envisage how the method could be modified to accommodate the LDAK SNP weights. For  
303 LDSC analyses, we used LDSC v.1.0.0 both for calculating LD Scores and estimating  $h_{\text{SNP}}^2$ .

304 **Accommodating very large effect loci:** Equation (2) assumes that all SNP effect sizes can be modeled by a single Gaussian distribution.  
305 Estimates are generally robust to violations of this assumption,<sup>9</sup> but problems can occur when individual SNPs have very large effect  
306 sizes, because a single Gaussian distribution cannot accommodate both these SNPs and the very many with small effect sizes. This is a  
307 common concern when analyzing autoimmune traits for which the major histocompatibility complex (MHC) can contribute substantial  
308 heritability. In response to this problem, some authors exclude MHC SNPs from analyses.<sup>7,30,52,53</sup> Another approach is to model effect  
309 sizes as a mixture of Gaussians,<sup>35,54</sup> but this is not computationally feasible for millions of SNPs and many thousands of individuals.  
310 Therefore, our proposed strategy is to first identify SNPs with  $P < 10^{-20}$  from single-SNP analysis, to prune these using a correlation  
311 squared threshold of 0.5, then to include those which remain as fixed-effect covariates. Thus in place of Equation (3), we assume  
312  $\mathbf{Y} \sim \mathcal{N}(\mathbf{Z}\boldsymbol{\theta} + \mathbf{T}\boldsymbol{\phi}, \mathbf{K}\sigma_g^2 + \mathbf{I}\sigma_e^2)$ , where columns of the matrix  $\mathbf{T}$  contain allele counts of the highly-associated SNPs (i.e.,  $\mathbf{T}$  is a  
313 submatrix of  $\mathbf{S}$ ), and the vector  $\boldsymbol{\phi}$  represents their effect sizes. In contrast to standard (non-SNP) covariates, the variance explained by  
314  $\mathbf{T}$  counts towards SNP heritability:  $h_{\text{SNP}}^2 = (\sigma_g^2 + \sigma_T^2) / (\sigma_g^2 + \sigma_T^2 + \sigma_e^2)$ , where  $\sigma_T^2 = (\mathbf{T}\boldsymbol{\phi})^T(\mathbf{T}\boldsymbol{\phi})$ . Supplementary Figures 21 & 22  
315 provides further details. In particular, we appreciate that our definition of highly-associated is somewhat arbitrary, so we confirm that  
316 estimates of  $h_{\text{SNP}}^2$  are almost unchanged if instead we use  $P < 5 \times 10^{-8}$ .

317 **Datasets and phenotypes:** When searching for GWAS datasets, we preferred those with sample size at least 4 000 to ensure reasonable  
318 precision of  $h_{\text{SNP}}^2$ .<sup>55</sup> In total, our datasets were constructed from 40 independent cohorts, all of which have been previously described  
319 (see Supplementary Tables 11 & 12 for references and details of how cohorts were merged to form datasets). For the UCLEB data, there  
320 were in total 28 quantitative traits with measurements recorded for at 7 000 individuals. For each of these, we quantile normalized, then  
321 applied a test for inflation due to genotyping errors (Supplementary Fig. 13). Specifically, our test, inspired by Bhatia *et al.*<sup>56</sup> and valid  
322 for quantitative phenotypes where individuals are recruited from multiple cohorts, first estimates  $h_{\text{SNP}}^2$  using only pairs of individuals  
323 in different cohorts, then using only pairs of individuals in the same cohort; a significant difference between the two estimates indicates  
324 possible inflation due to genotyping errors. We excluded five traits that showed evidence of inflation ( $P < 0.05/28$ ), leaving us with 23:  
325 height, weight, body mass index, waist circumference, forced vital capacity, one second forced vital capacity, systolic blood pressure  
326 (adjusted), diastolic blood pressure (adjusted), PR Interval, QT Interval, Corrected QT Interval, QRS Voltage Product, Sokolow Lyon,  
327 glucose, insulin, total cholesterol (adjusted), LDL cholesterol (adjusted), triglyceride (adjusted), viscosity, fibrinogen, Interleukin 6,  
328 C-reactive Protein and haemoglobin. Approximately 40% of individuals were receiving medication to reduce blood pressure, 25% to  
329 reduce lipid levels, so where indicated, phenotypes had been adjusted for this: for individuals on medication, their raw measurements  
330 had been increased either by adding on (blood pressure) or scaling by (lipid levels) a constant.<sup>57,58</sup> We note that some pairs of traits are  
331 highly correlated. However, as the overall correlation is not that extreme (we estimate the effective number of independent traits to be  
332 about 15), and most of our UCLEB analyses serve to support conclusions drawn from the GWAS traits, we decide to retain all 23 traits  
333 (rather than, say, consider only a subset). See Supplementary Note 3 for further details on phenotyping.

