

1 **Evolution of passwords for cost-free honest signalling**
2 **between symbionts and hosts**

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23 Abstract

24 Honest communication between potential partners with conflicting interests is generally
25 thought to require costly signals. Costly signalling can explain partner choice when it is
26 possible to link a strategic cost to an individual's quality, like in mate choice. However, in
27 mutualisms, it is usually impossible to link a cost to the likelihood that a potential partner will
28 behave cooperatively in the future. In fact, signals like Nod factors in rhizobial bacteria,
29 which form symbioses with leguminous plants, are evidence of cost-free, honest signals in
30 situations of potential conflict. How can such a signalling system evolve? We use a
31 population-genetics model to show that a cost-free, honest signal can evolve when the
32 receiver is under *soft selection*, which is when high juvenile mortality does not lead to a
33 corresponding reduction in fitness, a common occurrence in many species. Under soft
34 selection, senders evolve increasingly complex messages of identity, a system akin to a
35 password or a lock and key. Thus, a symbiont can signal that it shares a coevolutionary
36 history with a potential host, and if that history is mutualistic, then the host can believe that
37 the symbiont is mutualistic. Password signalling might also explain the acquisition of some
38 defensive symbionts and the evolution of complex species-recognition signals in mate choice.

39 **Key words:** evolutionary game theory, legume, horizontal transmission, mate choice,
40 microbiome, mutualism, Nod factor, partner choice, partner fidelity feedback, rhizobia,
41 signalling, defensive symbiosis, vertical transmission

42

43 “...Say now Shibboleth: and he said Sibboleth: for he could not frame to pronounce it right.
44 Then they took him, and slew him....” — Judges 12:6, King James Version

45 Introduction

46 *A ‘mechanism-design’ problem in mutualisms*

47 Let us think of communication between symbionts and hosts as a signalling problem. While
48 both mutualistic and parasitic partners have an incentive to enter a host, parasites decrease a
49 host's fitness. Hence, interests are not aligned, similar to what is found in mate choice, where
50 males of both high and low quality have an incentive to mate, and females have an incentive
51 to choose high-quality males. The host faces a ‘mechanism-design’ problem: how to design a
52 signalling system in which a mutualistic symbiont can uniquely identify itself as a mutualist
53 to a host. In other words, a symbiont must be able to signal its ‘mutualistic identity’ (i.e. that
54 the signaller will be reliably ‘nice’ to the host).

55 The difficulty with using the classic mechanism of costly signals (Grafen 1990; Maynard
56 Smith and Harper 2003) is that it is not readily apparent how a mutualistic nature can be
57 correlated with the strategic cost that is required for the maintenance of honest signalling in
58 situations of non-aligned interests. Costly signalling is arguably possible in a few cases like
59 big, symmetrical flowers, in which a signal of vigour *per se* can honestly signal that the
60 flower is likely carrying high amounts of rewards. But in most mutualisms, the mere
61 demonstration that a symbiont is *vigorous* does not demonstrate that the symbiont will also

62 *behave mutualistically* in the future (Edwards and Yu 2007). Thus, a strategic cost does not
63 seem to provide a good explanation for honest signalling in mutualisms.

64 Nonetheless, evolution is exceedingly clever and seems to have solved this mechanism-
65 design problem for Nod factors in legume-rhizobia symbioses (Oldroyd 2013). When the root
66 of a leguminous plant perceives Nod factor from rhizobial bacteria, the root initiates a
67 signalling cascade that results in the formation of an ‘infection thread’ to allow the bacteria to
68 colonize the host plant. All sorts of bacteria would benefit from gaining entry to a root, so
69 there is a strong temptation for pathogens to counterfeit Nod factor. Nonetheless, only
70 rhizobial bacteria appear to make Nod factors that are successfully recognised by the plant to
71 initiate signalling and infection events.

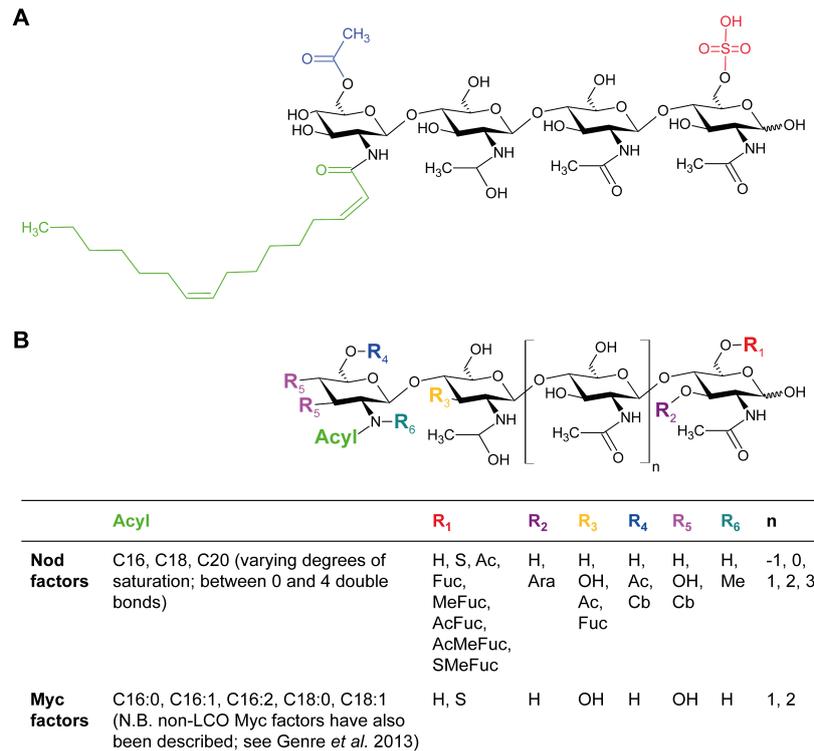
72 There are three general theoretical classes of honest signalling. We can rule out the ‘costly,
73 honest’ signal explanation of Nod factor, since there is no obvious *strategic* cost (additional
74 to the mere cost of producing the necessary molecules [Maynard Smith and Harper, 2003]) to
75 Nod factor. That is, Nod factor is not a bundle of ammonium molecules, serving as evidence
76 that the bacterium is capable of fixing nitrogen. We can also exclude ‘cheap talk’ signalling
77 (Crawford and Sobel 1982), which requires shared (partially aligned) interests between
78 signaller and receiver individuals, such as occurs between kin. But in horizontally transmitted
79 mutualisms, host and symbiont are different species and disperse separately, erasing shared
80 interests. The third class, ‘verifiable information,’ requires that the signal be true on its face.
81 In biology, verifiable-information signals exist within the concept of the *index* (Maynard
82 Smith and Harper 2003). For instance, claw marks high up on a tree trunk are a self-evident,
83 and thus believable, signal of a tiger’s large size. For Nod factor to be an index, it would need
84 to be a unique by-product of the same biochemical pathway that leads to the quality being
85 sought (nitrogen fixation), so that the mere presence of Nod factor would indicate a
86 mutualistic symbiont, but this is not the case. Nod factor can, in principle, be synthesized by
87 non-nitrogen-fixing rhizobial bacteria, which would appear to rule out verifiable-information
88 signals as well, but we will show how Nod factor can be included in this class.

89 *Cost-free, honest signals: passwords and Nod factors*

90 To start, we propose that Nod factors be thought of as ‘password signals,’ cost-free messages
91 of arbitrarily high complexity that can honestly convey identity. The idea that Nod factors
92 serve as passwords arises naturally from the many descriptions of Nod factors as chitin-based
93 chains adorned with multiple ‘decorations’ that vary across rhizobial species (**Figure 1**) and
94 the portrayal of Nod factors and Nod-factor receptors acting in a ‘lock-and-key’ manner
95 (Parniske and Downie 2003), such that different Nod factors are accepted by different plant
96 species (Perret et al. 2000), leading to a high degree of species-specificity in rhizobia-
97 hostplant associations.

98 Nod factors are famously variable in structure, with a lipochitooligosaccharide (LCO)
99 backbone of three to five N-acetylglucosamine residues to which multiple different
100 “decorations” can be added, e.g. addition of an acetyl, methyl, sulphate, or sugar moiety
101 (**Figure 1**).

