

1                   **Optimized but not maximized cue integration for 3D visual perception**  
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4 **Short title:** Optimized but not maximized cue integration for 3D visual perception  
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## 36 **Abstract**

37 Reconstructing three-dimensional (3D) scenes from two-dimensional (2D) retinal images is an ill-  
38 posed problem. Despite this, our 3D perception of the world based on 2D retinal images is  
39 seemingly accurate and precise. The integration of distinct visual cues is essential for robust 3D  
40 perception in humans, but it is unclear if this mechanism is conserved in non-human primates,  
41 and how the underlying neural architecture constrains 3D perception. Here we assess 3D  
42 perception in macaque monkeys using a surface orientation discrimination task. We find that  
43 perception is generally accurate, but precision depends on the spatial pose of the surface and  
44 available cues. The results indicate that robust perception is achieved by dynamically reweighting  
45 the integration of stereoscopic and perspective cues according to their pose-dependent  
46 reliabilities. They further suggest that 3D perception is influenced by a prior for the 3D orientation  
47 statistics of natural scenes. We compare the data to simulations based on the responses of 3D  
48 orientation selective neurons. The results are explained by a model in which two independent  
49 neuronal populations representing stereoscopic and perspective cues (with perspective signals  
50 from the two eyes combined using nonlinear canonical computations) are optimally integrated  
51 through linear summation. Perception of combined-cue stimuli is optimal given this architecture.  
52 However, an alternative architecture in which stereoscopic cues and perspective cues detected  
53 by each eye are represented by three independent populations yields two times greater precision  
54 than observed. This implies that, due to canonical computations, cue integration for 3D perception  
55 is optimized but not maximized.

56

## 57 **Author summary**

58 Our eyes only sense two-dimensional projections of the world (like a movie on a screen), yet we  
59 perceive the world in three dimensions. To create reliable 3D percepts, the human visual system  
60 integrates distinct visual signals according to their reliabilities, which depend on conditions such  
61 as how far away an object is located and how it is oriented. Here we find that non-human primates  
62 similarly integrate different 3D visual signals, and that their perception is influenced by the 3D  
63 orientation statistics of natural scenes. Cue integration is thus a conserved mechanism for  
64 creating robust 3D percepts by the primate brain. Using simulations of neural population activity,  
65 based on neuronal recordings from the same animals, we show that some computations which  
66 occur widely in the brain facilitate 3D perception, while others hinder perception. This work  
67 addresses key questions about how neural systems solve the difficult problem of generating 3D  
68 percepts, identifies a plausible neural architecture for implementing robust 3D vision, and reveals  
69 how neural computation can simultaneously optimize and curb perception.

## 70 Introduction

71 Three-dimensional (3D) visual perception is a significant achievement of the primate brain [1].  
72 Because the eyes detect two-dimensional (2D) projections of the world, like a movie on a screen,  
73 3D structure must be estimated. Creating 3D percepts from 2D images is a nonlinear optimization  
74 problem plagued by ambiguities and noise [2]. Human perceptual [3-5] and neuroimaging [6-9]  
75 studies show that integrating distinct visual cues resolves ambiguities and improves 3D estimates.  
76 The neural implementation of optimal cue integration is, theoretically, a linear process [10], but  
77 nonlinear computations such as quadratic nonlinearities and divisive normalization are also widely  
78 implicated in neural processing [11-17]. Because such nonlinearities reduce the independence of  
79 neuronal stimulus representations, they can conceivably impose limits on the precision of  
80 perception. We tested this hypothesis using psychophysics and computational modeling to  
81 evaluate how non-human primates (NHPs) perceptually integrate two visual cues which have  
82 prominent roles in human 3D vision: stereoscopic and perspective cues.

83 Stereoscopic cues arise from comparisons of left and right retinal images, which differ  
84 because the eyes are horizontally offset [18, 19]. Perspective cues originate from the projection  
85 of the 3D world onto 2D retinæ [20, 21]. The reliability of the 3D information carried by these cues  
86 depends on an object's spatial pose (i.e., position and orientation) [4, 5]. Specifically, stereoscopic  
87 cue reliability decreases with distance (**Fig 1A**) and perspective cue reliability increases with slant  
88 (**Fig 1B**). Human studies reveal that the integration of these cues is weighted according to their  
89 reliabilities [4, 5], but little is known about how NHPs perceptually integrate these cues.

90 Using an eight-alternative forced choice (8AFC) tilt discrimination task, we quantified how  
91 perception depends on planar surface pose. Contributions of stereoscopic and perspective cues  
92 to perception were evaluated using cue-isolating and combined-cue stimuli. For stereoscopic  
93 cues, performance decreased with distance from the fixation plane, consistent with geometric  
94 limitations of stereovision and the physiology of stereopsis [22]. For both cues, performance  
95 increased with slant. We further found evidence of a 3D analogue of the 'oblique effect' (more  
96 accurate and precise perception of cardinal than oblique tilts) [23-27], consistent with the influence  
97 of a prior for the 3D orientation statistics of natural scenes [28, 29].

98 Perception of combined-cue stimuli was consistent with an optimal integration strategy  
99 [30], with the cues dynamically reweighted according to their pose-dependent reliabilities. We  
100 found that perception was well explained by a neural architecture in which stereoscopic and  
101 perspective cues are represented by independent populations, with perspective signals from the  
102 two eyes combined via a quadratic nonlinearity and divisive normalization prior to their integration  
103 with stereoscopic cues. Cue integration was optimal given this architecture (population responses

104 were linearly summed [10]). However, an alternative architecture in which stereoscopic as well  
105 as left and right eye perspective cues are all represented independently yielded ~2 times greater  
106 precision. This indicates that 3D perception is optimized, but not maximized, and suggests that  
107 the precision of 3D perception is curbed by nonlinear canonical computations in the  
108 representation of perspective cues. Analogous limitations may exist for other sensory processes  
109 with multiple inputs signaling the same cue types, as occurs in audition, vestibular processing,  
110 and bimanual touch. Our findings suggest that cue integration is a conserved mechanism by  
111 which primates achieve robust 3D vision, and that the co-occurrence of multiple canonical  
112 computations (linear summation, quadratics, and divisive normalization) simultaneously optimizes  
113 and curbs perception.

114

## 115 **Results**

### 116 **Accuracy and precision of combined-cue 3D perception**

117 The 3D orientation of a planar surface can be described by two angular variables: tilt and slant  
118 [31, 32]. Tilt specifies the direction that the plane is oriented in depth (e.g., top-near), and slant  
119 specifies how much it is oriented in depth (i.e., its steepness; **Fig 1B**). We trained two rhesus  
120 macaques to perform an 8AFC tilt discrimination task. The monkeys reported a plane's tilt with an  
121 eye movement to a choice target (**Fig 2A**). Slant and distance were varied to evaluate how tilt  
122 perception changed with the plane's pose. We first quantified perception of combined-cue stimuli  
123 defined by stereoscopic and perspective cues (**Fig 2B** and **2C**).

124 Representative data characterizing tilt perception for combined-cue stimuli are shown in  
125 **Fig 3**. These data show error distributions of reported tilts ( $\Delta\text{Tilt} = \text{Reported Tilt} - \text{Presented Tilt}$ )  
126 calculated using all 8 tilts for 12 combinations of slant and distance. Perception was quantified in  
127 terms of accuracy (i.e., if there were systematic deviations between perceived and actual tilts)  
128 and precision (i.e., the variability of the percepts). Accuracy and precision were quantified using  
129 the bias ( $\mu$ ) and concentration ( $\kappa$ ) parameters of von Mises probability density function fits  
130 (equation 1, see **Methods**) to the error distributions [33]. No bias ( $\mu = 0^\circ$ ) indicates perfect  
131 accuracy, and larger values of  $\kappa$  (taller and narrower densities) indicate greater precision.

132 *Accuracy results.* All of the density functions shown in **Fig 3** peaked close to  $0^\circ$ , indicating  
133 that tilt perception was accurate. Indeed, across the 24 slant–distance combinations tested with  
134 Monkey L, the biases were centered close to  $0^\circ$  and narrowly distributed: circular mean  $\mu = 1.50^\circ$ ,  
135 circular standard deviation (SD) =  $3^\circ$ . The results were similar for Monkey F: mean  $\mu = 0.54^\circ$ , SD  
136 =  $2.33^\circ$  (N = 32 slant–distances). We repeated this analysis at each tilt individually, and again

137 found little bias (**S1A Fig**). These results indicate that tilt perception with combined-cue stimuli  
138 was accurate over a wide range of poses defined by distance, slant, and tilt.

139 Although the biases at each tilt were small, Monkey L showed an overall pattern consistent  
140 with an oblique effect for planar tilt. Across all cardinal tilts, the median absolute bias was  $3.66^\circ$   
141 ( $N = 24$  slant-distances  $\times 4$  tilts = 96), but  $8.21^\circ$  across all oblique tilts. Consistent with the  
142 influence of a prior for cardinal tilts, which are more frequent than oblique tilts in natural scenes  
143 [28, 29], the oblique biases were significantly larger than the cardinal biases (circular median test,  
144  $p = 5.32 \times 10^{-4}$ ). However, for Monkey F, the median absolute biases at cardinal ( $3.31^\circ$ ,  $N = 32 \times 4$   
145 = 128) and oblique ( $3.55^\circ$ ) tilts were not significantly different ( $p = 0.80$ ). Individual differences in  
146 the strength of the 2D oblique effect are similarly observed in humans [24, 25].

147 *Precision results.* The precision of combined-cue tilt perception depended on surface pose  
148 in two ways. First, precision increased monotonically with slant, as seen in the right marginals of  
149 **Fig 4**. This is also evident in **Fig 3** by comparing the density functions across columns. With  
150 increasing slant (left to right in the figure), the densities became taller and narrower (larger  $\kappa$ ).  
151 Second, precision showed an inverted U-shape as a function of distance, as shown in the top  
152 marginals of **Fig 4**. Likewise, this is seen in **Fig 3**, where the density functions at 57 cm (maroon  
153 curves) are taller and narrower than those at 37 cm (blue) or 137 cm (green). How precision  
154 depended on both slant and distance is summarized using heat maps in **Fig 4**. These precision  
155 landscapes reflect the interaction of the monotonic relationship between precision and slant and  
156 the inverted U-shape relationship between precision and distance, resulting in more gradual  
157 decreases in precision with distance at larger slants. Precision peaked at the largest slant ( $60^\circ$ )  
158 and  $\sim 20$  cm behind the plane of fixation for both monkeys. Although precision varied with surface  
159 pose, performance was above chance at all slant-distance combinations (Rayleigh test for  
160 circular uniformity, all  $p \leq 4.96 \times 10^{-14}$  and significant after correcting for 24 or 32 comparisons for  
161 Monkeys L and F, respectively).

162 We further found that precision did not differ significantly as a function of tilt for either  
163 monkey (**S2A Fig**), and that the results generalized to larger stimuli (**S3A Fig**). However, similar  
164 to the bias results, we found that Monkey L showed an oblique effect when we grouped precisions  
165 at cardinal and oblique tilts. For Monkey L, the median precision at cardinal tilts (6.95;  $N = 96$ )  
166 was significantly larger than at oblique tilts (4.78), Mann-Whitney U test ( $p = 5.99 \times 10^{-3}$ ). For  
167 Monkey F, the median precisions at cardinal (4.99,  $N = 128$ ) and oblique (5.01) tilts were not  
168 significantly different ( $p = 0.64$ ). These results parallel findings from human perceptual studies  
169 which indicate that the precision of 3D perception depends on the pose-dependent reliabilities of

170 the available visual cues [4, 5], and further suggest that there are individual differences in the  
171 extent to which the 3D orientation statistics of natural scenes impact perception.

172

### 173 **Contributions of stereoscopic cues to 3D perception**

174 Next, we assessed tilt perception using stimuli that isolated stereoscopic cues (**Fig 2D**). Control  
175 experiments confirmed that the stimuli contained no perspective cues that could be used to  
176 perform the task, and that performance was unaffected by a potential stereoscopic–perspective  
177 cue conflict [5] (**S4 Fig**). Error distributions of reported tilts were again calculated using all 8 tilts,  
178 and the accuracy and precision of perception were quantified using von Mises fits.

179 *Accuracy results.* For both monkeys, mean stereoscopic cue biases across all slant–  
180 distance combinations were again close to  $0^\circ$ , indicating that perception was generally accurate  
181 (Monkey L: mean  $\mu = -3.04^\circ$ , SD =  $19.64^\circ$ , N = 24; Monkey F: mean  $\mu = 1.87^\circ$ , SD =  $20^\circ$ , N = 32).  
182 However, the biases were broadly distributed. Examination of the biases at individual tilts  
183 suggested that this variability was due to geometric factors (**S1B Fig**). At surface poses with low  
184 stereoscopic cue reliability (i.e., combinations of large distances and small slants) precision was  
185 particularly poor, and the biases were correspondingly large. In contrast, performance was  
186 accurate at poses where the precision was reasonably high. Thus, perception was accurate so  
187 long as the cues were sufficiently reliable for the monkeys to perform the task well.