334 **Quality control:** We processed each of the 40 cohorts in identical fashion; see Supplementary Note 4 for full details. In summary,  
335 after excluding apparent population outliers, samples with extreme missingness or heterozygosity, and SNPs with MAF  $< 0.01$ , call-  
336 rate  $< 0.95$  or  $P < 10^{-6}$  from a test for Hardy-Weinberg Equilibrium, we imputed using IMPUTE2<sup>26</sup> and the 1000 Genome Phase 3  
337 (2014) Reference Panel.<sup>27</sup> When merging cohorts to construct the GWAS datasets, we retained only autosomal SNPs which in all cohorts  
338 have MAF  $> 0.01$  and  $r_j > 0.99$  (using IMPUTE2 `r2_type2` in place of  $r_j$  for directly genotyped SNPs). For the 8 UCLEB cohorts, we

339 applied these filters only after merging. We only relax quality control for the analyses of the UCLEB data where we explicitly examine  
340 the consequences of including lower-quality and rare SNPs. When possible, the matrix  $S$  contains expected allele counts (dosages); i.e.,  
341  $S_{i,j} = p_{i,j,1} + 2 \times p_{i,j,2}$ , where  $p_{i,j,1}$  and  $p_{i,j,2}$  denote the probabilities of allele counts 1 and 2, respectively. If hard genotypes are  
342 required, for example when using LDSC to compute LD Scores,<sup>10</sup> we round  $S_{i,j}$  to the nearest integer. As this was only necessary when  
343 considering high-quality SNPs ( $r_j > 0.99$ ), we expect this rounding to have negligible impact on results. For each trait, Table 1 reports  
344  $m$ , the total number of SNPs after imputation, and  $\sum_{j=1}^m w_j$ , the sum of SNP weights; the aim of these weights is to remove duplication  
345 of signal due to LD and their sum can loosely be interpreted as an effective number of independent SNPs. For the GWAS datasets,  $\sum w_j$   
346 ranges from 79 K to 125 K. By contrast, when restricted to only high-quality SNPs, the UCLEB data has  $\sum w_j = 39$  K, reflecting that  
347 the Metabochip directly captures a much smaller amount of genetic variation than standard genome-wide SNP arrays.

348 When analyzing quantitative traits, genotyping errors will tend only to be a concern when there are systematic differences between  
349 phenotypes across cohorts, and this is something we are able to explicitly test (Supplementary Fig. 13). However, for disease traits, when  
350 cases and controls have been genotyped separately (as is the design of most of our GWAS datasets), any errors will almost certainly  
351 correlate with phenotype and therefore cause inflation of  $h_{\text{SNP}}^2$ .<sup>9,29</sup> To test the effectiveness of our quality control for the GWAS traits,  
352 we construct a pseudo case-control study using two control cohorts; we confirm that the resulting estimate of  $h_{\text{SNP}}^2$  is not significantly  
353 greater than zero, suggesting that the quality control steps we use for the GWAS datasets are sufficiently strict (Supplementary Note 5).

354 Accurate estimation of  $h_{\text{SNP}}^2$  requires samples of unrelated individuals with similar ancestry. Prior to imputation, we removed  
355 ethnic outliers identified through principal component analyses (Supplementary Fig. 23). Post imputation, we computed (unweighted)  
356 allelic correlations using a pruned set of SNPs, then filtered individuals so that no pair remained with correlation greater than  $c$ , where  
357  $-c$  is the smallest observed pairwise correlation ( $c$  ranges from 0.029 to 0.038, depending on dataset). For our datasets, this filtering  
358 excluded relatively few individuals (on average 3.8%, with maximum 11.6%). For all analyses, we include a minimum of 30 covariates:  
359 the top 20 eigenvectors from the allelic correlation matrix just described, and projections onto the top 10 principal components computed  
360 from 1000 Genomes samples.<sup>27</sup> For the 19 GWAS traits, we also include sex as a covariate, while for intraocular pressure and wide range  
361 achievement test scores, we additionally include age. Supplementary Figure 24 reports the proportion of phenotypic variance explained  
362 by each covariate. To check our filtering and covariate choices, we estimate the inflation of  $h_{\text{SNP}}^2$  due to population structure and residual  
363 relatedness<sup>3</sup> (Supplementary Fig. 19). For the GWAS traits, we estimate that on average  $h_{\text{SNP}}^2$  estimates are inflated by at most 3.1%,  
364 with the highest observed for ischaemic stroke (7.1%). For the 23 UCLEB traits, the average inflation is 0.3% (highest 2.3%).