102



103

104

105 **Fig. 1. Structures of Nod and Myc factors.** **A.** The major Nod factor produced by
 106 *Sinorhizobium meliloti* consists of four N-acetylglucosamine residues (black) and a C16:2
 107 acyl group (green). This lipochitooligosaccharide (LCO) backbone is further decorated with
 108 sulphate (red) and acetyl (blue) groups. **B.** Generalised structure of Nod and Myc factors. The
 109 table shows some of the major decorations and length variants that have been characterised to
 110 date. **Ac:** acetyl, **Ara:** arabinosyl, **Cb:** carbamoyl, **Fuc:** fucosyl, **H:** hydrogen, **Me:** methyl,
 111 **OH:** hydroxyl, **S** sulphate, **AcFuc:** acetylated fucose, **MeFuc:** methylfucose, **AcMeFuc:**
 112 acetylated methylfucose, **SmeFuc:** sulphated methylfucose. Figure adapted from Perret et al.
 113 (2000), Wais et al. (2002), and Miller and Oldroyd (2011).

114

115 The set of Nod factor decorations varies across rhizobial species and biovars, as does the
 116 length of the LCO backbone and the acyl (fatty-acid) chain added to the non-reducing
 117 terminus. These degrees of freedom give rhizobia the capacity to produce a multitude of Nod-
 118 factor variants (Miller and Oldroyd 2011). For instance, if each of 7 positions on the LCO
 119 backbone can have one of three possible decorations (including no decoration), there are $3^7 =$
 120 2,187 possible variants, not counting length variation in the LCO backbone and acyl chains.
 121 The genetic architectures of Nod factors and Nod-factor-receptor genes both appear to favour
 122 rapid diversification. Known Nod-factor receptor genes are found in tandem arrays, and this
 123 might allow rapid evolution of these genes via recombination (Parniske and Downie 2003).
 124 Similarly, the diversity of *nod* genes allows rhizobia to add multiple and different decorations
 125 to the LCO backbone and to simultaneously produce multiple different Nod factor molecules
 126 (Miller and Oldroyd 2011).

127 However, what are the conditions under which passwords evolve via natural selection? Let us
128 say that a bacterial lineage has co-evolved with a plant and that during this time, the
129 symbiosis has progressed from an ancestral state of close association at the root surface (e.g.
130 the plant root secretes carbohydrate compounds into its immediate surroundings and captures
131 ammonium by diffusion and/or active transport) to the more intimate state of endosymbiosis,
132 via the formation of an infection thread and root nodule. The benefit of evolving
133 endosymbiosis is increased efficiency of nitrogen fixation in an oxygen-free environment,
134 and the exclusion of free-riders is a possible additional benefit. *Nod*-gene duplication and
135 mutation allowed rhizobia to evolve Nod factors of greater and greater complexity (more
136 decorations added to the LCO chain). Nod factor does not have to be costly, except in the
137 trivial sense that it needs some energy to be synthesized, and indeed Nod factor does not
138 seem to have a *strategic* cost. However, Nod factor does need to be uniquely recognizable,
139 which could explain why it is complex.

140 During the evolution of Nod factor, we assume that the host plant also had ancestral
141 physiological mechanisms for shedding or withdrawing resources from worthless roots that
142 have failed to take up fixed nitrogen (Partner Fidelity Feedback PFF, Weyl et al. 2010) or had
143 even evolved punishment (Host Sanctions HS, Kiers et al. 2003). Either way, *rhizobial*
144 *lineages with the correct Nod factor have been subjected to selection for nitrogen fixation*
145 *and against pathogenicity*. The usefulness of passwords is that they can truthfully signal that
146 a bacterial lineage shares a co-evolutionary history with a host lineage, and if that history is
147 mutualistic, then the plant is selected to engage in symbiosis with bearers of the password.
148 Thus, passwords signal a particular evolutionary history (an ‘identity’), and since that history
149 is written into the bacterial symbiont’s genome, the genome enforces a particular behaviour.

150 This is what a person does with a password on a bank website: he credibly signals to the
151 bank that they share a specific history of transactions, and if those transactions have been
152 acceptable to the bank, there is a good chance that the person will continue to act acceptably,
153 so the bank should allow continued transactions. (Passwords are an alternative to repeated
154 games. By definition, players of repeated games build up interaction histories with other
155 players and thus need to recognize individuals in order to apply the correct history to each
156 player, but with passwords, it is possible for a member of a host lineage to recognize a
157 member of a specific symbiont lineage, even if that pair of individuals has never met.)

158 For a password to be a reliable signal of identity, the password needs to be complex.
159 Otherwise, a password that identifies one lineage could easily be evolved *de novo* by other
160 lineages. Thus, our challenge becomes one of explaining why there is directional selection for
161 signal complexity. We also need to explain ‘strictness’ in receivers, where strict means that
162 the receiver rejects passwords with (too many) errors. It is the combination of a sufficiently
163 complex signal and a sufficiently strict receiver that renders it essentially impossible for
164 parasites to evolve a working password *de novo*.

165 *Passwords can evolve under soft selection*

166 In the context of mutualistic interactions, the process we envisage leading to the evolution of
167 cost-free but still honest signalling is the following. Consider a population of hosts

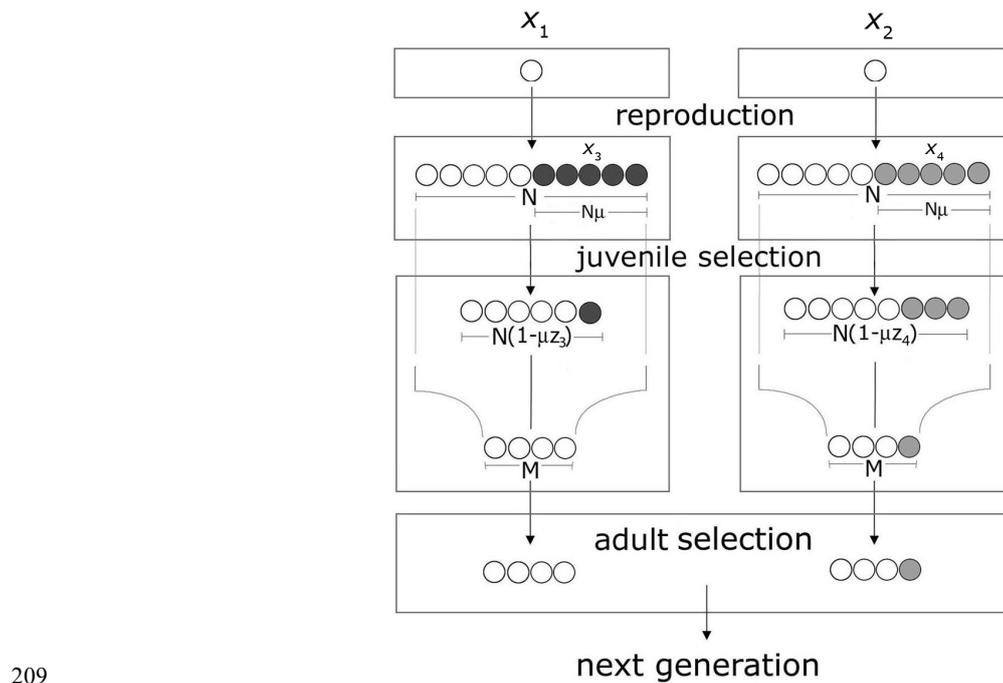
168 harbouring only mutualistic symbionts. *Mutations in the symbionts* lead to a slightly different
169 (simpler or more complex) password or to a parasitic phenotype. We assume that double
170 mutations are negligible, meaning that a symbiont will not simultaneously evolve a new
171 password and a newly parasitic phenotype. We also let *mutations in the hosts* lead them to
172 accept a slightly different (simpler or more complex) password. Hosts accepting the original
173 password will therefore accept both mutualistic and the new mutant parasitic partners, but
174 hosts with mutations that cause them to accept a different password will reject parasitic
175 symbionts, which still use the original password under the assumption that double mutations
176 are negligible.