188 For the stereoscopic cue stimuli, Monkey L once again showed a pattern of biases  
189 consistent with an oblique effect. Across all cardinal tilts, the median absolute bias was  $7.76^\circ$   
190 (N = 96), but  $15.58^\circ$  across all oblique tilts (circular median test,  $p = 3.89 \times 10^{-3}$ ). Monkey F did not  
191 show this pattern: the median absolute biases at cardinal ( $8.19^\circ$ , N = 128) and oblique ( $8.26^\circ$ ) tilts  
192 were not significantly different ( $p = 1$ ). However, when precision was low, both monkeys showed  
193 a bias towards reporting bottom-near ( $270^\circ$ ) tilts (**S1B Fig**). This bias is consistent with the  
194 influence of a prior for ground planes, which are preponderant in natural scenes [28, 29].

195 *Precision results.* Precision landscapes over slant and distance are shown for the  
196 stereoscopic cue stimuli in **Fig 5A**. The overall patterns resembled the combined-cue landscapes  
197 (Monkey L:  $r = 0.96$ ,  $p = 3.37 \times 10^{-13}$ ; Monkey F:  $r = 0.81$ ,  $p = 1.49 \times 10^{-8}$ ). There was a monotonic  
198 relationship between precision and slant, indicating that stereoscopic cue reliability increases with  
199 slant. There was also an inverted U-shape relationship between precision and distance which is  
200 explained by geometric and physiological factors. The falloff in precision with distance is  
201 consistent with the decreasing reliability of stereoscopic cues (**Fig 1A**). The falloff in precision  
202 with distance from the fixation plane (both toward and away from the monkey) is consistent with  
203 the limited range of horizontal disparities represented by the visual system [22]. While the



204 stereoscopic and combined-cue precision landscapes were similar in pattern, precision was  
205 significantly lower for the stereoscopic cue stimuli than the combined-cue stimuli (Wilcoxon  
206 signed-rank test; Monkey L:  $p = 1.82 \times 10^{-5}$ ,  $N = 24$ ; Monkey F:  $p = 7.95 \times 10^{-7}$ ,  $N = 32$ ). Indeed, at  
207 combinations of large distances and small slants, performance with stereoscopic cue stimuli was  
208 not significantly different from chance (outlined in black in **Fig 5A**; Rayleigh test for circular  
209 uniformity, corrected for multiple comparisons). At greater distances, performance was at chance  
210 levels even with larger stimuli (**S3B Fig**). Thus, stereoscopic cues did not contribute to tilt  
211 perception beyond ~137 cm (less for small slants), indicating that perspective cues mediated  
212 above chance performance with combined-cue stimuli at those poses (**Fig 4** and **S3A Fig**).

213 We further found that precision did not differ significantly as a function of tilt (**S2B Fig**), or  
214 between cardinal (median  $\kappa = 3.46$  and  $1.80$  for Monkeys L and F, respectively) and oblique  
215 (median  $\kappa = 3.18$  and  $1.76$  for Monkeys L and F, respectively) tilts for either monkey (Mann-  
216 Whitney U test; Monkey L:  $p = 0.37$ ,  $N = 96$ ; Monkey F:  $p = 0.99$ ,  $N = 128$ ). Together, these results  
217 indicate that the contributions of stereoscopic cues to 3D perception are constrained by a  
218 combination of viewing geometry and physiology.

219

## 220 **Contributions of perspective cues to 3D perception**

221 Next, we assessed tilt perception using stimuli that isolated perspective cues (**Fig 2E**). To  
222 eliminate stereoscopic cues, we presented single eye views of combined-cue stimuli to the  
223 appropriate eye, and only the fixation target to the other eye. Performance was comparable with  
224 the two eyes (**S5 Fig**), so responses to left and right eye stimulus presentations were pooled  
225 together. Error distributions of reported tilts were calculated using all 8 tilts, and the accuracy and  
226 precision of tilt perception were quantified using von Mises fits.

227 *Accuracy results.* The perspective cue biases were centered close to  $0^\circ$  and narrowly  
228 distributed across all slant–distance combinations (Monkey L: mean  $\mu = 0.31^\circ$ ,  $SD = 3.04^\circ$ ,  
229  $N = 24$ ; Monkey F: mean  $\mu = 2.19^\circ$ ,  $SD = 2.69^\circ$ ,  $N = 32$ ), indicating that perception was accurate  
230 irrespective of the surface pose. Indeed, there was little bias at any individual tilt (**S1C Fig**).  
231 Although the biases at each tilt were small, Monkey L showed an overall pattern of biases  
232 consistent with an oblique effect for planar tilt: the median absolute bias at oblique tilts ( $8.71^\circ$ ,  
233  $N = 96$ ) was significantly larger than the median absolute bias at cardinal tilts ( $4.10^\circ$ ), circular  
234 median test ( $p = 1.50 \times 10^{-3}$ ). Monkey F did not show this pattern: median absolute biases at  
235 oblique ( $6.27^\circ$ ,  $N = 128$ ) and cardinal ( $5.41^\circ$ ) tilts were not significantly different ( $p = 0.32$ ). These  
236 results indicate that perspective cues support accurate perception of 3D orientation across a wide  
237 range of surface poses, and further suggest that there are individual differences in the extent to

238 which the 3D orientation statistics of natural scenes influence perception based on perspective  
239 cues.

240 *Precision results.* Precision landscapes over slant and distance are shown for the  
241 perspective cue stimuli in **Fig 5B**. At all poses, performance was above chance. There was a  
242 monotonic relationship between precision and slant (greater precision at higher slants), consistent  
243 with the slant-dependent reliability of perspective cues (**Fig 1B**). Precision was independent of  
244 distance, reflecting that the perspective cues in our stimuli signaled orientation but not distance  
245 due to the elimination of absolute size cues (see **Methods**). For both monkeys, precision was  
246 significantly lower for the perspective cue stimuli than the combined-cue stimuli (Wilcoxon signed-  
247 rank test; Monkey L:  $p = 7.48 \times 10^{-4}$ ; Monkey F:  $p = 1.86 \times 10^{-6}$ ). Across all poses, the perspective  
248 and stereoscopic cue precisions were not significantly different (Wilcoxon signed-rank test;  
249 Monkey L:  $p = 0.49$ ; Monkey F:  $p = 0.15$ ). However, the relative precisions for the two cue types  
250 were distance dependent. For distances at or just behind the fixation plane (57, 77, and 87 cm),  
251 precision was significantly higher with stereoscopic cues (Wilcoxon signed-rank test; Monkey L:  
252  $p = 4.88 \times 10^{-4}$ ,  $N = 12$ ; Monkey F:  $p = 4.88 \times 10^{-3}$ ,  $N = 12$ ). For nearer (37 cm) and further distances  
253 ( $> 87$  cm), precision was significantly lower with stereoscopic cues (Wilcoxon signed-rank test;  
254 Monkey L:  $p = 6.84 \times 10^{-3}$ ,  $N = 12$ ; Monkey F:  $p = 1.03 \times 10^{-4}$ ,  $N = 20$ ).

255 We further found that precision did not differ substantially as a function of tilt (**S2C Fig**),  
256 and that the results generalized to larger stimuli (**S3C Fig**). However, similar to the bias results,  
257 we found that Monkey L showed an oblique effect when we grouped precisions at cardinal and  
258 oblique tilts. For Monkey L, the median precision at cardinal tilts (4.32;  $N = 96$ ) was significantly  
259 larger than at oblique tilts (3.12), Mann-Whitney U test ( $p = 0.011$ ). For Monkey F, the median  
260 precisions at cardinal (2.77,  $N = 128$ ) and oblique (3.0) tilts were not significantly different  
261 ( $p = 0.12$ ). Together, these results indicate that both stereoscopic and perspective cues contribute  
262 to 3D perception within peripersonal space, and that perspective cues extend 3D perception  
263 beyond the range supported by stereoscopic cues.

264

### 265 **Perceptual cue integration**

266 The previous sections showed that perception was more precise for combined-cue than cue-  
267 isolated stimuli, and that the relative precisions for cue-isolated stimuli were pose-dependent.  
268 Given these results, we next tested if the cues were integrated optimally. That is, if stereoscopic  
269 and perspective cues were dynamically reweighted according to their pose-dependent reliabilities  
270 to maximize the precision of combined-cue stimulus perception. To test this hypothesis, we used  
271 cue integration theory to derive optimal predictions of the combined-cue bias ( $\hat{\mu}_c$ ) and precision



272 ( $\hat{\kappa}_c$ ) from the cue-isolated data (see **Methods**) [30]. We then compared the observed and optimal  
273 combined-cue biases and precisions to determine if the two cues were optimally reweighted on a  
274 trial-by-trial basis.

275         Representative error distributions and von Mises fits are shown for cue-isolated and  
276 combined-cue stimuli along with optimal predictions in **Fig 6A–D**. Observed (blue curves) and  
277 optimal (dashed black curves) combined-cue performances were highly similar. Across all slant–  
278 distance combinations, the observed and optimal biases were not significantly different from each  
279 other (circular median test for multiple samples,  $p = 0.13$ ,  $N = 56$ , both monkeys). Likewise, the  
280 observed and optimal precisions were highly correlated ( $r = 0.94$ ,  $p = 1.39 \times 10^{-27}$ ,  $N = 56$ ),  
281 distributed along the identity line (**Fig 6E**), and not significantly different from each other (Wilcoxon  
282 signed-rank test,  $p = 0.12$ ,  $N = 56$ ). The results were similar with larger stimuli (**S3D Fig**), and  
283 when cue integration was assessed separately for cardinal and oblique tilts (**S6 Fig**). These  
284 results suggest that, like humans, monkeys achieve robust 3D visual perception through the  
285 optimal integration of stereoscopic and perspective cues. Since all of the stimulus conditions were  
286 interleaved and presented pseudo-randomly, cue reweighting had to occur dynamically to match  
287 the vagaries of cue reliabilities that occurred with trial-to-trial changes in surface pose.

288

### 289 **Neuronal models of 3D visual cue integration**

290 We found optimal integration of stereoscopic and perspective cues, consistent with previous  
291 human results [4, 5]. However, previous studies did not consider that combined-cue stimuli  
292 actually contain three cues: stereoscopic, left eye perspective, and right eye perspective [34]. A  
293 distinction between left and right eye perspective cues may seem surprising, but the two retinal  
294 projections of 3D stimuli can differ enough to yield significantly different discrimination  
295 performance [35]. In order to understand how the visual system integrates these three sources of  
296 information, we modeled different neural architectures and compared the model results to the  
297 observed data. If the visual system represented all three cues independently, then the precision  
298 of 3D perception could be greater than observed in this study, and elsewhere [3-5]. This raises  
299 two questions. First, what neural architecture can account for the observed perceptual results?  
300 Second, given the individual cue sensitivities, how close does the visual system come to  
301 maximizing the precision of 3D perception for combined-cue stimuli?

302         To address these questions, we used Bayesian decoding of simulated neuronal population  
303 responses [10]. Responses to stereoscopic, left eye perspective, and right eye perspective cues  
304 were simulated for each monkey based on recordings from neurons in the caudal intraparietal  
305 (CIP) area of the same animal (see **Methods**). Area CIP neurons are implicated in 3D perception

306 since they are selective for 3D surface orientation, and their activity functionally correlates with  
307 behavioral reports of 3D orientation [32, 36-40]. We tested three neural architectures for  
308 combining cue-isolated responses (**Fig 7A**), and decoded the resulting combined-cue  
309 representation to simulate perceptual data (**Fig 7B**). Since the precision (but not the accuracy) of  
310 the simulated perceptual data depended on the architecture, we compared the decoded model  
311 precisions to the observed precisions in the monkey data (**Fig 7C**).

312 The first architecture assumed three independent neuronal populations, each of which  
313 represents tilt based on one of the three cues. Optimal integration is achieved by summing the  
314 three population responses [10] (**Fig 7A**, top). When we compared the monkey and decoded  
315 precisions, we found that the model was significantly more sensitive than the monkeys, Wilcoxon  
316 signed-rank test,  $p = 4.5 \times 10^{-9}$  (**Fig 7C**, orange points). Indeed, if the three cues were represented  
317 independently and optimally integrated, the precision of combined-cue perception would have  
318 been, on average, 2.04 times greater than observed (ratio of decoded/observed precisions). This  
319 confirms our hypothesis that 3D tilt perception is less precise than theoretically possible with three  
320 independent cue representations. However, it is possible that observed tilt perception results from  
321 optimal integration of non-independent neuronal representations [3].