365 **Single-SNP analysis:** Supplementary Figure 25 provides Manhattan Plots from logistic (case-control traits) and linear regression (quan-  
366 titative traits), performed using PLINK v.1.9. These analyses provide the summary statistics required by LDSC. For the GWAS traits,  
367 we identified highly-associated SNPs ( $P < 10^{-20}$ ) within the MHC for 6 of the GWAS traits (rheumatoid arthritis, type 1 diabetes,  
368 psoriasis, ulcerative colitis, celiac disease and multiple sclerosis), while rs2476601, a SNP within *PTPN22*, is highly associated with  
369 both rheumatoid arthritis and type 1 diabetes.<sup>59,60</sup> For the UCLEB traits, we find highly associated SNPs within *SCN10A* (PR Interval),  
370 *APOE* (total cholesterol, LDL cholesterol and C-reactive protein) and *ZPPI* (triglyceride levels). For heritability analysis, these SNPs  
371 were pruned, then included as additional fixed-effect covariates as described above.

372 **Computational requirements:** The most time-consuming aspect of analysis was genotype imputation; for a typically-sized cohort  
373 ( $\sim 3000$  individuals) this took approximately one CPU-year (i.e., a few days on a 100-node cluster). Next is computation of SNP  
374 weights, which for the GWAS traits ( $\sim 4$  M SNPs) took approximately one CPU-month (again, this can be near-perfectly parallelized).  
375 Finally, solving the mixed-model via REML would take between a few minutes for the smaller traits ( $\sim 5000$  individuals) and a few hours  
376 for the largest ( $\sim 14000$  individuals). Memory-wise, the most onerous task is solving the mixed-model, for which memory demands  
377 scale with  $n^2$ ; however, even for the largest dataset, this was less than 5 Gb (when using multiple kinship matrices, LDAK allows for  
378 these to be read on-the-fly, so that the memory demands are no higher than when using only one).

- 380 1. Yang, J. *et al.* Common SNPs explain a large proportion of the heritability for human height. *Nat. Genet.* **42**, 565–569 (2010).
- 379 2. Maher, B. Personal genomes: the case of the missing heritability. *Nature* **456**, 18–21 (2008).
- 381 3. Speed, D. *et al.* Describing the genetic architecture of epilepsy through heritability analysis. *Brain* **137**, 26802689 (2014).
- 382 4. Henderson, C., Kempthorne, O., Searle, S. & von Krosigk, C. The estimation of environmental and genetic trends from records  
383 subject to culling. *Biometrics* **15**, 192–218 (1959).
- 384 5. Falconer, D. & Mackay, T. *Introduction to Quantitative Genetics (4th Edition)* (Longman, 1996).
- 385 6. Yang, J. *et al.* Genomic partitioning of genetic variation for complex traits using common SNPs. *Nat. Genet.* **43**, 519–525 (2011).
- 386 7. Gusev, A. *et al.* Partitioning Heritability of Regulatory and Cell-Type-Specific Variants across 11 Common Diseases. *Am. J. Hum.*  
387 *Genet.* **95**, 535–552 (2014).
- 388 8. Lee, S., Yang, J., Goddard, M., Visscher, P. & Wray, N. Estimation of pleiotropy between complex diseases using SNP-derived  
389 genomic relationships and restricted maximum likelihood. *Bioinformatics* **28**, 2540–2542 (2012).
- 390 9. Speed, D., Hemani, G., Johnson, M. & Balding, D. Improved heritability estimation from genome-wide SNP data. *Am. J. Hum.*  
391 *Genet.* **91**, 1011–1021 (2012).
- 392 10. Bulik-Sullivan, B. *et al.* LD score regression distinguishes confounding from polygenicity in genome-wide association studies. *Nat.*  
393 *Genet.* **47**, 291–295 (2014).
- 394 11. Bulik-Sullivan, B. Relationship between LD Score and Haseman-Elston Regression (2015). Preprint available on BioRxiv.
- 395 12. Corbeil, R. & Searle, S. Restricted maximum likelihood (REML) estimation of variance components in the mixed model. *Techno-*  
396 *metrics* **18**, 31–38 (1976).
- 397 13. Golan, D., Lander, E. & Rosset, S. Measuring missing heritability: Inferring the contribution of common variants. *PNAS* **111**,  
398 E5272E5281 (2014).
- 399 14. Lee, S. *et al.* Estimation of SNP-heritability from dense genotype data. *Am. J. Hum. Genet.* **93**, 1151–1155 (2013).
- 400 15. Yang, J. *et al.* Genetic variance estimation with imputed variants finds negligible missing heritability for human height and body  
401 mass index. *Nat. Genet.* **47**, 1114–1120 (2015).
- 402 16. Chen, G. *et al.* Estimation and partitioning of (co)heritability of inflammatory bowel disease from GWAS and immunochip data.  
403 *Hum. Mol. Genet.* **23**, 4710–4720 (2014).
- 404 17. Ek, W. *et al.* Germline genetic contributions to risk for esophageal adenocarcinoma, Barretts Esophagus, and gastroesophageal  
405 reflux. *J. Natl. Cancer Inst.* **105**, 1711–1718 (2013).
- 406 18. Bevan, S. *et al.* Genetic heritability of ischemic stroke and the contribution of previously reported candidate gene and genomewide  
407 associations. *Stroke* **43**, 3161–3167 (2012).
- 408 19. Keller, M. *et al.* Using genome-wide complex trait analysis to quantify 'missing heritability' in parkinson's disease. *Hum. Mol.*  
409 *Genet.* **21**, 4996–5009 (2012).
- 410 20. Yin, X. *et al.* Common variants explain a large fraction of the variability in the liability to psoriasis in a han chinese population.  
411 *BMC Genomics* **15** (2014).
- 412 21. Lee, S. *et al.* Estimating the proportion of variation in susceptibility to schizophrenia captured by common SNPs. *Nat. Genet.* **44**,  
413 247–250 (2012).
- 414 22. Stahl, E. *et al.* Bayesian inference of the polygenic architecture of rheumatoid arthritis. *Nat. Genet.* **44**, 483–489 (2012).
- 415 23. Robinson, E. *et al.* The genetic architecture of pediatric cognitive abilities in the Philadelphia Neurodevelopmental Cohort. *Mol.*  
416 *Psychiatry* **20**, 454–458 (2015).
- 417