177 Mutant *hosts* will have higher fitness than the resident hosts if the benefit of avoiding
178 parasites is higher than the cost of the difficulty of finding the rarer partners carrying the right
179 mutant password (i.e. some mutant hosts fail to find a symbiont partner). If this is the case,
180 mutant *symbionts* with the slightly different (simpler or more complex) passwords will also
181 have higher fitness, because mutualistic symbionts with the original password suffer some of
182 the cost of PFF or HS triggered by the co-colonising parasitic mutants (because the host
183 expends energy to trigger PFF or HS and because of the opportunity cost of mutualists having
184 lost out on colonisation opportunities to parasites). Then, we must only explain why the
185 mutant symbionts with the more complex password have higher fitness than the mutants with
186 the simpler password. We will show that, *provided that the hosts undergo soft selection*, the
187 hosts that mutated to accept the more complex passwords have a selective advantage, and as
188 a result, the signallers that mutated to slightly more complex passwords will also have a
189 selective advantage. Repeating this scenario over time leads to increasingly complex
190 passwords that are honest and cost-free.

191 The critical, non-intuitive step in this scenario is the contribution of *soft selection* (Buchholz
192 1922; Wallace 1981; Klekowski 1988) (**Figure 2**). Soft selection occurs when juveniles are
193 produced in excess of available carrying capacity for adults. For instance, a plant produces
194 many more seeds than there are patches in the environment to support adults, and as a result,
195 juvenile populations are unavoidably bottlenecked by this exogenous mechanism, or to put it
196 another way, when a parent loses some fecundity, its fitness remains virtually unaffected,
197 since some offspring would have died no matter what. Since all offspring contain *de novo*
198 mutations, which, if non-silent, are more likely to be deleterious than beneficial, soft-selected
199 lineages evolve genes that are hypersensitive to mutation (i.e. antirobust); that is, soft-
200 selected genes evolve to suffer large losses in function after mutation, causing the offspring
201 that carry more mutations to be much more likely to die, which reduces competition with
202 their more fortunate siblings that carry fewer mutations. Parental fitness benefits more from
203 this filtering out of mutated offspring than is lost to reduced fecundity, since most offspring
204 are destined to die anyway in a soft-selection scenario.

205 In what follows, we use a population-genetics model to show that soft selection leads to the
206 evolution of more complex passwords. We stick with the scenario in which the signaller is a
207 bacterium and the receiver is a host plant, but the model applies in general to partner choice.

208



209

210 **Figure 2. Soft selection leads to antirobustness.** Alleles x_1 and x_2 have the same fitness.
 211 However, x_1 's mutants x_3 happen to have lower fitness than do x_2 's mutants x_4 ($z_3 > z_4$, so $(1-$
 212 $\mu z_3) < (1-\mu z_4)$), and hence, x_1 is less robust than is x_2 . If there is a juvenile selection stage, x_3
 213 mutants are more likely to be eliminated before the adult stage; if juvenile selection does not
 214 cause a loss of fecundity (because only a small fraction of the progeny goes on to the adult
 215 stage anyway), the adult progeny of x_1 (M) will have a selective advantage over the adult
 216 progeny of x_2 , and x_1 will increase in frequency.

217

218 Model

219 Consider a population of nitrogen-fixing rhizobial bacteria producing a password composed
 220 of separate elements a , b , c , etc. and allow each of these elements to arise or disappear in a
 221 stepwise fashion by a mutation that can add or remove one element (a mutation is denoted by
 222 adding or removing a letter from the password). For simplicity of notation, the order of the
 223 elements does not matter; that is, passwords of increasing complexity are denoted by a , ab ,
 224 abc, \dots , etc., and $ab=ba$, $abc=acb$, etc. Note also that any element of a password can mutate
 225 (e.g. $abc \rightarrow tbc \rightarrow tbu$), but we ignore these scenarios here because our focus is on the origin
 226 of complexity *per se*, which we are representing by a longer password. In other words, we
 227 focus on explaining how to get from passwords of low to high complexity ($a \rightarrow abcde$), and
 228 we ignore the diversification of passwords ($abcde \rightarrow uewix$). Our way of denoting more
 229 complex passwords mimics the idea that Nod factor has evolved complexity by the
 230 proliferation of additional *nod* genes that have added more "decorations" to the basic LCO
 231 backbone (Miller and Oldroyd 2011).

232 In each generation, a fraction μ_p of the population produces a simpler password, and a
 233 fraction μ_p produces a more complex password. For instance, in a population of bacteria

234 sending password ab , some mutants will send the password a , and some will send the
235 password abc . Bacteria can also evolve to be parasitic by failing to export fixed nitrogen to
236 the host plant. Hence, the population will also have a fraction μ_D of parasitic ab^* bacteria.
237 Assuming that double mutations are rare, we ignore a^* and abc^* mutants.

238 Finally, we denote plants that accept a given password by capital letters; each plant type only
239 accepts its own corresponding bacterium type (AB accepts only ab and ab^* ; A accepts only
240 a ; ABC accepts only abc). This assumption of strictness in the plants is here made only for
241 explanatory convenience, and we explicitly model the evolution of strictness in the next
242 section. We start with a population fixed on AB plants and ab bacteria, with a low frequency
243 of mutants ABC , A , abc , a , and ab^* .

244 *Signaller (Bacteria)*

245 Selection coefficients s_i against the different bacterial types i in the **adult phase** are as
246 follows (See **Appendix**).

247 $s_{ab} > 0$ because AB plants can be colonized by ab and ab^* bacteria. ab bacteria suffer some
248 fitness loss due to ab^* 's cheating, as a consequence of the costs of host response against ab^*
249 (direct costs of nodule senescence via HS or PFF in mixed nodules, indirect costs of
250 senescence due to plant expenditure of energy, and opportunity costs of the plant failing to
251 have hosted ab bacteria in all nodules, which were excluded via competition with ab^*
252 bacteria for limited colonization opportunities).

253 $s_{ab^*} > 0$ for the same reasons as above. If the plant reacts indiscriminately against ab or ab^* ,
254 then $s_{ab^*} < s_{ab}$, since ab^* do not pay the cost of nitrogen fixation. If HS or PFF is preferentially
255 targeted toward ab^* -inhabited nodules, then $s_{ab^*} > s_{ab}$.

256 $s_{abc} = 0$ because there is no costly host response by ABC plants, since there are no abc^*
257 bacteria. Strictly, $s_{abc} = 0$ when abc and ABC have zero additional difficulty in finding each
258 other, relative to ab and AB finding each other. The assumption is reasonably upheld under
259 soft selection because ABC plants make many ABC seedlings, which sample the bacterial
260 population, and only some need to pick up abc bacteria. Note that this is an origin scenario
261 for abc passwords, so a successful colonization of ABC plants only has to happen once in
262 history for abc bacteria to start to be selected.

263 $s_a = 0$ for the same reason that $s_{abc} = 0$. There is no costly host response by A plants since there
264 are no a^* bacteria in the population, due to our assumption of no double mutations.

265 In the simplest scenario, if we assume that there is no juvenile selection during the dispersal
266 stage and that the (rare) mutant a and abc symbionts find their partners with no loss of
267 fecundity, it is trivial to show that simple and complex passwords (a and abc) have the same
268 fitness, which is greater than that of ab and ab^* , and that a and abc will have the same
269 frequencies (0.5) at equilibrium, driving ab and ab^* to extinction.

270 Of course, in a general scenario, there can be selection during dispersal, and rare types will be
271 less likely to find the appropriate host, resulting in loss of fecundity. The juvenile selection
272 coefficients z_i against the different bacterial types i measure the degree of selection against
273 each allele i in the dispersal phase (during which the symbionts must find a suitable host).

274 The degree of soft selection $\phi_B = M/N$ (where N is the number of offspring before juvenile
275 selection and M is the maximum number of individuals that can go on to the adult phase)
276 measures the buffering effect of soft selection. When $\phi_B = 1$, selection is hard, and losses
277 during the dispersal phase result in reduced fitness. At the other extreme, when $\phi_B = 0$,
278 selection is soft, and any loss during the dispersal phase is fully recouped, resulting in no loss
279 of fecundity.