322 The second architecture tested this possibility, and assumed two independent populations  
323 that represent tilt based on either stereoscopic cues or perspective cues (regardless of the  
324 stimulated eye). For the perspective cue population, when both eyes are stimulated, the left and  
325 right eye driven responses are combined with a quadratic nonlinearity and divisively normalized  
326 (**Fig 7A**, middle). A similar model describes V1 responses to compound stimuli, and the  
327 operations combining the responses are widely implicated in neural processing [11-17]. As a  
328 consequence of divisive normalization, the independence of the two perspective cue  
329 representations is reduced, thereby decreasing the improvement in perceptual precision that  
330 results from having two cues. Optimal integration with this architecture is achieved by summing  
331 the stereoscopic and perspective cue population responses. When we compared the observed  
332 and decoded precisions (**Fig 7C**, green points), we found that they were not significantly different  
333 (Wilcoxon signed-rank test,  $p = 0.25$ ). Thus, the perceptual results are consistent with a neural  
334 architecture in which two independent populations represent stereoscopic cues and perspective  
335 cues (from both eyes, combined using nonlinear canonical neural computations).

336 Lastly, we considered the possibility that a single neuronal population estimates tilt from  
337 both stereoscopic and perspective cues. When both eyes are stimulated, responses driven by  
338 each of the three cues are combined with a quadratic nonlinearity and divisively normalized  
339 (**Fig 7A**, bottom). As such, none of the cues are represented independently, and no explicit cue

340 integration is required. When we compared the observed and decoded precisions, we found that  
341 the model was significantly less sensitive than the monkeys, Wilcoxon signed-rank test,  
342  $p = 4.5 \times 10^{-9}$  (**Fig 7C**, magenta points).

343 These results identify a plausible neural architecture that can account for perceptual cue  
344 integration findings in both humans and monkeys, and rule out alternatives. They further identify  
345 the processing of left and right eye perspective cues within a single neuronal population as a  
346 potential factor limiting the precision of 3D perception, and demonstrate that 3D perception is  
347 optimized but not maximized as a result of canonical neural computations.

348

## 349 **Discussion**

350 We evaluated the contributions of stereoscopic and perspective cues to 3D perception in  
351 macaque monkeys. Since the reliability of 3D cues is strongly affected by changes in depth or  
352 slant (**Fig 1**), we used an eight-alternative forced choice tilt discrimination task as a proxy for  
353 estimating how 3D sensitivity depends on object pose (i.e., orientation and position). We found  
354 that 3D perception was generally accurate across a wide range of poses. Instances of poor  
355 accuracy were largely restricted to stereoscopic cue stimuli with particularly low cue reliability.  
356 Thus, poor accuracy presumably reflected low certainty about the plane's tilt and difficulty  
357 performing the task. Precision showed a clear pose dependence. For stereoscopic cues, precision  
358 increased with slant and decreased with distance from the fixation plane. For perspective cues,  
359 precision increased with slant and was independent of distance. At large distances and small  
360 slants, perspective cues were the sole contributor to 3D tilt perception, indicating that perspective  
361 cues extend 3D perception beyond the range supported by stereopsis.

362

## 363 **Evidence for a 3D oblique effect**

364 The oblique effect for 2D tilt is characterized by larger biases and lower precisions at oblique  
365 compared to cardinal tilts [23-25, 27], and thought to reflect a prior for natural scene statistics [41].  
366 A recent human study similarly found an oblique effect for 3D tilt with natural scene patches [27],  
367 consistent with a prior for the statistics of planar tilt [28, 29]. We examined if monkeys show an  
368 oblique effect for planar tilt, while testing for individual differences and cue-specific dependencies.  
369 Monkey L had larger biases at oblique than cardinal tilts in all three cue conditions, and lower  
370 precision at oblique than cardinal tilts in the combined-cue and perspective cue conditions. Such  
371 tilt dependencies were not as evident in Monkey F, indicating individual differences in the 3D  
372 oblique effect, similar to those for the 2D oblique effect in humans [24, 25]. Given the extensive  
373 training with both cardinal and oblique tilts, it is unlikely that training accounts for the oblique effect

374 in Monkey L. Furthermore, both monkeys showed systematic biases with stereoscopic cue stimuli  
375 when precision was low, such that their reports were pulled towards ‘bottom-near’. This is  
376 consistent with the influence of a prior for ground planes, which occur in preponderance in natural  
377 scenes [28, 29]. Thus, Monkey F may have a weaker prior than Monkey L, such that only the  
378 ground plane component had an observable impact on perception. It is unlikely that the bottom-  
379 near bias reflected a preference for making downward saccades since horizontal eye movements  
380 are more accurate than vertical eye movements in humans [42, 43], and the oculomotor systems  
381 of macaques and humans are highly similar [44]. The results thus suggest that 3D perception in  
382 both humans and non-human primates is influenced by a prior for the 3D orientation statistics of  
383 natural scenes, and that the strength of that influence differs across individuals.

384

### 385 **Optimal cue integration**

386 We used cue integration theory to predict combined-cue performance from stereoscopic and  
387 perspective (left and right eyes pooled) cue performances, and found that the cues were optimally  
388 integrated to achieve robust 3D perception. While this is consistent with previous human studies  
389 [4, 5], it is somewhat surprising since the theory assumes independent cue representations, but  
390 complete independence is unlikely (e.g., due to common retinal processing) [3]. The finding thus  
391 implies that the major sources of noise in 3D tilt estimation based on stereoscopic and perspective  
392 cues are largely independent. This could occur if the two estimates are created within different  
393 neuronal populations. Since the stimuli were interleaved, our finding of an optimal integration  
394 strategy further implies that the cues are dynamically reweighted to match the vagaries of cue  
395 reliabilities that occur with moment-to-moment changes in viewing conditions, such as happens  
396 every time the eyes move. Together with previous human studies [3-9], the current findings  
397 suggest that reliability-dependent cue integration is a conserved mechanism by which primates  
398 achieve robust 3D vision, and validate the macaque monkey as an ideal model system for  
399 studying the neural basis of 3D cue integration.

400

### 401 **Canonical computations optimize, but do not maximize 3D perception**

402 We found that the observed perceptual results were optimal for a neural architecture in which two  
403 independent populations represent 3D tilt based on stereoscopic cues and perspective cues. To  
404 account for the perceptual results, it was essential that left and right eye perspective cue  
405 responses be combined with a quadratic nonlinearity and divisively normalized. Due to divisive  
406 normalization, the contributions of the two eyes’ perspective cues to perception will range from  
407 averaging (both signals contribute equally) when they are equally reliable to winner-take-all (only

408 the more reliable signal contributes) when they differ substantially [11]. Thus, the model accounts  
409 for previous human cue integration findings showing cue averaging for balanced perspective cues  
410 [4, 5], and winner-take-all behaviors for imbalanced perspective cues [35]. Since divisive  
411 normalization reduces the independence of the two perspective cue responses, the computation  
412 imposes limits on the precision of perception. Our simulations showed that independent  
413 representations of stereoscopic, left eye perspective, and right eye perspective cues would double  
414 the precision of 3D perception.

415 Why would evolution not select for an integration strategy with higher precision? One  
416 possibility is the biological inefficiency associated with the sheer number of neurons required to  
417 maintain three independent cue representations, and the duplication of computational units to  
418 separately estimate 3D information from left and right eye signals. Another possibility is that 3D  
419 estimates derived from perspective cues are noisy, and combining left and right eye signals with  
420 divisive normalization attenuates that noise. The result suggests that 3D perception is optimized  
421 (through linear combinations of independent stereoscopic and perspective cue population  
422 responses), but is not maximized (due to divisive normalization of left and right eye perspective  
423 signals). We are currently testing this hypothesis with electrophysiological studies in the same  
424 monkeys. The results further serve as a reminder that “optimal” is in the eye of the beholder, and  
425 is most meaningful in the context of a specific neural architecture. We predict that analogous  
426 processes exist in other sensory systems which have multiple inputs sensitive to the same  
427 signals, as occurs in audition, vestibular processing, and bimanual touch.

428

## 429 **Methods**

### 430 **Subjects and preparation**

431 All surgeries and experimental procedures were approved by the Institutional Animal Care and  
432 Use Committee (IACUC) at the University of Wisconsin–Madison (Protocol G005229), and were  
433 in accordance with the National Institutes of Health’s Guide for the Care and Use of Laboratory  
434 Animals. All efforts were taken to ensure the well-being of the animals, including daily enrichment.  
435 Two male rhesus monkeys (*Macaca mulatta*) participated (Monkey L: 5 years of age, ~7.8 kg in  
436 weight; Monkey F: 4 years of age, ~5.5 kg in weight). A Delrin ring for stabilizing the head during  
437 training and experimental sessions was attached to the skull under general anesthesia [32, 38,  
438 39]. After recovery, each monkey was trained to sit in a custom primate chair with head restraint,  
439 and to fixate a visual target within 2° version and 1° vergence windows for a liquid reward. We  
440 verified the ability to perceive stereoscopically-defined depth by having the monkeys fixate

441 simulated targets between -20 and 40 cm of depth from the screen [45]. Binocular eye position  
442 was monitored optically at a sampling rate of 1,000 Hz (EyeLink 1000 plus, SR Research).

443

#### 444 **Experimental control and stimulus presentation**

445 Experimental control was performed using an open-source, network-based parallel processing  
446 framework [45]. Stimuli were created in MATLAB using Psychtoolbox 3 [46], and rendered using  
447 an NVIDIA GeForce GTX 970 graphics card on a Linux workstation (Ubuntu 16.04 LTS, Intel  
448 Xeon Processor, 24 GB RAM). A DLP LED projector (VPixx Technologies, Inc.) was used to rear  
449 project the stimuli onto a polarization preserving screen (Stewart Film Screen, Inc.). Stimuli were  
450 projected at 1,280 x 720 pixel resolution with a 240 Hz refresh rate. The screen distance was ~57  
451 cm. The projected area subtended ~70° x 43° of visual angle. Stereoscopic presentation was  
452 achieved by sequencing the presentation of stimulus ‘half-images’ to each eye (120 Hz/eye) using  
453 a circular polarizer synchronized to the projector. Polarized glasses were worn.

454

#### 455 **Visual stimuli**

456 Planar surfaces were defined using random dot patterns (N = 250 dots). At the plane of fixation,  
457 dots subtended 0.35° of visual angle. The dots were bright (37.8 cd/m<sup>2</sup>) on a gray (12.3 cd/m<sup>2</sup>)  
458 background, measured through the polarized glasses (PR-524 LiteMate, Photo Research). On  
459 the screen, the stimuli were circular in shape and subtended 20° of visual angle.

460 Planes were presented at all combinations of eight tilts (0° to 315° in 45° steps), four slants  
461 (15° to 60° in 15° steps), and either six (Monkey L: 37, 57, 77, 87, 107, and 137 cm) or eight  
462 (Monkey F: 37, 57, 77, 87, 97, 107, 117, and 137 cm) distances. At 37 cm, all dots were in front  
463 of the plane of fixation. At 57 cm, dots were distributed in front of and behind the plane of fixation.  
464 At 77 cm and beyond, all dots were behind the plane of fixation. Presenting the stimuli at distances  
465 where the dots were entirely in front of, distributed about, or entirely behind the plane of fixation  
466 prevented the monkeys from relying on local absolute disparity cues to perform the task, ensuring  
467 that they judged the tilt of the plane [5, 40]. Under natural viewing conditions, changes in slant or  
468 distance affect the retinotopic area subtended by an object. To not confound 2D retinal features  
469 and 3D structure, we held the retinotopic area constant for all stimuli [32, 38, 39].

470 Stimuli were defined by both stereoscopic and perspective cues (‘combined-cue’; **Fig 2B**  
471 and **2C**), stereoscopic cues (**Fig 2D**), or perspective cues (**Fig 2E**). Combined-cue stimuli had a  
472 uniform distribution of dots across the plane. Left and right eye half images were rendered by  
473 using perspective geometry to project each dot onto the appropriate screen position for the given  
474 eye. The perspective cues thus included retinal density gradients, foreshortening, and scaling. To



475 ensure that the perspective cues only provided orientation information [5], the dots were scaled  
476 according to the plane's distance such that their screen size depended only on the slant and tilt.  
477 Stereoscopic cue stimuli were created by defining a uniform distribution of dots on the screen and  
478 using ray tracing to assign each dot to a location on the plane. All dots had a circular shape and  
479 subtended  $0.35^\circ$ , irrespective of the pose. As such, the stereoscopic cue stimuli were designed  
480 to not contain any perspective cues that could be used to judge orientation. This was verified  
481 perceptually (**S4A Fig**). The combined-cue and stereoscopic cue stimuli were presented to both  
482 eyes. The perspective cue stimuli were the same as the combined-cue stimuli, but only one eye  
483 saw the planar stimulus (pseudo-randomly selected each trial) to eliminate stereoscopic cues.  
484 Both eyes saw the fixation target.