- 418 24. Shah, T. *et al.* Population genomics of cardiometabolic traits: Design of the University College London-London School of Hygiene  
419 and Tropical Medicine-Edinburgh-Bristol (UCLEB) Consortium. *PLoS One* **8**, e71345 (2013).
- 420 25. Voight, B. *et al.* The Metabochip, a custom genotyping array for genetic studies of metabolic, cardiovascular, and anthropometric  
421 traits. *PLoS Genet.* **8**, e1002793 (2012).
- 422 26. Howie, B., Marchini, J. & Stephens, M. Genotype imputation with thousands of genomes. *G3* **1**, 457–470 (2011).
- 423 27. The 1000 Genomes Project Consortium. A map of human genome variation from population-scale sequencing. *Nature* **467**, 1061–  
424 1073 (2010).
- 425 28. Dempster, E. & Lerner, I. Heritability of threshold characters. *Genetics* **35**, 212–236 (1950).
- 426 29. Lee, S., Wray, N., Goddard, M. & Visscher, P. Estimating missing heritability for disease from genome-wide association studies.  
427 *Am. J. Hum. Genet.* **88**, 294–305 (2011).
- 428 30. Lee, J. Y. S., Goddard, M. & Visscher, P. GCTA: a tool for genome-wide complex trait analysis. *Am. J. Hum. Genet.* **88**, 76–82  
429 (2011).
- 430 31. Bethesda (MD): National Library of Medicine (US), N. C. f. B. I. The ncbi handbook [internet] (2002).
- 431 32. Finucane, H. *et al.* Partitioning heritability by functional annotation using genome-wide association summary statistics. *Nat. Genet.*  
432 **47**, 1228–1235 (2015).
- 433 33. Hastie, T., Tibshirani, R. & Friedman, J. *The Elements of Statistical Learning* (Springer, 2001).
- 434 34. Habier, D., Fernando, R., Kizilkaya, K. & Garrick, D. Extension of the Bayesian alphabet for genomic selection. *BMC Bioinform-*  
435 *atics* **186**, 186–197 (2011).
- 436 35. Moser, G. *et al.* Simultaneous discovery, estimation and prediction analysis of complex traits using a bayesian mixture model. *PLoS*  
437 *Genet.* **11**, e1004969 (2015).
- 438 36. Lippert, C. *et al.* FaST linear mixed models for genome-wide association studies. *Nat. Methods* **8**, 833–835 (2011).
- 439 37. Zhou, X. & Stephens, M. Genome-wide efficient mixed-model analysis for association studies. *Nat. Genet.* **44**, 821–824 (2012).
- 440 38. Yang, J., Zaitlen, N., Goddard, M., Visscher, P. & Price, A. Advantages and pitfalls in the application of mixed-model association  
441 methods. *Nat. Genet.* **46**, 100–106 (2014).
- 442 39. Cross-Disorder Group of the Psychiatric Genomics Consortium. Genetic relationship between five psychiatric disorders estimated  
443 from genome-wide SNPs. *Nat. Genet.* **45**, 984–994 (2013).
- 444 40. Bulik-Sullivan, B. *et al.* An atlas of genetic correlations across human diseases and traits. *Nat. Genet.* **47**, 1236–1241 (2015).
- 445 41. Gazal, S. *et al.* Linkage disequilibrium dependent architecture of human complex traits reveals action of negative selection (2016).  
446 Preprint available on BioRxiv.
- 447 42. The ENCODE Project Consortium. An integrated encyclopedia of dna elements in the human genome. *Nature* **489**, 57–74 (2012).
- 448 43. Kumar, S., Feldman, M., Rehkopf, D. & Tuljapurkar, S. Limitations of GCTA as a solution to the missing heritability problem.  
449 *PNAS* **113**, E61E70 (2015).
- 450 44. Molloy, A. *et al.* A common polymorphism in HIBCH influences methylmalonic acid concentrations in blood independently of  
451 cobalamin. *Am. J. Hum. Genet.* **5**, 869–882 (2016).
- 452 45. Hayes, B., Visscher, P. & Goddard, M. Increased accuracy of artificial selection by using the realized relationship matrix. *Genet.*  
453 *Res.* **91**, 47–60 (2009).