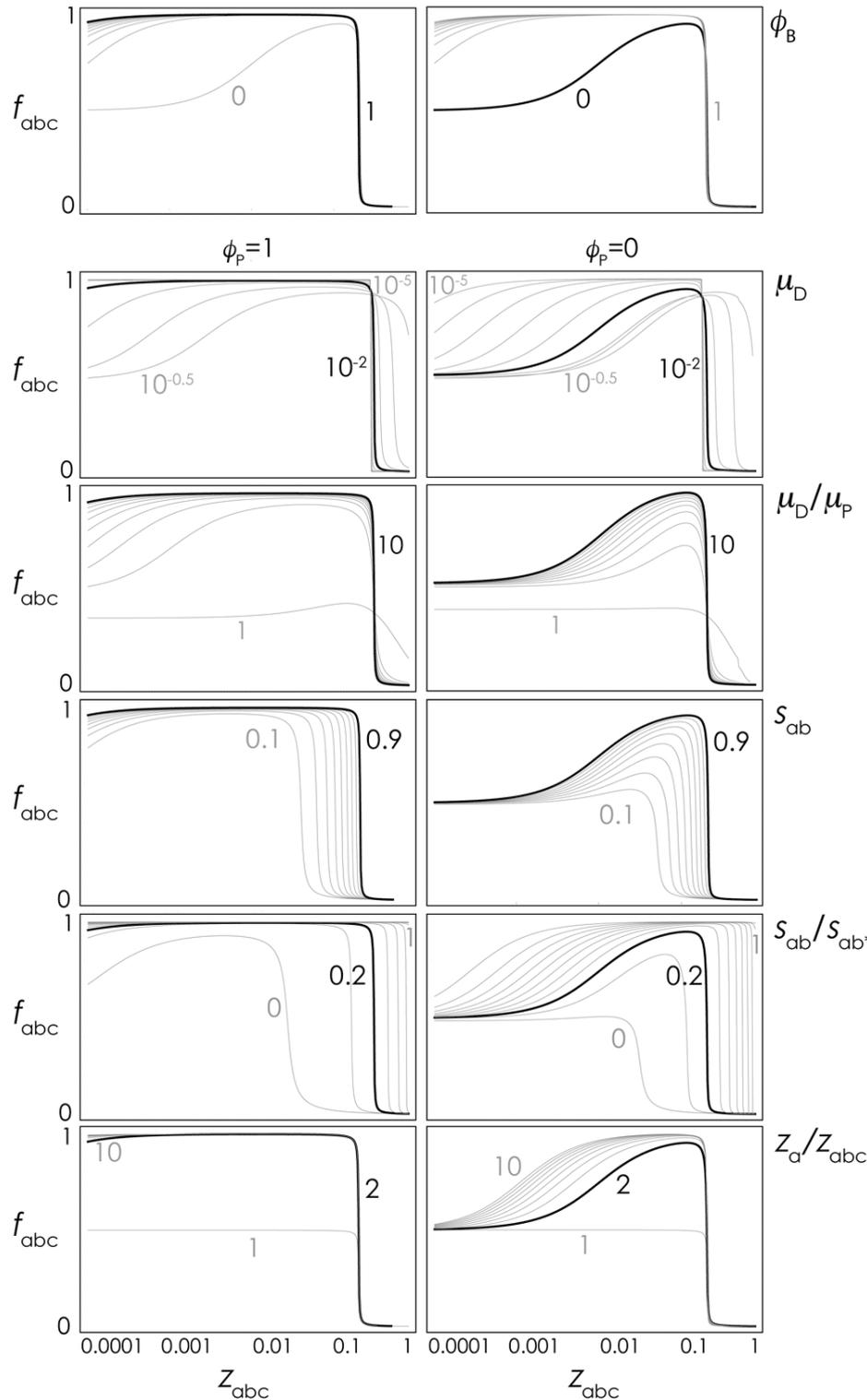
280 To account for difficulty of finding rare hosts, we assume that there is no juvenile selection
281 against the common alleles ab and ab^* but that the alleles with new passwords a and abc are
282 less likely to find the appropriate but rare host (A and ABC). Therefore, $z_{ab} = z_{ab^*} = 0$ and $z_a,$
283 $z_{abc} > 0$. However, once the a and abc types have found a host, there is no selection against
284 them in the adult phase, whereas there is selection in the adult phase against ab^* , which also
285 induces selection against ab (as we outline above: ab^* elicits nodule senescence in plants
286 and compete for nodules, both of which therefore reduce fitness for ab as well).

287 Figure 3 shows the parameter space in which selection in the juvenile phase makes complex
288 passwords increase in frequency. Complex passwords (abc) evolve as long as selection
289 against the rare type during dispersal is lower (approximately) than selection against cheating
290 mutants ab^* (and especially if selection occurs against ab as well) in the adult phase. Note
291 that the degree of soft selection ϕ_B is largely irrelevant (left vs. right columns, Figure 3); that
292 is, soft selection is not necessary for the complex password to evolve. Increasing μ_D and
293 reducing μ_P increases the frequency of the abc allele. However, lower μ_D and higher μ_P
294 enable the abc allele to increase in frequency for higher values of z_{abc} .

295 *The complex password evolves even if there is mortality in the juvenile (dispersal) phase for*
296 *the mutant passwords a and abc ; the abc allele increases in frequency (and may go to*
297 *fixation) if $z_a > z_{abc}$. If $z_a = z_{abc}$, a and abc have frequency 0.5 at equilibrium (the exact z_a/z_{abc}*
298 *ratio is rather irrelevant, unless $\phi_B = 0$).*

299 In this scenario, both the simpler password a and the more complex password abc have an
300 advantage against the original password because selection against ab^* and ab in the adult
301 phase offsets the loss of fecundity in the juvenile phase ($z_i > 0$, due to the difficulty in finding
302 an appropriate host). Whether a or abc prevails is determined by whether the coefficients of
303 juvenile selection for the two types are different, which depends on the rarity (hence on the
304 fitness) of the respective hosts. If ABC plants increase in the population, then abc bacteria
305 will have an advantage over a ($z_a > z_{abc}$), and selection will lead to more complex passwords.
306 So now we have to explain why ABC plants evolve, rather than A plants.

307 In other words, so far, we have simply recovered the well-known finding that selection
308 induced by parasites induces diversity in a population. We are left to explain why this
309 diversity leads directionally to increased signal complexity. Our solution in the next section
310 will invoke soft selection.



311

312 **Figure 3. Frequency of the complex password (symbiont) at equilibrium.** The
 313 equilibrium frequency of *abc* as a function of its coefficient of juvenile selection (z_{abc}). The
 314 black curve shows the equilibria for $\mu_D=10^{-2}$, $\mu_P=10^{-3}$, $s_{ab^*}=0.9$, $s_{ab}=0.2s_{ab^*}$, $z_a=2z_{abc}$ and for
 315 $\phi_B=0$ (right: no loss of fecundity in the juvenile phase) or $\phi_B=1$ (left: full loss of fecundity in
 316 the juvenile phase). The grey curves show the equilibria for other values of the parameters.
 317 There is no selection in the *adult* phase against *a* and *abc* ($s_a=s_{abc}=0$) and no selection in the
 318 *juvenile* phase against *ab* and *ab** ($z_{ab}=z_{ab^*}=0$).

319 However, before going on, it is possible to outline a simpler but perhaps less general scenario
320 for the evolution of password signalling. It might be the case that $s_a > 0$ (i.e. a suffers a fitness
321 cost) if there are a^* bacteria in the population, which elicit PFF response in plants, and this
322 response harms a , as in the argument for ab having $s_{ab} > 0$. This could occur if a passwords are
323 simple enough that they can be evolved *de novo* by parasitic bacteria. In this case, it seems
324 obvious that abc will go to evolutionary fixation, as long as the benefit of being a pathogen is
325 not greater than the cost of PFF. Then, ABC plants will increase simply because they accept
326 only abc bacteria, whereas A and AB plants accept some pathogenic bacteria (a^* and ab^*).

327 In the following, let us retain the more conservative assumption that $s_a = 0$ (i.e. parasites
328 induce diversity in the host, but this diversity is not biased towards higher or lower
329 complexity). The next part of our explanation is to show why hosts evolving to accept higher
330 complexity symbionts (ABC) have a selective advantage over hosts evolving to accept lower
331 complexity symbionts (A).