485

#### 486 **Tilt discrimination task**

487 The monkeys were trained to discriminate planar tilt in an 8AFC task. Task training began once  
488 a monkey could fixate a target on a blank screen for 2 s. They first learned to perform a two-  
489 alternative (right-near vs. left-near) task with all cue conditions and distances interleaved. The  
490 correct choice target was initially presented at a higher contrast than the distractor, and the  
491 contrast difference was reduced with training. Once an 80% correct rate with equal target  
492 contrasts was reached, all four cardinal tilts were introduced with a target contrast difference.  
493 Once a 50% correct rate was reached with equal contrasts, we started alternating training days  
494 between four cardinal and four oblique tilts. Once a 70% accuracy rate was reached with both  
495 cardinal and oblique tilts, all eight tilts were introduced together. Data collection began after  
496 performance in the 8AFC task stabilized.

497 In the task (**Fig 2A**), a monkey first acquired fixation of a target at the center of an  
498 otherwise blank screen. The target was a red circular dot ( $10.6 \text{ cd/m}^2$  through the polarized  
499 glasses) subtending  $0.3^\circ$  of visual angle. After fixating for 300 ms, a plane centered on the target  
500 was presented for 1,000 ms. Fixation was then held for an additional 500 to 1,500 ms (pseudo-  
501 random duration) with no plane present. The fixation target then disappeared, and eight choice  
502 targets appeared at an eccentricity of  $11^\circ$  with polar angles ranging from  $0^\circ$  to  $315^\circ$  in  $45^\circ$  steps.  
503 The side of the plane nearest the monkey was reported with a saccade to the choice target at the  
504 corresponding polar angle for a liquid reward. A trial was aborted if fixation was broken before the  
505 choice targets appeared or if a choice was not made within 500 ms of their appearance. During  
506 the task, version and vergence were enforced with  $2^\circ$  windows. Offline, we calculated the time-  
507 averaged vergence error during the stimulus presentation for each trial. A  $1^\circ$  vergence window  
508 was then used to eliminate trials with errors  $\geq 0.5^\circ$  in magnitude.

509 Stimuli were presented in a pseudo-random order using a block design. A block consisted  
510 of one repetition of each combination of tilt, slant, distance, and cue type (Monkey L: 576  
511 trials/block; Monkey F: 768 trials/block). The data set included 23,942 trials for Monkey L and  
512 55,726 trials for Monkey F.

513

## 514 **Analyses**

515 To quantify performance, we computed probability density functions describing the errors in  
516 reported tilts as follows. First, we took the difference between the reported tilt and the presented  
517 tilt for each trial:  $\Delta\text{Tilt} = \text{Reported Tilt} - \text{Presented Tilt}$ . Second, we created an error distribution of  
518 reported tilts by calculating the probability that the monkey reported  $\Delta\text{Tilt}$ . This was performed  
519 separately for each combination of slant, distance, and cue type. Depending on the analysis, error  
520 distributions were calculated using data at: (i) one tilt, (ii) cardinal ( $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$ ) or  
521 oblique ( $45^\circ$ ,  $135^\circ$ ,  $225^\circ$ , and  $315^\circ$ ) tilts, or (iii) all tilts. Third, a von Mises probability density  
522 function was fit to each error distribution using maximum likelihood estimation:

523

$$524 \quad VM(\Delta\text{Tilt}) = e^{(\kappa \cdot \cos(\Delta\text{Tilt} - \mu))} / (2\pi \cdot I_0(\kappa)) \quad (\text{equation 1}).$$

525

526 This function has two parameters: the mean ( $\mu$ ) and concentration ( $\kappa$ ), which capture the  
527 accuracy and precision of perception, respectively. The closer  $\mu$  is to 0, the more accurate (less  
528 biased) the judgments. The larger  $\kappa$ , the more concentrated (taller and narrower) the distribution,  
529 indicating more precise judgments. A modified Bessel function of order 0,  $I_0(\kappa)$ , normalizes the  
530 function to have unit area. The tilt sampling resolution limits the maximum  $\kappa$  that can be reliably  
531 estimated. We set an upper bound of  $\kappa = 18$  in the maximum likelihood estimation routine, which  
532 corresponds to ~90% of the probability density function falling within the  $45^\circ$  tilt sampling interval.

533 To evaluate the integration of stereoscopic and perspective cues, we compared the  
534 observed combined-cue bias ( $\mu_c$ ) and precision ( $\kappa_c$ ) to predictions derived from cue integration  
535 theory for circular variables [30]. The predictions were created using the stereoscopic and  
536 perspective cue biases ( $\mu_s$  and  $\mu_p$ , respectively) and precisions ( $\kappa_s$  and  $\kappa_p$ , respectively) taken  
537 from the von Mises fits. The optimal combined-cue parameters (bias:  $\hat{\mu}_c$ ; precision:  $\hat{\kappa}_c$ ) are:

538

$$539 \quad \hat{\mu}_c = \tan^{-1} \left( \frac{\kappa_s \cdot \sin(\mu_s) + \kappa_p \cdot \sin(\mu_p)}{\kappa_s \cdot \cos(\mu_s) + \kappa_p \cdot \cos(\mu_p)} \right) \quad (\text{equation 2})$$

540

541 
$$\text{and } \hat{\kappa}_c = \sqrt{\kappa_s^2 + \kappa_p^2 + 2 \cdot \kappa_s \cdot \kappa_p \cdot \cos(\mu_p - \mu_s)} \quad (\text{equation 3}).$$

542

543 Circular statistics were used for  $\mu$  [47]. Linear statistics were used for  $\kappa$ . All reported  
544  $p$ -values are two-tailed. When multiple statistical comparisons were performed,  $p$ -values were  
545 adjusted using Bonferroni correction.

546

### 547 **Stereoscopic cue controls**

548 We performed two controls to test if perception of the stereoscopic cue stimuli was affected by  
549 perspective cues. First, we tested if the stereoscopic cue stimuli contained perspective cues that  
550 could be used to perform the tilt discrimination task (**S4A Fig**). To elicit stereoscopic percepts,  
551 the stereoscopic cue stimuli were presented binocularly (both eyes saw the planar stimulus). To  
552 eliminate stereoscopic cues, they were presented monocularly (only one eye saw the planar  
553 surface, both eyes saw the fixation point). Above chance performance with monocularly viewed  
554 stimuli would indicate the presence of usable perspective cues. To maximize the potential  
555 perspective cue, the stimuli were presented at the largest tested slant (60°). They were presented  
556 at 57 cm. Parameters were otherwise the same as in the main experiment. All stimuli were  
557 presented interleaved. Monkey L completed 675 trials. Monkey F completed 1,819 trials.

558 Second, we considered if the stereoscopic cue precisions were affected by a potential  
559 stereoscopic–perspective cue conflict (**S4B Fig**). For the stereoscopic cue stimuli, the constant  
560 size, shape, and screen density of the dots can be interpreted as signaling zero slant. For  
561 stereoscopically defined non-zero slants, this could result in a perceived cue conflict which would  
562 increase with dot number since more isotropic dots provide more evidence of zero slant [5]. We  
563 therefore assessed precision with the stereoscopic cue stimuli as a function of dot number. A  
564 decrease in precision at larger dot numbers would indicate a cue conflict. To maximize the  
565 potential conflict, the stimuli were presented at the largest tested slant (60°). They were presented  
566 at 57 cm for Monkey L, and at 57 and 97 cm for Monkey F. Eleven dot numbers ranging from 5  
567 to 250 (in steps of 25 starting at 25) were used. Parameters were otherwise the same as in the  
568 main experiment. All stimuli were presented interleaved. Monkey L completed 2,309 trials.  
569 Monkey F completed 5,844 trials at 57 cm, and 5,225 trials at 97 cm.

570

### 571 **Perspective cue control**

572 We evaluated the perception of perspective cue stimuli after pooling responses to left and right  
573 eye stimulus presentations. To test the underlying assumption that perception was comparable

574 for the two eyes, we independently fit von Mises probability density functions to the error  
575 distributions for each eye. Accuracies and precisions for the two eyes were then compared  
576 (S5 Fig).

577

### 578 **Neuronal models of cue integration**

579 We used Bayesian decoding of model neuronal populations based on recordings from 3D surface  
580 orientation selective neurons in the caudal intraparietal (CIP) area to test if different neural  
581 architectures can account for the perceptual findings [10, 17]. Assuming independent neurons  
582 with Poisson spike count statistics, the probability that tilt ( $t$ ) elicits population response ( $\mathbf{r}$ ) is:

583

$$584 \quad p(\mathbf{r}|t) = \prod_i \frac{e^{-f_i(t)} f_i(t)^{r_i}}{r_i!} \quad (\text{equation 4}).$$

585

586 Here  $f_i(t)$  and  $r_i$  are the  $i^{\text{th}}$  neuron's tilt tuning curve and response, respectively. By Bayes  
587 rule and assuming a uniform prior, the posterior,  $p(t|\mathbf{r})$ , describing the likelihood that  $t$  was  
588 presented given  $\mathbf{r}$  is proportional to equation 4. As the number of neurons increases,  $p(t|\mathbf{r})$   
589 converges to a Gaussian, and assuming  $p(t|\mathbf{r})$  guides behavior, the precision of perception ( $1/\sigma^2$   
590 for that Gaussian) is proportional to the gain of the population activity ( $g$ ). The constant of  
591 proportionality ( $\lambda$ ) depends on the number of neurons and their tuning widths [10].

592 We simulated populations of monkey-specific neuronal tuning curves based on 3D surface  
593 orientation tuning curves measured in area CIP of the same animal (Monkeys L: N = 175; Monkey  
594 F: N = 94). The stimuli used in the neuronal recordings were the same as in the current study,  
595 except that only combined-cue stimuli were shown and the distances were 37, 57, 97, and 137  
596 cm. We fit the tilt tuning curves at each slant–distance combination with a von Mises function [38].  
597 Using these fits, we calculated the mean response amplitude and tuning width across neurons for  
598 each slant–distance combination and monkey, and linearly interpolated the values for untested  
599 distances. Using these parameters (DC offsets were not included in the model), we simulated 72  
600 CIP neurons for each monkey, with 5° increments in tilt preference.

601 To determine the proportionality constant ( $\lambda$ ) relating population gain to perceptual  
602 precision, we decoded the simulated population activity after scaling the responses by  $\lambda(\kappa_{\text{TW}})$ ,  
603 which depended on the pose-specific tuning width ( $\kappa_{\text{TW}}$ ) of the model neurons. We tried several  
604 functions to describe the relationship between  $\lambda$  and  $\kappa_{\text{TW}}$  (linear, exponential, double exponential,  
605 and two phase exponential). For each model, the parameters were fit to minimize the difference  
606 between the distributions of tilt errors made by the monkey and the decoded model posteriors.

607 Fitting was performed separately for the two monkeys. Akaike's Information Criterion was used  
 608 to select the best fitting  $\lambda$  function. The best fit was provided by the exponential function (Monkey  
 609 L:  $r = 0.89$ ,  $p = 5.3 \times 10^{-7}$ ; Monkey F:  $r = 0.82$ ,  $p = 9.4 \times 10^{-7}$ ),  $\lambda = DC + G \cdot \exp(\alpha \cdot \kappa_{TW})$ , which was  
 610 used in the simulations (Monkey L:  $DC = -1.6 \times 10^{-3}$ ,  $G = 0.93$ ,  $\alpha = -1.87$ ; Monkey F:  $DC = 2 \times 10^{-4}$ ,  
 611  $G = 0.49$ ,  $\alpha = -1.89$ ).

612 Next, we found response amplitudes for the cue-isolated conditions that minimized the  
 613 difference between the distributions of tilt errors made by the monkey and the decoded model  
 614 posteriors. For this, we assumed that the neuronal tuning widths did not differ across the visual  
 615 cue conditions since CIP tilt tuning widths are similar regardless of the defining cue [36, 37]. The  
 616 tilt tuning curves of the simulated neurons were thus:

$$617 \quad f_i(t|s, d, c) = g_{s,d,c} \cdot \exp(\kappa_{s,d} \cdot [\cos(t - \mu_i) - 1]) \quad (\text{equation 5}).$$

618  
 619 Here  $f_i(t|s, d, c)$  is the  $i^{\text{th}}$  neuron's tilt tuning curve for a given slant ( $s$ ), distance ( $d$ ), and  
 620 visual cue ( $c$ ). The gain ( $g_{s,d,c}$ ) depended on slant, distance, and visual cue. The tuning width  
 621 ( $\kappa_{s,d}$ ) depended on slant and distance. The preferred tilt is  $\mu_i$ .