- 454 46. Habier, D., Fernando, R. & Dekkers, J. The impact of genetic relationship information on genome-assisted breeding values. *Genetics*  
455 **177**, 2389–2397 (2007).
- 456 47. Speed, D. & Balding, D. Relatedness in the post-genomic era: is it still useful? *Nat. Rev. Genet.* **16**, 33–44 (2014).
- 457 48. Hardy, G. Mendelian proportions in a mixed population. *Science* **28**, 49–50 (1908).
- 458 49. Weinberg, W. Über den Nachweis der Vererbung beim Menschen. *Jahreshefte des Vereins für Vaterländische Naturkunde in*  
459 *Württemberg* **64**, 368–382 (1908).
- 460 50. Lee, S. & van der Werf, J. An efficient variance component approach implementing an average information REML suitable for  
461 combined LD and linkage mapping with a general complex pedigree. *Genet. Sel. Evol.* **38**, 25–43 (2006).
- 462 51. World Health Organization. Global tuberculosis report (2014).
- 463 52. Gusev, A. *et al.* Quantifying missing heritability at known GWAS loci. *PLoS Genet.* **9**, e1003993 (2013).
- 464 53. Speed, D. & Balding, D. MultiBLUP: improved SNP-based prediction for complex traits. *Gen. Res.* **24**, 1550–1557 (2014).
- 465 54. Zhou, X., Carbonetto, P. & Stephens, M. Polygenic modeling with Bayesian sparse linear mixed models. *PLoS Genet.* **9**, e1003264  
466 (2013).
- 467 55. Visscher, P. *et al.* Statistical power to detect genetic (co)variance of complex traits using snp data in unrelated samples. *PLoS Genet.*  
468 **10**, e1004269 (2014).
- 469 56. Bhatia, G. *et al.* Haplotypes of common SNPs can explain missing heritability of complex diseases (2016). Preprint available on  
470 BioRxiv.
- 471 57. Tobin, M., Sheehan, N., Scurrah, K. & Burton, P. Adjusting for treatment effects in studies of quantitative traits: antihypertensive  
472 therapy and systolic blood pressure. *Stat. Med.* **24**, 2911–2935 (2005).
- 473 58. Asselbergs, F. *et al.* Large-scale gene-centric meta-analysis across 32 studies identifies multiple lipid loci. *Am. J. Hum. Genet.* **91**,  
474 8230838 (2012).
- 475 59. Todd, J. *et al.* Robust associations of four new chromosome regions from genome-wide analyses of type 1 diabetes. *Nat. Genet.* **39**,  
476 857–864 (2007).
- 477 60. Plenge, R. *et al.* TRAF1-C5 as a risk locus for rheumatoid arthritis—a genomewide study. *N. Engl. J. Med.* **20**, 1199–1209 (2007).