332 *Receivers (Hostplants)*

333 Let us consider a locus with four alleles: ABC , A , ABC^* , A^* , coding for the receiver's
334 recognition system. We only need to model two types (A and ABC) and their nonsense or
335 missense mutants (A^* , ABC^*):

- 336 • Allele ABC always accepts password abc
- 337 • Allele ABC^* is a missense mutant of allele ABC
- 338 • Allele A always accepts password a
- 339 • Allele A^* is a missense mutant of allele A

340 The key assumption is that a mutant of A (A^*) will be more likely to still accept its password
341 a than a mutant of allele ABC (ABC^*) will accept its password abc . In other words, we posit
342 that allele A is *robust* to mutation and that allele ABC is *antirobust* to mutations. Note that
343 robustness is unrelated to viability. A and ABC have exactly the same fitness (since abc and a
344 bacteria have the same effect on the plant's fitness); it is their mutants (A^* and ABC^*) that
345 have different fitnesses.

346 The mechanism behind our assumption could simply be that an ABC receiver system is
347 necessarily made up of more numerous or complex molecules that interact with each other,
348 since password abc is physically more complex than password a . Thus, even a mutation of
349 'small effect' in one component of ABC might prevent the abc signal molecule from fitting
350 properly in the other component(s) of ABC and thus prevent the different receptor molecules
351 from interacting correctly with each other to trigger a signalling cascade. A complexly
352 interacting receptor is also more likely to be a strict receiver, since even a small change in the
353 Nod factor would cause it fit differently in one or more of the receptor's molecules and thus
354 interfere with the interaction of those molecules.

355 We thus look for evidence that the Nod-factor receptor complex is composed of multiple
356 molecules that need to interact with each other in order to trigger a proper signalling cascade.
357 As it happens, there is considerable empirical support for this model (Oldroyd 2013). The
358 Nod-factor receptor complex is made up of two separately produced receptor-like kinases

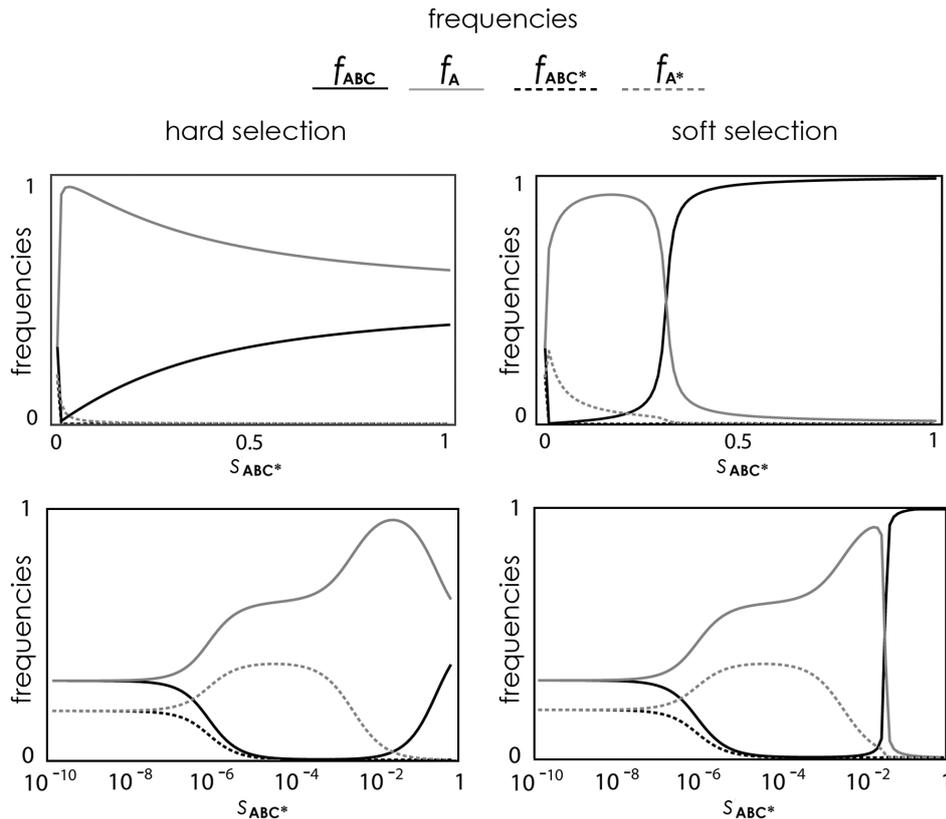
359 (NFR1 and NFR5, using *Lotus japonicus* nomenclature), each of which carries extracellular
360 LysM motifs that bind to the *N*-acetylglucosamine backbone of the Nod-factor backbone
361 (Broghammer et al. 2012). The two receptor-like kinases heterodimerise *in vivo* (Madsen et
362 al. 2011), and *mutation in either of the two kinases prevents rhizobial infection* (Radutoiu et
363 al. 2003). Importantly, NFR5 is a non-functional kinase (this is known because it lacks
364 essential protein subdomains), and thus, *NFR5 can act only via its interaction with NFR1*
365 (Madsen et al. 2011). The activated NFR1/5 complex then appears to activate a third
366 receptor-like kinase (SYMRK), which is necessary for downstream propagation of the
367 infection pathway (Radutoiu et al. 2003). Finally, Morieri et al. (2013) have shown that the
368 removal of just one acetyl group decoration from the *Sinorhizobium meliloti* Nod factor is
369 enough to prevent calcium influx in its host, *Medicago truncatula*, and this failed influx
370 prevents the initiation of an infection thread. Morieri *et al.* propose a model in which *only the*
371 *correct Nod factor is able to bring about “cooperative interactions” between receptor*
372 *proteins “such that the resulting interaction alters the kinase activity or specificity of the*
373 *receptor complex”*, triggering the calcium influx that is needed for successful infection-thread
374 initiation. In short, the Nod factor receptor is clearly a machine of many interdependent parts,
375 and thus, of many points of failure.

376 The evidence that an *A* receptor would be more robust to mutation is sparser, because all
377 known Nod-factor receptors (and Nod factors) are complex (Perret et al. 2000; Madsen et al.
378 2011; Miller and Oldroyd 2011; Broghammer et al. 2012). However, the Myc factors
379 produced by arbuscular mycorrhizal (AM) fungi are structurally simpler (Maillet et al. 2011),
380 and multiple, distantly related plant species will enter into symbiosis with the same AM
381 fungal genotype, despite the reasonable expectation that the Myc receptor complexes from
382 different plant lineages have mutated during plant diversification. However, because Myc-
383 factor receptors are not yet characterised, it is not yet possible to rule out the alternative
384 hypothesis that individual plant species produce multiple, species-specific Myc receptors.

385 Now, the evolution of password complexity requires that *ABC* increase in frequency over *A*.
386 The question therefore is: why should *ABC* increase in frequency, given that alleles *ABC* and
387 *A* are neutral? If anything, it seems that allele *A* should increase, since it is more robust to
388 mutation and will therefore have a higher rate of back mutations from allele *A*^{*}, whereas
389 allele *ABC*^{*} is less likely to survive and will provide fewer back mutants to *ABC* (Wagner et
390 al. 1997; Hermisson et al. 2002; de Visser et al. 2003).

391 We can see this effect in **Figure 4** left column, where we assume the standard hard-selection
392 scenario in which juvenile mortality reduces adult fitness ($z_{A^*}=z_{ABC^*}=0$). The frequency (f_A)
393 of the robust allele *A* increases, provided that the selection coefficient s_{ABC^*} against the
394 defective mutant *ABC*^{*} is high enough (approximately higher than the mutation rate). If
395 selection is too weak compared to the mutation rate, the differential amount of back
396 mutations (to *ABC* and *A* from *AB*) is negligible, and the two alleles *A* and *ABC* maintain the
397 same frequencies. If selection is strong enough, however, with no soft selection, the robust
398 allele *A* increases in frequency over *ABC* because it receives more back mutations from *A*^{*}
399 than *ABC* receives from *ABC*^{*}.