622 The model tuning curves were used to simulate neuronal population responses with  
 623 Poisson variability for the cue-isolated conditions: stereoscopic ( $r_S$ ), left eye perspective ( $r_{P_L}$ ),  
 624 and right eye perspective ( $r_{P_R}$ ). We tested three strategies for integrating these responses to  
 625 create a combined-cue representation ( $r_C$ ) that was decoded after scaling the responses by  
 626  $\lambda(\kappa_{TW})$ , see **Fig 7**. Since equation 4 converges to a Gaussian, the decoded posteriors were fit  
 627 with Gaussian probability functions. To allow for a direct comparison, we refit the monkey tilt error  
 628 distributions with Gaussians. Precision estimates from the von Mises and Gaussian fits were  
 629 highly correlated (Monkey L:  $r = 1.0$ ,  $p = 9.8 \times 10^{-25}$ ; Monkey F:  $r = 0.99$ ,  $p = 3.9 \times 10^{-26}$ ).

630 *Three populations.* This model assumes three independent neuronal populations, each of  
 631 which represents tilt based on a different cue: stereoscopic, left eye perspective, or right eye  
 632 perspective. Optimal cue integration was achieved by summing the three cue-isolated population  
 633 responses to create a combined-cue representation,  $r_C$  [10]:  
 634

$$635 \quad r_C = \overset{\text{stereo. pop.}}{\widehat{r}_S} + \overset{\text{left persp. pop.}}{\widehat{r}_{P_L}} + \overset{\text{right persp. pop.}}{\widehat{r}_{P_R}} \quad (\text{equation 6}).$$

636  
 637 *Two populations.* This model assumes two independent neuronal populations which  
 638 represent tilt based on stereoscopic cues or perspective cues from both eyes. The response of  
 639

640 the perspective population to combined-cue stimuli is the divisively normalized sum of the squared  
641 single eye responses. Quadratic nonlinearities and divisive normalization are canonical  
642 computations that are broadly implicated in neural processing [11-17]. Optimal cue integration  
643 was achieved by summing the two cue-isolated population responses to create a combined-cue  
644 representation,  $r_C$  [10]:

$$645 \quad r_C = \overbrace{\widehat{r}_S}^{\text{stereo. pop.}} + \frac{\overbrace{r_{P_L}^2 + r_{P_R}^2}^{\text{single persp. pop.}}}{r_{P_L} + r_{P_R}} \quad (\text{equation 7}).$$

646  
647 *One population.* This model assumes a single population of neurons that estimates tilt  
648 from both stereoscopic and perspective cues. The response of this population to combined-cue  
649 stimuli is the divisively normalized sum of the squared cue-isolated responses:

$$650 \quad r_C = \frac{\overbrace{r_S^2 + r_{P_L}^2 + r_{P_R}^2}^{\text{one pop.}}}{r_S + r_{P_L} + r_{P_R}} \quad (\text{equation 8}).$$

651  
652  
653 There are no free parameters in any of the models, so they can be directly compared. The  
654 responses of all three models to cue-isolated stimuli are equivalent, so distinguishing between  
655 them requires that their combined-cue predictions be compared to the performance of the  
656 monkeys.

657  
658

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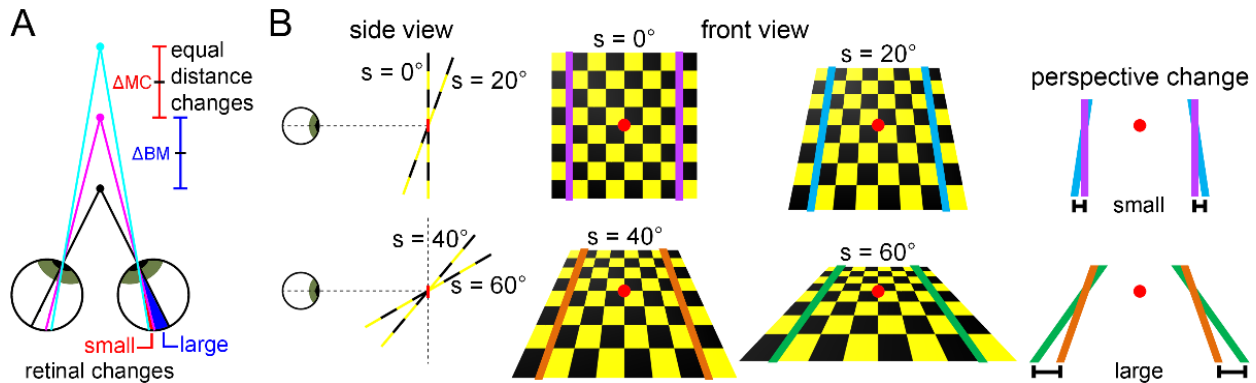
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761 **Figures and figure legends**

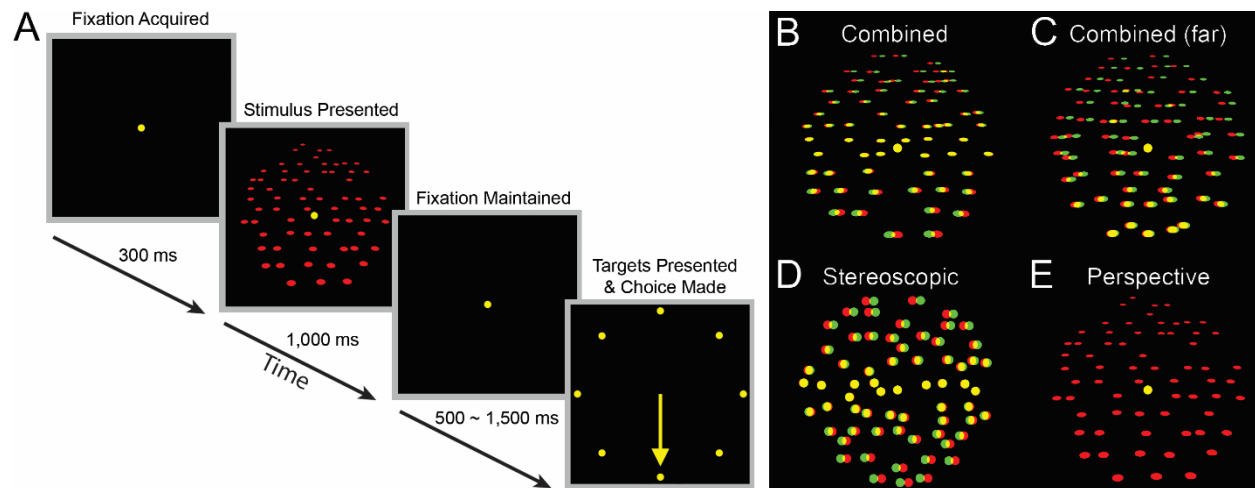
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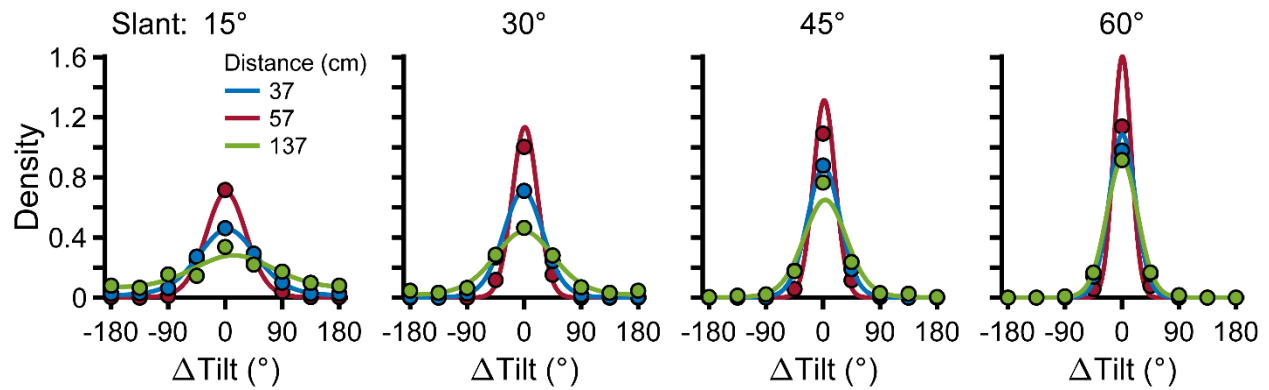
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765 **Fig 1. 3D cue reliabilities depend on object pose. (A)** Stereoscopic cue reliability decreases  
 766 with distance. Equivalent changes in distance result in smaller retinal image changes at greater  
 767 distances. The distance between the black and magenta dots ( $\Delta BM$ ) is equal to the distance  
 768 between the magenta and cyan dots ( $\Delta MC$ ), but the retinal change is larger for  $\Delta BM$  than  $\Delta MC$ .  
 769 **(B)** The reliability of perspective cues increases with orientation in depth (slant). The rate at which  
 770 parallel lines converge in a 2D projection increases with slant. This is illustrated with a  
 771 checkerboard rotated about the horizontal axis passing through the red dot. Colored lines are  
 772 parallel in the world. A  $20^\circ$  slant ( $s$ ) rotation produces a smaller perspective change between  
 773 slants of  $0^\circ$  and  $20^\circ$  (top row) than slants of  $40^\circ$  and  $60^\circ$  (bottom row).



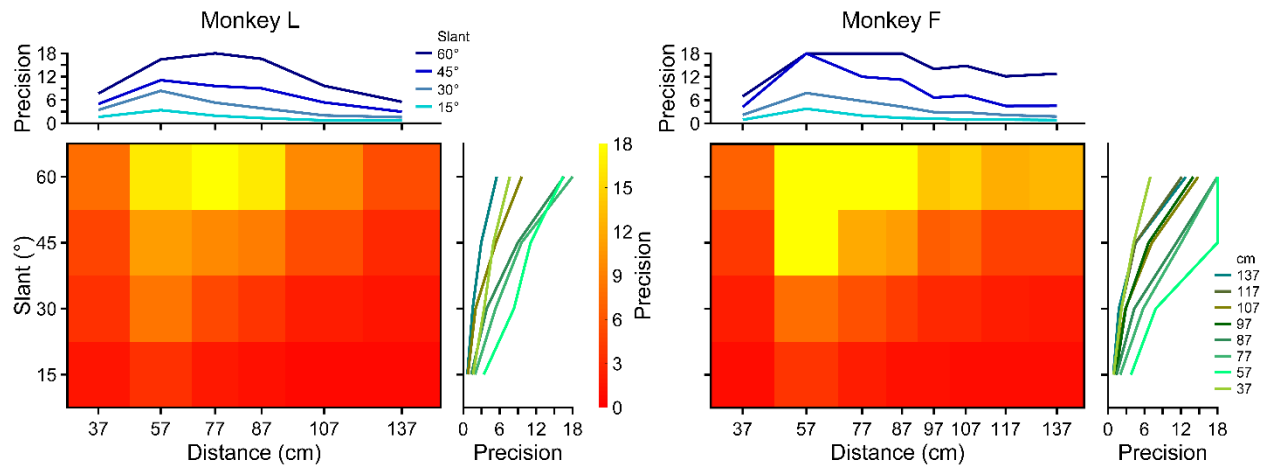
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776 **Fig 2. Discrimination task and stimuli.** (A) Eight alternative tilt discrimination task. Fixation was  
777 maintained on a target for 300 ms. A plane centered on the target was then presented for 1,000  
778 ms. Fixation was then held for an additional 500-1,500 ms before the target disappeared and  
779 eight choice targets appeared. The tilt of the plane was reported through a saccade to the  
780 corresponding choice target. For example, the bottom target for a bottom-near (tilt =  $270^\circ$ ) plane.  
781 (B-E) Example planes (tilt =  $270^\circ$ , slant =  $60^\circ$ ). For clarity, dot size is exaggerated and dot number  
782 reduced from the actual experiments. Stimuli are illustrated as red-green anaglyphs.  
783 (B) Combined-cue stimulus at 57 cm (fixation distance). (C) Combined-cue stimulus at 77 cm (all  
784 dots behind fixation). (D) Stereoscopic cue stimulus at 57 cm. (E) Perspective cue stimulus at 57  
785 cm (left eye presentation).