400



401

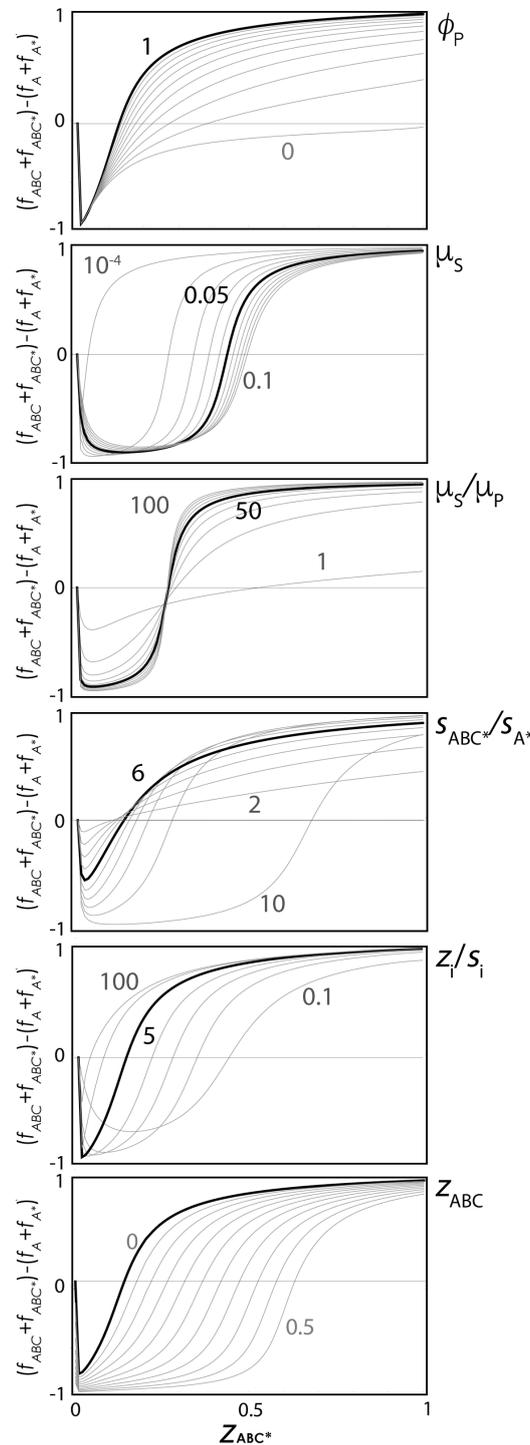
402 **Figure 4. Frequency of the four types of receiver (host) at equilibrium.** Equilibrium
 403 frequencies ($f_A, f_{ABC}, f_{A^*}, f_{ABC^*}$) of the four receptor alleles as a function of selection, with
 404 hard selection (left, $z_i=0$) or with soft selection (right, $z_i=s_i$); $\phi_p=1$, $s_{A^*}/s_{ABC^*}=1/10$; $s_A=s_{ABC}=0$;
 405 $\mu_S=10^{-4}$, $\mu_P=10^{-7}$; s_{A^*} and s_{ABC^*} are selection coefficients against dysfunctional A and ABC
 406 alleles, which arise with probability μ_S . The horizontal axis in the two bottom panels is
 407 logarithmic, to highlight the equilibrium frequencies with weak selection.

408

409 However, *with soft selection*, antirobust alleles increase in frequency over robust alleles
 410 (**Figures 4, 5, Appendix 1**). In fact, if the selection coefficients are high enough, the
 411 antirobust allele ABC can go to fixation. Strong selection is necessary, but not high mutation
 412 rates. Contrary to the evolution of robustness, the magnitude of the mutation rate does not
 413 make any relevant difference to the evolution of anti-robustness because the driving force is
 414 not the rate of back mutation, but the soft-selection process that eliminates the non-functional
 415 ABC mutants (Otto and Hastings 1998). Lower rates of mutation to non-functional ABC
 416 mutants do enable complex passwords to evolve for lower levels of soft selection (**Figure 5**).

417 The fraction of complex passwords at equilibrium increases as the ratio between the mutation
 418 rates for passwords and the mutation rate to defective alleles decreases (**Figure 5**), because if
 419 the mutation rate to defective alleles is too high, the advantage derived from soft selection is
 420 offset by partial loss of fecundity. A similar effect is observed for z_i/s_i (**Figure 5**), because
 421 when juvenile selection is stronger, defective alleles are more easily eliminated from the
 422 offspring (hence, the original antirobust allele is more likely to increase in frequency). The

423 influence of s_{ABC^*}/s_{A^*} is more complex, as it depends on the value of z_{ABC^*} (**Figure 5**). Note
 424 that soft selection can lead to an increase in the complex (antirobust) allele even if
 425 complexity itself has a cost ($z_{ABC^*} > 0$).



426 **Figure 5. Frequency of the antirobust recognition system (host) at equilibrium.**
 427 Equilibrium frequency of ABC as a function of its coefficient of juvenile selection (z_{ABC^*}). The
 428 black curve shows the equilibria for parameter values $\phi_p=0$, $s_{ABC^*}/s_{A^*}=10$, $s_i/z_i=1$, $\mu_s=10^{-2}$,
 429 $\mu_s/\mu_p=50$. The grey curves show the equilibria for other values of the parameters. There is no

430 selection in the adult phase against A and ABC ($s_A=s_{ABC}=0$) and no selection in the juvenile
431 phase against A ($z_A=0$).

432

433 Discussion

434 *Passwords: an honest, verifiable-information signal*

435 Game theory recognizes three classes of honest signalling models: costly signals, cheap talk,
436 and verifiable information. Passwords can be thought of as a variant of the verifiable-
437 information class. By virtue of their complexity and thus the low likelihood that they can
438 evolve *de novo*, they serve as signals of lineage identity that are self-evidently true, so long as
439 the receiver can recognise the password. In the context of horizontally transmitted
440 mutualisms, passwords can evolve to reliably signal a shared coevolutionary history, and a
441 coevolutionary history between mutualistic lineages strongly implies that the individual
442 sending the password is itself a mutualist or a recent descendent of a mutualist.

443 We emphasise that the role of signalling is only to allow the receiver to distinguish different
444 types, here, mutualistic and parasitic so that the receiver can associate preferentially with one
445 of them. The source of the natural selection for mutualistic types in the first place is some
446 form of partner fidelity feedback or host sanctions, and the source of parasitic types is
447 mutation. There is a superficial similarity of password signalling with green-beard signals
448 (Jansen and van Baalen 2006), which allow kin to identify each other, but green beards are a
449 within-species mechanism, and passwords can act between species. Green-beard signals can
450 also be simple, due to their linkage with cooperation loci.

451 In short, we argue that password signalling allows hosts to engage in successful Partner
452 Choice. As it happens, it has been shown experimentally that legume plants are able to
453 associate preferentially with ‘more mutualistic’ (nitrogen-fixing) rhizobial bacteria (Heath
454 and Tiffin 2009; Gubry-Rangin et al. 2010; Sachs et al. 2010). Importantly, these studies used
455 mutualistic and parasitic rhizobial bacteria that had been isolated from the same soil as the
456 host plant, and we predict that the rejected parasitic bacterial lineages used in these
457 experiments were producing Nod factors that had diverged from the mutualistic lineages that
458 were accepted.

459 Our proposed scenario for the evolution of password signalling derives from the observation
460 that plants are subject to a non-trivial degree of soft selection, since plants generally make
461 many more juveniles than can possibly grow into adults. As a result, juvenile mortality in
462 plants, due to some combination of failure to find bacterial symbionts and of selection against
463 missense mutations, does not result in fitness loss.

464 We then posit that mutations in the genes for complex Nod-factor receptors (ABC) are
465 inherently more likely to result in non-functional receptors than are mutations in the genes for
466 simple-signal receptors (A), because more complex Nod-factor receptors are likely
467 constructed from more interdependent parts. Thus, only fully functional ABC -receptors are
468 likely to be represented in adult plants, because ABC -mutant juveniles will have died due to

469 an inability to recruit rhizobia. In contrast, when the genes for *A*-type receptors mutate, the
470 receptors are more likely to retain some function because they are simple, and some of these
471 lower fitness mutants will thus be represented in the adult stage. Competition between *ABC*
472 and *A* adults will then favour *ABC*, and thus, complex rhizobial passwords (*abc*) will also be
473 favoured, and the system will evolve toward complex signals of identity. After enough
474 rounds, Nod factor will have evolved to a high enough degree of complexity that it will be
475 essentially impossible for a bacterium to evolve a working Nod factor *de novo*.

476 We also recall our first, and simpler, scenario for the evolution of complex signalling
477 passwords, which relies on the possibility that simple Nod factors can evolve *de novo* in non-
478 mutualistic bacteria (*a**). In this situation, bacteria that evolve more complex passwords (*abc*)
479 are favoured over those that evolve simpler passwords (*a*), as the former will find themselves
480 in parasite-free hostplants, at least until *abc* bacteria evolve parasitic behaviour *abc**. This
481 scenario also relies on soft selection, in that many *ABC* juvenile plants will die before finding
482 a suitable *abc* partner, but as long as there are lots of *ABC* juveniles, some will be successful,
483 and these will form the next generation.

484 *Limits to complexity in passwords*

485 In either scenario, we expect a natural upper limit to the complexity of passwords because
486 there will be physical limits on the reliable functioning of complex-signal receptors ($S_{ABC} >$
487 0), and receptors that evolve beyond these limits will likely fail to perceive any symbionts,
488 which reduces the effect of soft selection (**Figure 5**). Thus, the evolution of complexity in
489 passwords cannot escape indefinitely from the evolution of parasitic genotypes within
490 rhizobial lineages. There must also be mechanisms to senesce nodules that have been
491 colonised by parasites (Kiers et al. 2003; Weyl et al. 2010). And of course, such mechanisms
492 were necessary to proliferate the mutualistic genotypes of rhizobia in the first place, or there
493 would have been no mutualistic lineages for the plant to recognise.

494 Once a combination of password signalling and selective nodule senescence has evolved,
495 mutualistic strains of rhizobia should grow to dominate soils. As a result, it is possible to
496 imagine situations where some hosts will evolve to relax the strictness of association or
497 evolve to accept multiple passwords. As one example, some leguminous tree species, and the
498 non-legume Cannabaceae plant genus *Parasponia*, are early-successional species that
499 colonise low-nutrient soils, and they can be colonised by multiple rhizobial genera, including
500 strains from different continents (Behm et al. 2014). Under such conditions, we expect that
501 the risk of being colonised by non-productive or even pathogenic bacteria is outweighed by
502 the benefit being able to fix nitrogen. It will be interesting to see if these species have
503 evolved multiple Nod factor receptors, or if their receptors are less strict (which should make
504 them more robust to mutation). Indeed, in *Parasponia andersonii*, it appears that the latter
505 might be true, because this species uses the same receptor for both Myc and Nod factors (Op
506 den Camp et al. 2011).

507 Myc factors, which consist of simple, almost entirely undecorated LCOs (Maillet et al. 2011),
508 provide an interesting counterexample to complex Nod factors. Why have AM fungi not
509 evolved complex passwords? Part of the answer is likely due to the fact that plants are

510 colonized by multiple AM fungal species, and by doing so, plants make the fungi compete for
511 plant carbon, thereby reducing the carbon cost of AM-provided phosphorous (Argüello et al.
512 2016). A plant that evolved a more complex receptor would reduce its diversity of fungal
513 partners and thus reduce the number of competing fungal suppliers. It is also possible that
514 each AM fungal genotype benefits from colonizing multiple plant species, if plants vary
515 temporally in the photosynthate that they are able to transfer to their fungal partners. An
516 individual AM fungus that evolved a more complex Myc factor recognised only by the rare
517 plant genotype that had also evolved a matching receptor would not be able to create to create
518 such networks.

519 *Extensions to the model*

520 Our model has some limitations. First, it only shows that more complex passwords will,
521 under certain conditions, increase in frequency over less complex passwords, without
522 explicitly showing that they will become more and more complex over time. Increasing
523 complexity, however, is obvious if we assume that more complex alleles arise by mutation
524 after the fixation of the complex password (i.e. *ABC* will increase in frequency over *AB*, then
525 *ABCD* will increase in frequency over *ABC*, and so on). If mutants arise before fixation, the
526 dynamics will be more complex, but the logic remains the same: more complex passwords
527 will have an advantage over less complex ones under soft selection. Second, our model does
528 not allow for the fitness of an allele being dependent on the proportions of its partner in the
529 population. It does, however, allow juvenile selection against rare alleles (the phase in which
530 partner choice occurs). While it might be interesting to analyse frequency-dependent, soft
531 selection coefficients, there seems to be no compelling reason why this should select against
532 signal complexity. Third, we have ignored the fact that, as alleles become more complex,
533 asymmetries in mutation rates may arise (e.g. mutations from *ABC* to *A* might be more likely
534 than mutations from *ABC* from *A*). Preliminary results with asymmetric mutation rates,
535 however, did not reveal significant differences in the results, as long as the differences are not
536 extreme.

537 *Password signalling in mate recognition and defensive-symbiont acquisition*

538 Soft selection occurs in practically all vascular plants, and also in many cryptogamic plants
539 and in animals (Buchholz 1922; Wallace 1981; Klekowski 1988; Archetti 2009). Although
540 detailed treatments are outside the scope of this paper, we hypothesise that password
541 signalling can evolve in other recognition systems. For example, polymorphic toxin systems
542 (PTS) comprise complex, multi-domain molecules that exhibit high levels of allelic diversity.
543 Hillman and Goodrich-Blair (2016) have proposed that eukaryotic hosts can directly identify
544 suitable defensive symbionts to acquire by sensing the PTS produced by those symbionts,
545 and they review evidence that hosts produce PTS-receptors that are specific to particular
546 symbiont lineages. Another possible class of password signals are post-mating recognition
547 systems in gametes, which could require either a complex or a simple signal to differentiate
548 conspecifics from heterospecifics. If the complex recognition system is more antirobust to
549 mutation, then gametes that have suffered mutation will die unmated. However, those that
550 survive will only have accepted conspecifics. In contrast, gametes that accept a simple signal

551 might be robust to mutation and thus accept heterospecifics, producing hybrids. Under the
552 twin assumptions that soft selection is acting (most juveniles die before achieving adulthood)
553 and that hybrids have lower fitness, there could be selection for a mate-recognition system
554 that requires a complex signal.

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643 Appendix

644 Plants

645 Consider a locus with four alleles: ABC , A , ABC^* , A^* (with frequencies f_{ABC} , f_A , f_{ABC^*} , f_{A^*} ,
646 respectively:

- 647 • Allele ABC always accepts password abc
- 648 • Allele ABC^* is a missense mutant of allele ABC
- 649 • Allele A always accepts password a
- 650 • Allele A^* is a missense mutant of allele A

651

652 We assume that alleles A and ABC can mutate to each other with the same probability μ_P and
653 that each can mutate to dysfunctional alleles (respectively A^* and ABC^*) with probability μ_S .
654 We assume that all alleles have the same total mutation rate, hence A^* and ABC^* have other
655 mutants (A^0 and ABC^0) with frequency μ_P that have zero fitness -- see **Table A1**.

656 The recurrence equations for this system are:

$$657 T \cdot f_{ABC}' = f_{ABC}(1-S_{ABC})(1-\mu_S-\mu_P)(1-Z_{ABC})/\alpha_P + f_A(1-S_A)\mu_P(1-Z_{ABC})/\beta_P + f_{ABC^*}(1-S_{ABC^*})\mu_S(1-Z_{ABC})/\chi_P$$

$$658 T \cdot f_A' = f_{ABC}(1-S_{ABC})\mu_P(1-Z_A)/\alpha_P + f_A(1-S_A)(1-\mu_S-\mu_P)(1-Z_A)/\beta_P + f_{A^*}(1-S_{A^*})\mu_S(1-Z_A)/\delta_P$$

$$659 T \cdot f_{ABC^*}' = f_{ABC}(1-S_{ABC})\mu_S(1-Z_{ABC^*})/\alpha_P + f_{ABC^*}(1-S_{ABC^*})(1-\mu_S-\mu_P)(1-Z_{ABC^*})/\chi_P$$

$$660 T \cdot f_{A^*}' = f_A(1-S_A)\mu_S(1-Z_{A^*})/\beta_P + f_{A^*}(1-S_{A^*})(1-\mu_S-\mu_P)(1-Z_{A^*})/\delta_P$$

661

662 where T is a normalizing factor obtained by summing the right-hand side of the four above
663 equations; α_P , β_P , χ_P , δ_P are normalizing factors of the offspring frequencies (see Table 1) ;

664 $\phi_P = M/N$ is the degree of soft selection, where N is the number of offspring before soft
665 selection (the same for all alleles) and M ($< N$) is the maximum number of individuals that can
666 go on to the adult phase after soft selection. N_i' is the number of offspring of an individual
667 with allele i after soft selection:

$$668 N_{ABC}' = N[(1-\mu_S-\mu_P)(1-z_{ABC}) + \mu_P(1-z_A) + \mu_S(1-z_{ABC^*})]$$

$$669 N_A' = N[(1-\mu_S-\mu_P)(1-z_A) + \mu_P(1-z_{ABC}) + \mu_S(1-z_{A^*})]$$

$$670 N_{ABC^*}' = N[(1-\mu_S-\mu_P)(1-z_{ABC^*}) + \mu_S(1-z_{ABC})]$$

$$671 N_{A^*}' = N[(1-\mu_S-\mu_P)(1-z_{A^*}) + \mu_S(1-z_A)]$$

672

673 The effect of soft selection on offspring frequencies is that frequencies are normalized
674 (because we are assuming no loss of viability) after juvenile selection by dividing them by
675 the total frequencies of the surviving offspring (α_P , β_P , χ_P , δ_P as appropriate, see **Table 1**).
676 Individuals with these normalized frequencies go on to the adult phase, where another round
677 of (hard) selection occurs. Selection in the juvenile phase has no effect on fecundity if $z_i < 1 - \phi$.