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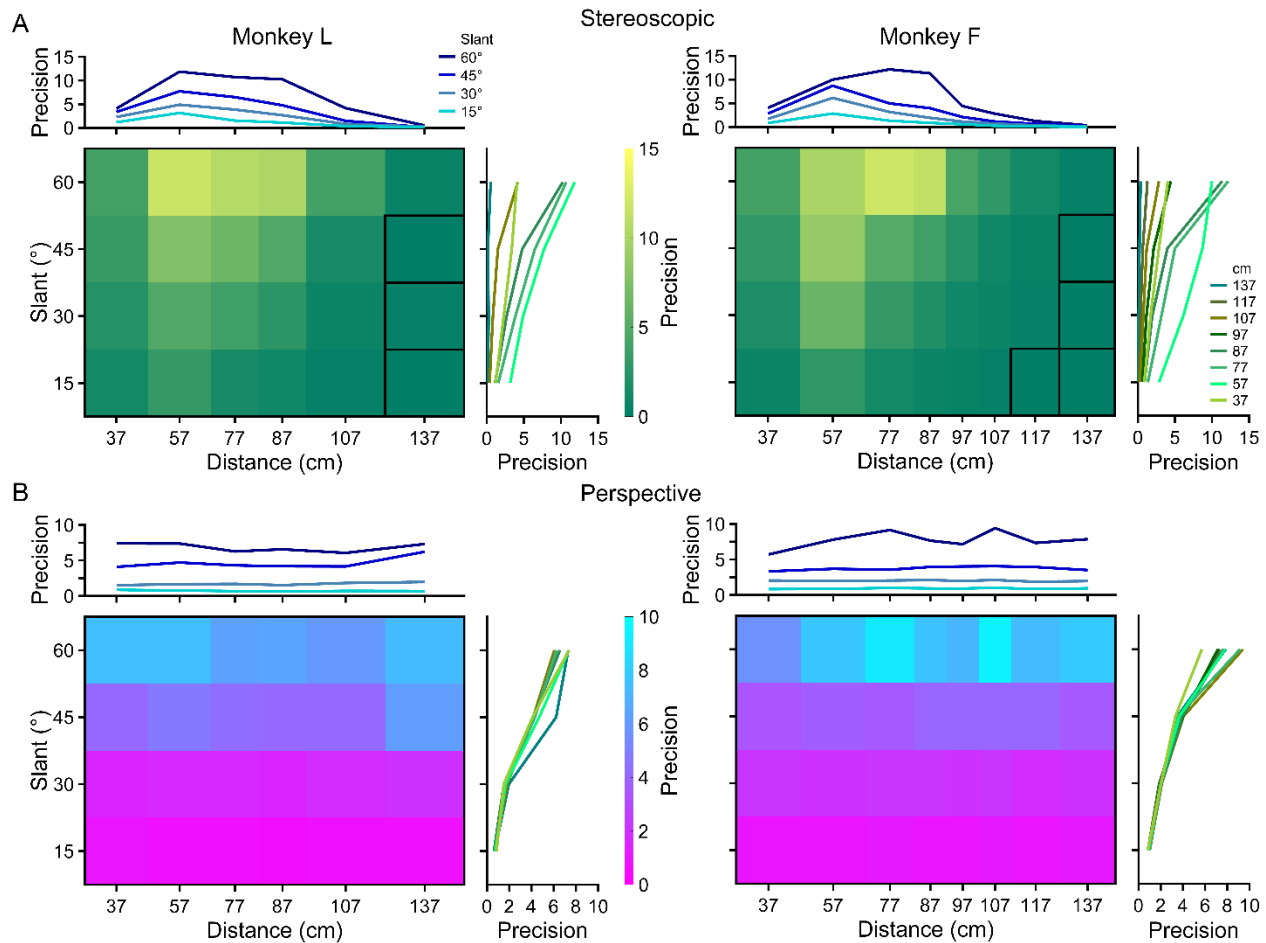
787 **Fig 3. Tilt discrimination with combined-cue stimuli.** Each curve shows a probability density  
788 function describing the errors in reported tilts made by Monkey L with combined-cue stimuli,  
789 calculated using all 8 tilts. Each column shows densities at a given slant, and colors correspond  
790 to different distances. Correct choices correspond to  $\Delta\text{Tilt} = 0^\circ$ . The probability that an error of a  
791 given  $\Delta\text{Tilt}$  was made is shown with a point. Solid curves are von Mises density fits used to  
792 quantify the accuracy and precision of perception. All curves peaked close to  $\Delta\text{Tilt} = 0^\circ$ , indicating  
793 that performance was accurate. Taller and narrower densities indicate greater precision. Precision  
794 increased monotonically with slant (curves grow taller from left to right), and showed an inverted  
795 U-shape as a function of distance (red curves are taller than blue and green curves).



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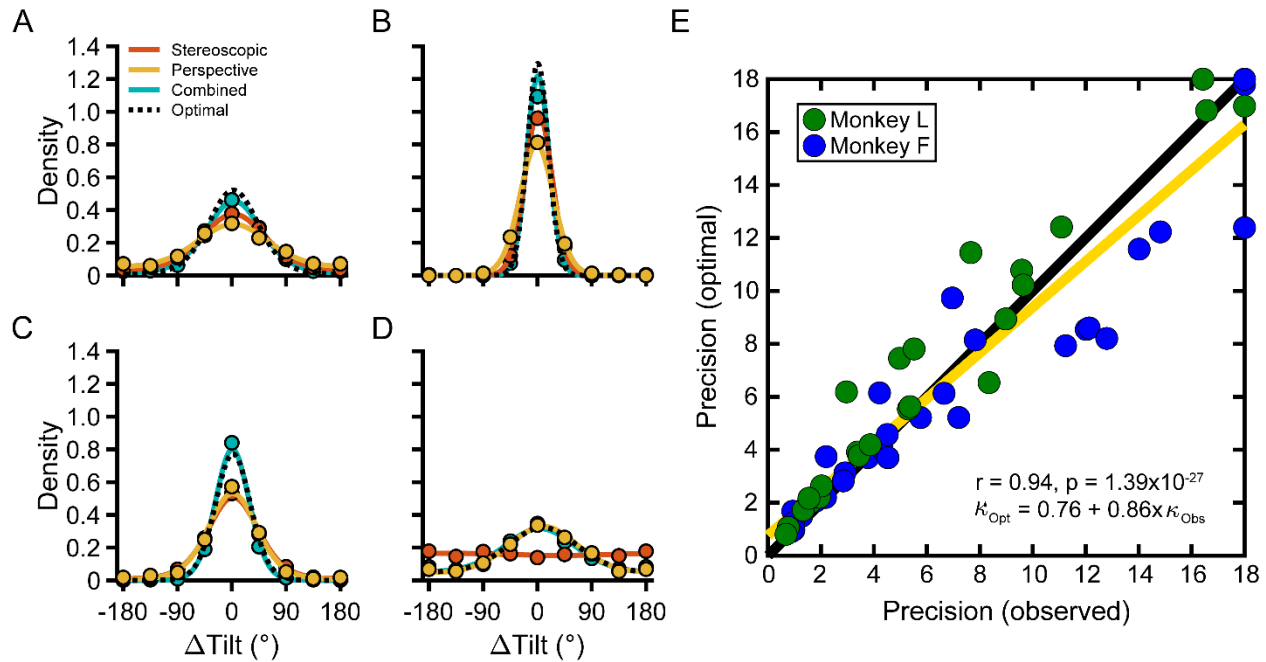
798 **Fig 4. Precision of tilt perception with combined-cue stimuli.** Heat maps showing the  
799 precision ( $\kappa$ ) of tilt perception as a function of slant and distance for Monkeys L (left) and F (right),  
800 calculated using all 8 tilts. Red hues indicate lower precision and yellow hues indicate higher  
801 precision. Precision peaked at the largest tested slant ( $60^\circ$ ) and 20 cm behind the plane of fixation  
802 ( $57\text{ cm}$ ). The larger the slant, the slower performance fell off with distance, giving a wedge-shaped  
803 appearance to the precision landscapes. Right marginal curves show  $\kappa$  as a function of slant for  
804 each distance. Precision increased monotonically with slant. Upper marginal curves show  $\kappa$  as a  
805 function of distance for each slant. Precision showed an inverted U-shape as a function of  
806 distance. An upper bound of 18 was set on  $\kappa$  (see **Methods**).





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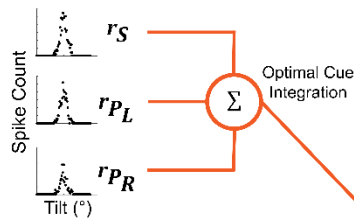
809 **Fig 5. Precision of tilt perception with cue-isolating stimuli. (A)** Stereoscopic cue stimuli.  
810 Precision ( $\kappa$ ) increased monotonically with slant, and showed an inverted U-shape as a function  
811 of distance. Performance was at chance levels for combinations of small slants and large  
812 distances (outlined in black). **(B)** Perspective cue stimuli. Precision increased monotonically with  
813 slant, and was largely independent of distance. Plotted as in **Fig 4**.



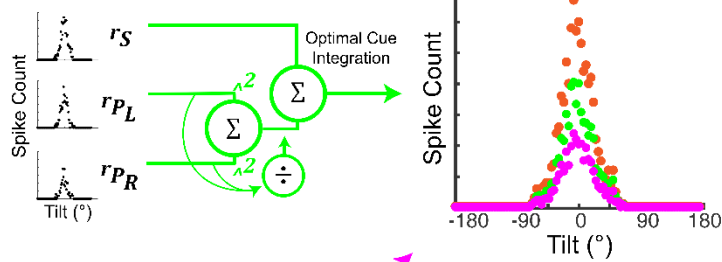
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816 **Fig 6. Optimal cue integration.** (A-D) Representative densities. Solid curves show von Mises  
817 fits for each cue type (colors) and dotted back lines show optimal combined-cue performance.  
818 (A) Slant = 15°, distance = 77 cm (Monkey F). Stereoscopic  $\kappa >$  perspective  $\kappa$ . (B) Slant = 60°,  
819 distance = 87 cm (Monkey L). Stereoscopic  $\kappa >$  perspective  $\kappa$ . (C) Slant = 60°, distance = 107 cm  
820 (Monkey L). Stereoscopic  $\kappa \approx$  perspective  $\kappa$ . (D) Slant = 45°, distance = 117 cm (Monkey F).  
821 Stereoscopic  $\kappa \ll$  perspective  $\kappa$ . Combined-cue perception depended entirely on perspective  
822 cues. (E) Each point shows the optimal vs. observed combined-cue precision ( $\kappa$ ) for a single  
823 slant–distance combination and monkey. The type-II regression line is shown in yellow ( $\kappa = 18$   
824 were excluded from the fit). Combined-cue precision was well predicted by optimal cue integration  
825 across a broad range of poses with different relative isolated-cue precisions.

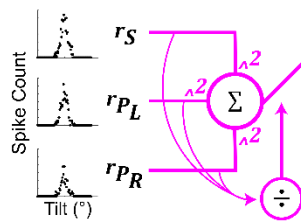
## A Three Populations



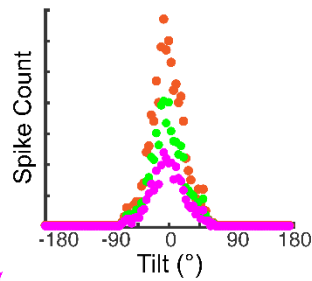
## Two Populations



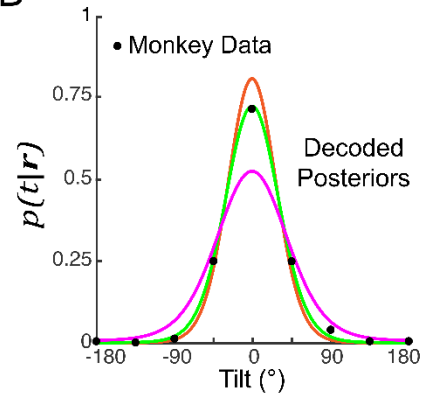
## One Population



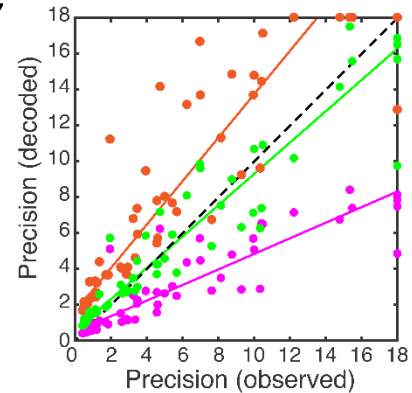
Combined-Cue Representation



## B



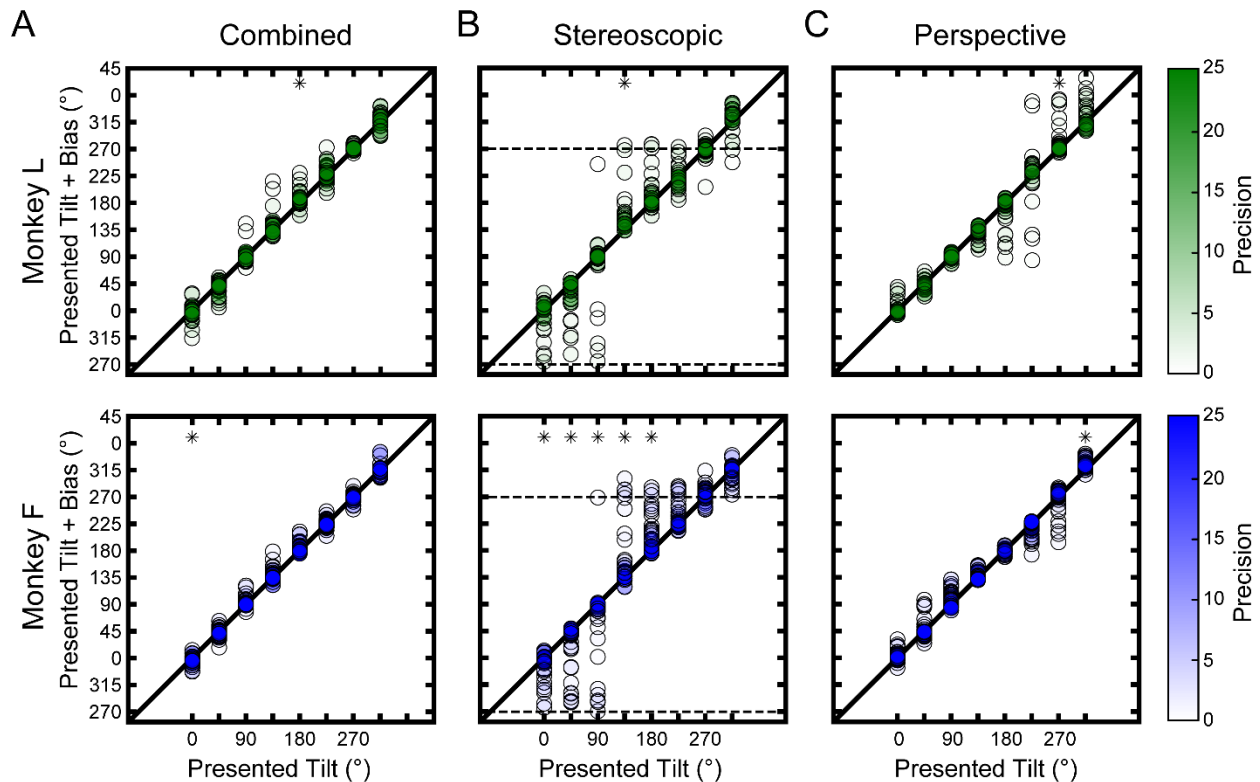
## C



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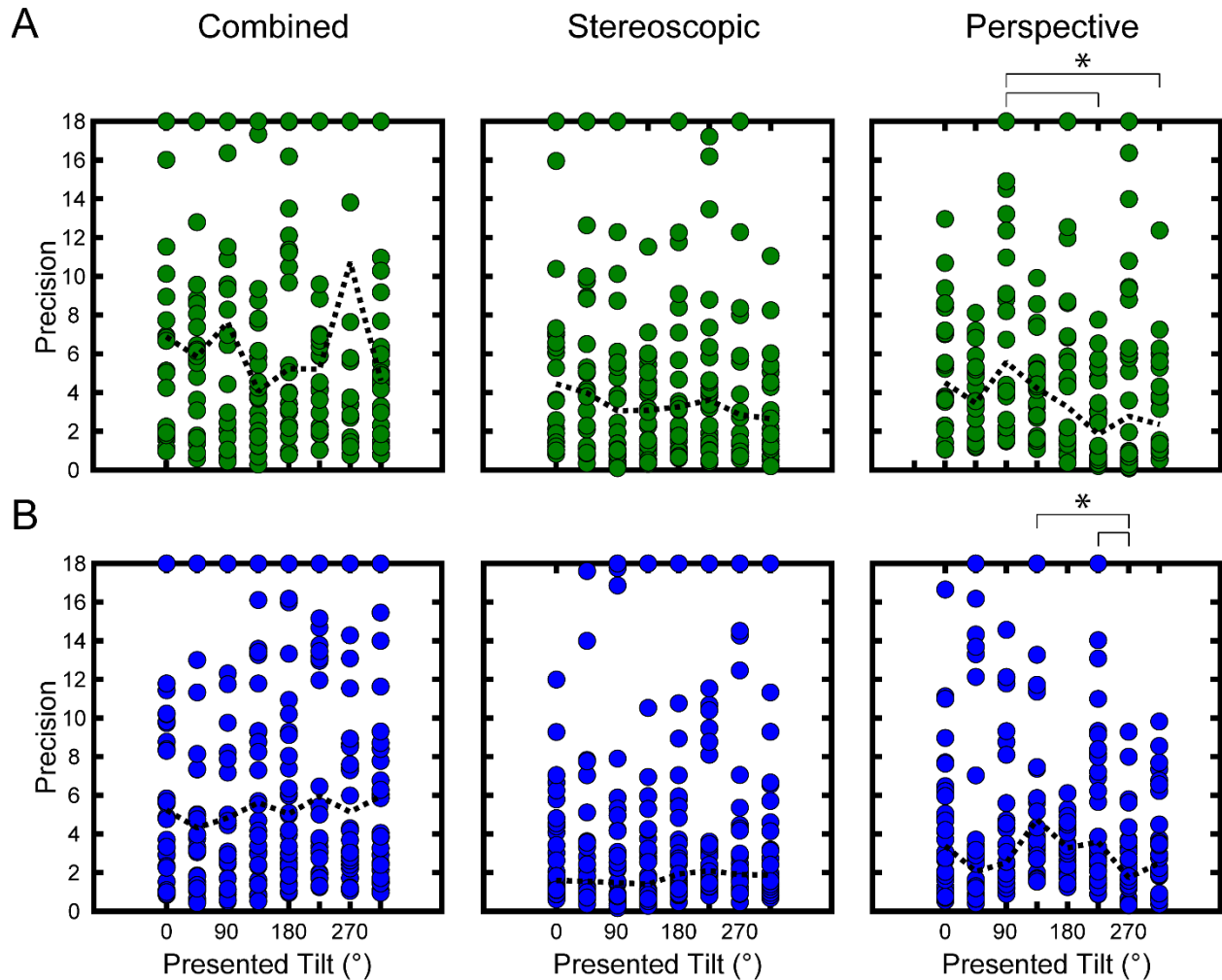
828 **Fig 7. Cue integration is optimized but not maximized. (A)** Schematics of three possible neural  
829 architectures for integrating population responses to stereoscopic cues ( $r_S$ ), left eye perspective  
830 cues ( $r_{PL}$ ), and right eye perspective cues ( $r_{PR}$ ). Top: Three independent populations represent  
831 each cue (orange). Middle: Two independent populations represent stereoscopic cues and  
832 perspective cues regardless of the stimulated eye (green). Bottom: One population estimates tilt  
833 using all three cues (magenta). Resulting combined-cue representations for each architecture are  
834 shown to the right. **(B)** Tilt posteriors,  $p(t|r)$ , decoded from the combined-cue representations.  
835 Black dots show corresponding data from Monkey L. Given the same cue-isolated responses,  
836 precision was greatest for the three population model and lowest for the one population model.  
837 Note the close correspondence between the monkey data and the two population model's  
838 decoded posterior. **(C)** Comparison of decoded model precisions and observed monkey  
839 precisions. Each point shows precision for a single pose. The three population model was more  
840 precise than the monkeys (nearly all points are above the identity line, black dashed). The two  
841 population model closely matched the monkeys' performance (points are distributed about the  
842 identity line). The one population model was less precise than the monkeys (nearly all points are  
843 below the identity line). Solid lines show type-II regressions ( $\kappa = 18$  excluded).

844 **Supporting information**



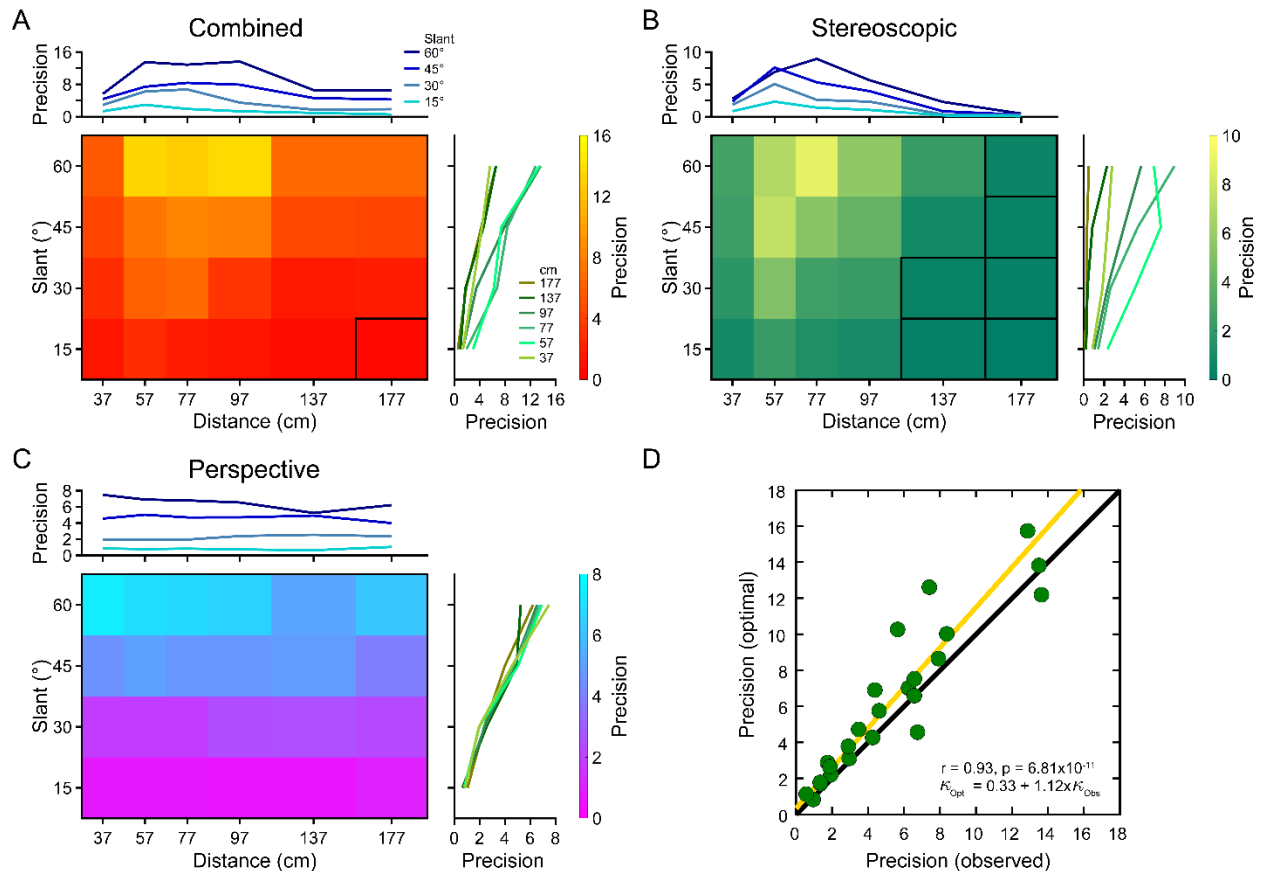
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847 **S1 Fig. Bias as a function of tilt for each monkey and cue condition.** The abscissa indicates  
848 the presented tilt and the ordinate indicates the presented tilt plus bias ( $\mu$ ). Black diagonals are  
849 identity lines. Greater vertical distance from identity indicates greater bias. Each point indicates  
850 the bias for a single pose. The fill opacity indicates the precision ( $\kappa$ ). A circular median test  
851 was used to assess if the biases at each tilt were significantly different from  $0^\circ$ , corrected for multiple  
852 comparisons ( $N = 8$  tilts). Asterisks mark significant biases. **(A)** Combined-cue stimuli. Significant  
853 biases occurred at  $180^\circ$  (median  $\mu = 4.89^\circ$ ,  $p = 2.77 \times 10^{-4}$ ) for Monkey L, and at  $0^\circ$  (median  
854  $\mu = -4.20^\circ$ ,  $p = 5.35 \times 10^{-4}$ ) for Monkey F. In both cases, the median biases were small compared  
855 to the  $45^\circ$  tilt sampling interval. Absolute bias and precision were negatively correlated: Spearman  
856  $r = -0.64$ ,  $p = 3.20 \times 10^{-52}$  ( $N = 448$  slant  $\times$  distance  $\times$  tilt combinations, both monkeys).  
857 **(B)** Stereoscopic cue stimuli. Significant biases occurred at  $135^\circ$  (median  $\mu = 16.03^\circ$ ,  
858  $p = 2.77 \times 10^{-4}$ ) for Monkey L, and at  $0^\circ$  (median  $\mu = -6.40^\circ$ ,  $p = 2.10 \times 10^{-3}$ ),  $45^\circ$  (median  $\mu = -4.80^\circ$ ,  
859  $p = 2.10 \times 10^{-3}$ ),  $90^\circ$  (median  $\mu = -7.61^\circ$ ,  $p = 2.10 \times 10^{-3}$ ),  $135^\circ$  (median  $\mu = 15.69^\circ$ ,  $p = 1.13 \times 10^{-4}$ ),  
860 and  $180^\circ$  (median  $\mu = 19.26^\circ$ ,  $p = 1.93 \times 10^{-5}$ ) for Monkey F. Biases were most prevalent at low  
861 precisions, and the direction of the biases was consistent with perception being pulled towards a  
862 tilt of  $270^\circ$  (bottom-near). Absolute bias and precision were negatively correlated: Spearman  
863  $r = -0.67$ ,  $p = 1.29 \times 10^{-59}$  ( $N = 448$ ). **(C)** Perspective cue stimuli. Significant biases occurred at  
864  $270^\circ$  (median  $\mu = 3.07^\circ$ ,  $p = 1.54 \times 10^{-3}$ ) for Monkey L, and at  $315^\circ$  (median  $\mu = 8.92^\circ$ ,  
865  $p = 2.46 \times 10^{-7}$ ) for Monkey F. Absolute bias and precision were negatively correlated: Spearman  
866  $r = -0.65$ ,  $p = 1.79 \times 10^{-54}$ ;  $N = 448$ ).