678 The equilibrium frequencies of the four alleles can be found by specifying the parameters
 679 μ_S , μ_P , s_1 and z_1 for the system above and calculating the leading eigenvector. The situation
 680 in which allele 1 is less robust than allele 2, is given by $s_{ABC^*} > s_{A^*} > 0$ and $z_{ABC^*} > z_{A^*} > 0$.

681

682 **Table A1.** Offspring and fitness for the four plant types.

		offspring			
	fitness	f_{ABC}	f_A	f_{ABC^*}	f_{A^*}
f_{ABC}	$1-s_{ABC}$	$(1-\mu_S-\mu_P)(1-z_{ABC})/\alpha_P$	$\mu_P(1-z_A)/\alpha_P$	$\mu_S(1-z_{ABC^*})/\alpha_P$	0
f_A	$1-s_A$	$\mu_P(1-z_{ABC})/\beta_P$	$(1-\mu_S-\mu_P)(1-z_A)/\beta_P$	0	$\mu_S(1-z_{A^*})/\beta_P$
f_{ABC^*}	$1-s_{ABC^*}$	$\mu_S(1-z_{ABC})/\chi_P$	0	$(1-\mu_S-\mu_P)(1-z_{ABC^*})/\chi_P$	0
f_{A^*}	$1-s_{A^*}$	0	$\mu_S(1-z_A)/\delta_P$	0	$(1-\mu_S-\mu_P)(1-z_{A^*})/\delta_P$

683

684 $\alpha_P = (1 - \phi_P) [(1 - \mu_S - \mu_P)(1 - z_{ABC}) + \mu_P(1 - z_A) + \mu_S(1 - z_{ABC^*})] + \phi_P$ [1]

685 $\beta_P = (1 - \phi_P) [(1 - \mu_S - \mu_P)(1 - z_A) + \mu_P(1 - z_{ABC}) + \mu_S(1 - z_{A^*})] + \phi_P$ [1]

686 $\chi_P = (1 - \phi_P) [(1 - \mu_S - \mu_P)(1 - z_{ABC^*}) + \mu_S(1 - z_{ABC})] + \phi_P$ [1]

687 $\delta_P = (1 - \phi_P) [(1 - \mu_S - \mu_P)(1 - z_{A^*}) + \mu_S(1 - z_A)] + \phi_P$ [1]

688

689 *Bacteria*

690 Consider a locus with four alleles: a , ab , abc , ab^* , with frequencies $f_a, f_{ab}, f_{abc}, f_{ab^*}$,
 691 respectively

- 692 • Allele ab codes for a password of intermediate complexity
- 693 • Allele a codes for a password of lower complexity
- 694 • Allele abc codes for a password of higher complexity
- 695 • Allele ab^* codes for a password of intermediate complexity in a cheater bacterium

696

697 We assume that alleles a and abc can mutate to ab only among the alleles that can enter
 698 plants but that the total mutation rate is the same for all alleles. Hence, a and abc also have
 699 other mutants with frequency μ_P that produce either a password that has no match in the plant
 700 population or a defective bacterium (and therefore zero fitness) – see **Table A2**.

701 The recurrence equations for this system are:

702 $T \cdot f_{ab}' = f_{ab}(1-s_{ab})(1-2\mu_P-\mu_D)(1-z_{ab})/\alpha_B + f_a(1-s_a)\mu_P(1-z_{ab})/\beta_B + f_{abc}(1-s_{abc})\mu_P(1-z_{ab})/\chi_B + f_{ab}^*(1-$
 703 $s_{ab}^*)\mu_D(1-z_{ab})/\delta_B$

704 $T \cdot f_a' = y_{ab}(1-s_{ab}) \mu_P(1-z_a)/\alpha_B + f_a(1-s_a)(1-2\mu_P-\mu_D)(1-z_a)/\beta_B$

705 $T \cdot f_{abc}' = y_{ab}(1-s_{ab}) \mu_P(1-z_{abc})/\alpha_B + f_{abc}(1-s_{abc})(1-2\mu_P-\mu_D)(1-z_{abc})/\chi_B$

706 $T \cdot f_{ab}^*' = y_{ab}(1-s_{ab}) \mu_D(1-z_{ab}^*)/\alpha_B + f_{ab}^*(1-s_{ab}^*)(1-2\mu_P-\mu_D)(1-z_{ab}^*)/\delta_B$

707

708 Where α_B , β_B , χ_B , δ_B are normalizing factors of the offspring frequencies (see Table 2) and ϕ_B
 709 $=m/n$ is the degree of soft selection, where n is the number of offspring before soft selection
 710 (the same for all alleles) and m ($<n$) is the maximum number of individuals that can go on to
 711 the adult phase after soft selection. n_i' is the number of offspring of an individual with allele i
 712 after soft selection:

713 $n_{ab}' = n[(1-2\mu_P-\mu_D)(1-z_{ab}) + \mu_P(1-z_a) + \mu_P(1-z_{abc}) + \mu_D(1-z_{ab}^*)]$

714 $n_a' = n[(1-2\mu_P-\mu_D)(1-z_a) + \mu_P(1-z_{ab})]$

715 $n_{abc}' = n[(1-2\mu_P-\mu_D)(1-z_{abc}) + \mu_P(1-z_{ab})]$

716 $n_{ab}^*' = n[(1-2\mu_P-\mu_D)(1-z_{ab}^*) + \mu_D(1-z_{ab})]$

717

718 **Table A2.** The offspring and fitness of the four types of bacteria

freq.	fitness	offspring			
		f_{ab}	f_a	f_{abc}	f_{ab}^*
f_{ab}	$1-s_{ab}$	$(1-2\mu_P-\mu_D)(1-z_{ab})/\alpha_B$	$\mu_P(1-z_a)/\alpha_B$	$\mu_P(1-z_{abc})/\alpha_B$	$\mu_D(1-z_{ab}^*)/\alpha_B$
f_a	$1-s_a$	$\mu_P(1-z_{ab})/\beta_B$	$(1-2\mu_P-\mu_D)(1-z_a)/\beta_B$	0	—
f_{abc}	$1-s_{abc}$	$\mu_P(1-z_{ab})/\chi_B$	0	$(1-2\mu_P-\mu_D)(1-z_{abc})/\chi_B$	0
f_{ab}^*	$1-s_{ab}^*$	$\mu_D(1-z_{ab})/\delta_B$	0	0	$(1-2\mu_P-\mu_D)(1-z_{ab}^*)/\delta_B$

719

720 $\alpha_B = (1-\phi_B)[(1-2\mu_P-\mu_D)(1-z_{abc}) + \mu_P(1-z_a) + \mu_P(1-z_{abc}) + \mu_D(1-z_{ab}^*)] + \phi_B$

721 $\beta_B = (1-\phi_B)[(1-2\mu_P-\mu_D)(1-z_a) + \mu_P(1-z_{ab})] + \phi_B$

722 $\chi_B = (1-\phi_B)[(1-2\mu_P-\mu_D)(1-z_{abc}) + \mu_P(1-z_{ab})] + \phi_B$

723 $\delta_B = (1-\phi_B)[(1-2\mu_P-\mu_D)(1-z_{ab}^*) + \mu_D(1-z_{ab})] + \phi_B$