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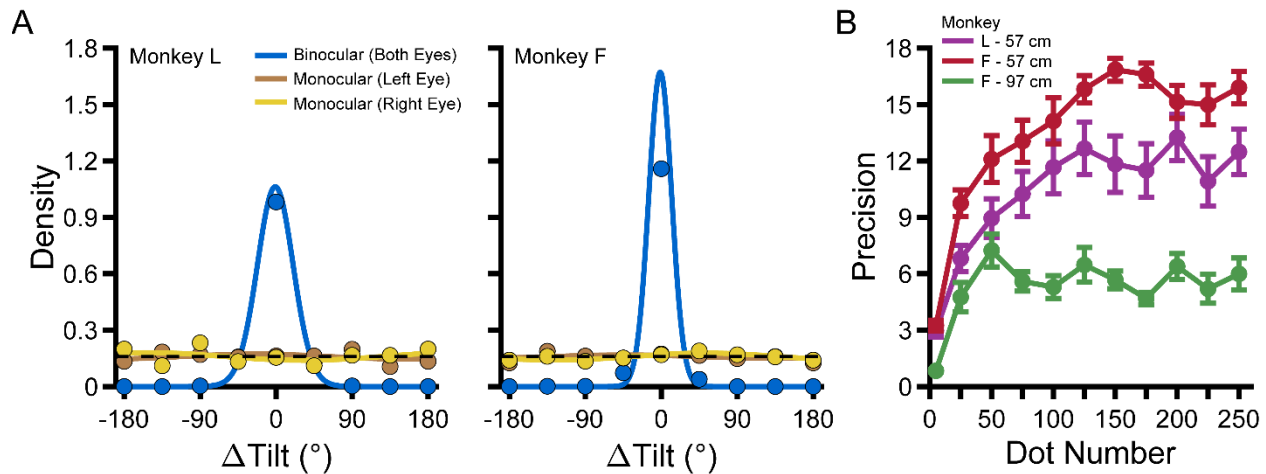
869 **S2 Fig. Precision as a function of tilt for each monkey and cue condition.** The abscissa  
870 indicates the presented tilt and the ordinate indicates the precision of tilt perception ( $\kappa$ ). Each  
871 point shows the precision for a single pose. Dashed black lines trace the median precisions. A  
872 Kruskal-Wallis test followed by Tukey's honestly significant difference test was used to assess  
873 pairwise differences. Bracketed asterisks mark significant differences. **(A)** Combined-cue stimuli.  
874 Precision did not depend significantly on tilt (Monkey L:  $p = 0.15$ ; Monkey F:  $p = 0.90$ ).  
875 **(B)** Stereoscopic cue stimuli. Precision did not depend significantly on tilt (Monkey L:  $p = 0.69$ ;  
876 Monkey F:  $p = 0.25$ ). **(C)** Perspective cue stimuli. For Monkey L, the precision at  $90^\circ$  was  
877 significantly greater than at  $225^\circ$  ( $p = 9.27 \times 10^{-4}$ ) and  $315^\circ$  ( $p = 0.04$ ). For Monkey F, the precision  
878 at  $270^\circ$  was significantly lower than at  $135^\circ$  ( $p = 1.06 \times 10^{-3}$ ) and  $225^\circ$  ( $p = 0.02$ ).



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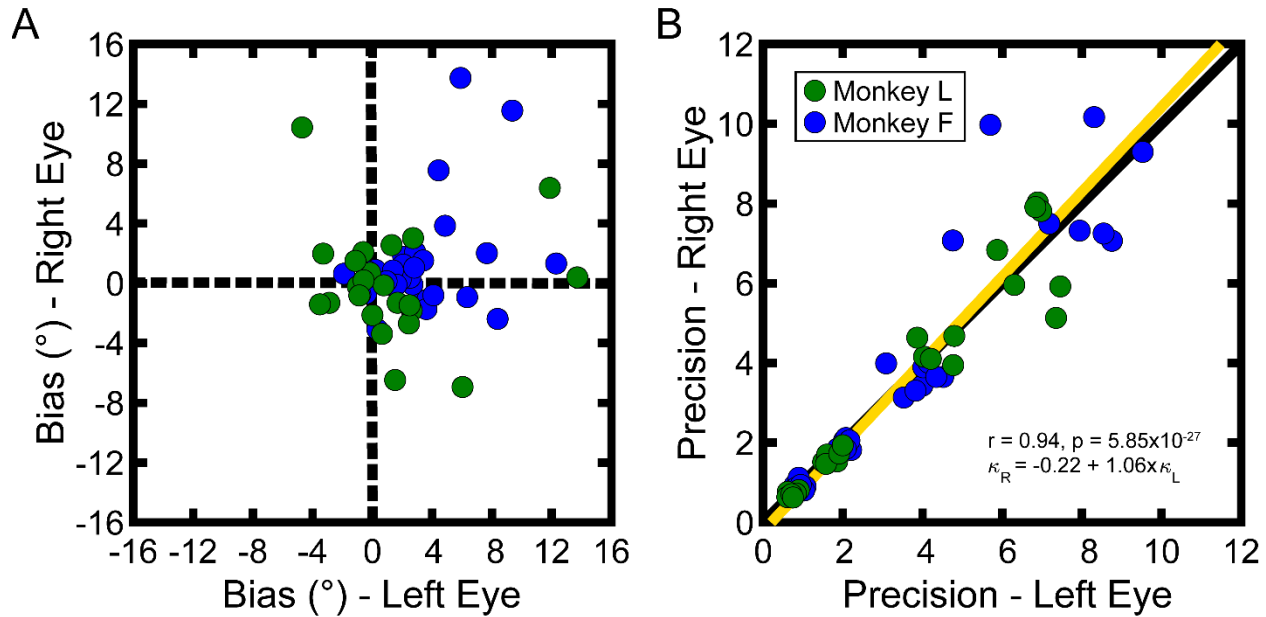
881 **S3 Fig. Precision and cue integration with larger stimuli.** Monkey L also performed the tilt  
 882 discrimination task with 30° stimuli defined by 550 dots. Four slants (15°, 30°, 45°, and 60°) and  
 883 six distances (37, 57, 77, 97, 137, and 177 cm) were presented. A total of 7,508 trials were  
 884 completed. **(A–C)** Heat maps showing precision ( $\kappa$ ) as a function of slant and distance for each  
 885 cue condition. Poses at which performance was not different from chance (Rayleigh test for  
 886 circular uniformity, corrected for multiple comparisons) are outlined in black. **(A)** Combined-cue  
 887 stimuli. **(B)** Stereoscopic cue stimuli. By 177 cm, performance was at chance levels for all slants.  
 888 **(C)** Perspective cue stimuli. **(D)** Cue integration. Each point shows the optimal vs. observed  
 889 combined-cue precision ( $\kappa$ ) for a single pose. The yellow line is the type-II regression line.  
 890 Combined-cue precision was well predicted by optimal cue integration across a broad range of  
 891 poses with different relative isolated-cue precisions.





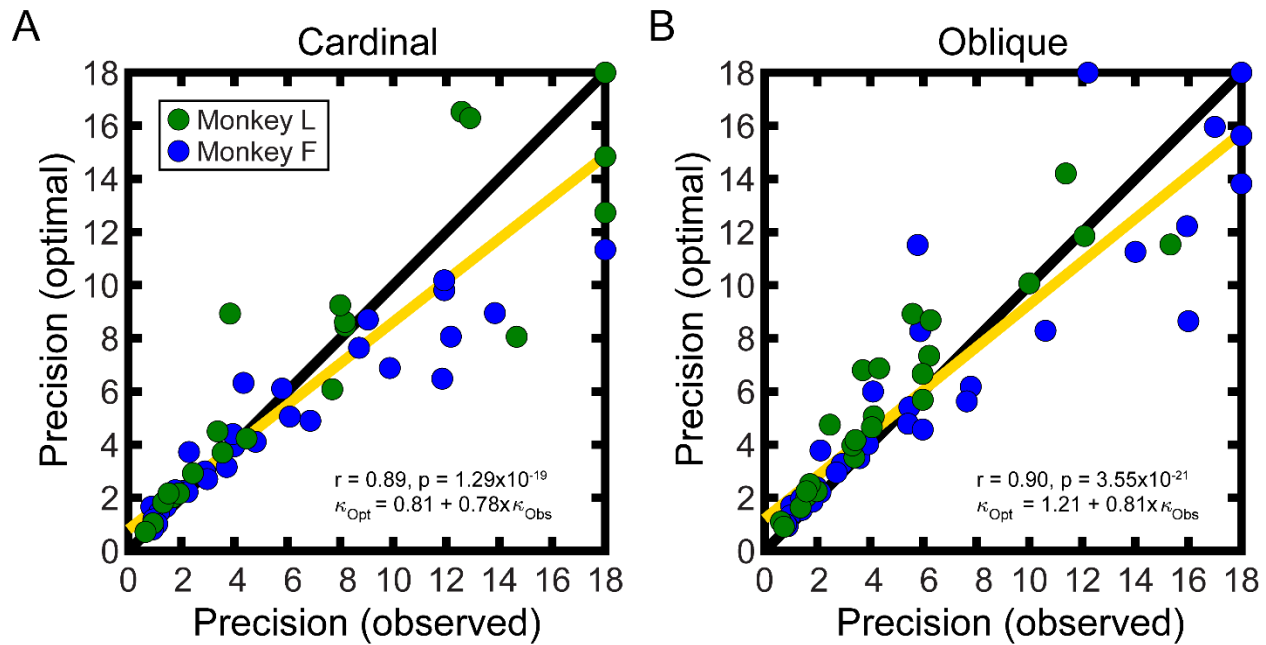
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894 **S4 Fig. Stereoscopic cue controls.** (A) Probability density functions with von Mises fits  
895 describing the errors in reported tilts made by Monkey L (left) and Monkey F (right) with  
896 stereoscopic cue stimuli, calculated using all 8 tilts. Stimuli were viewed binocularly (both eyes  
897 saw the planar stimulus; blue curves) or monocularly (one eye saw the planar stimulus; right eye  
898 stimulated: yellow; left eye stimulated: orange). Chance performance is indicated by the black  
899 dashed line. As expected, a Rayleigh test for circular uniformity confirmed significant binocular  
900 performance (Monkey L:  $p = 1.62 \times 10^{-230}$ ; Monkey F:  $p = 2.23 \times 10^{-308}$ ). In the monocular viewing  
901 conditions, performance was not significantly different from chance (Monkey L: left eye  $p = 0.72$ ,  
902 right eye  $p = 0.62$ ; Monkey F: left eye  $p = 0.69$ , right eye:  $p = 0.21$ ). Thus, the stimuli contained  
903 no usable perspective information for performing the task. (B) Precision ( $\kappa$ ) as a function of dot  
904 number, tested at 57 cm (Monkey L: purple; Monkey F: red) and 97 cm (Monkey F: green). Error  
905 bars show SEM across sessions. Precision depended significantly on dot number (Kruskal-Wallis  
906 test; Monkey L:  $p = 1.02 \times 10^{-7}$ ; Monkey F:  $p = 3.82 \times 10^{-12}$  at 57 cm; Monkey F:  $p = 8.71 \times 10^{-11}$  at 97  
907 cm). The initial increase with dot number was expected since more dots provide greater signal for  
908 performing the task. To test if any differences were the result of a decrease in precision, we ran  
909 pairwise comparisons using Tukey's honestly significant difference test. In each case of a  
910 significant difference, the precision at the larger dot number was greater than at the smaller dot  
911 number. There were no significant differences between dot numbers  $\geq 75$ . Thus, precision  
912 increased monotonically with dot number, suggesting that our stereoscopic cue precision  
913 estimates were not affected by a stereoscopic–perspective cue conflict.



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**S5 Fig. Perspective cue control.** Each point corresponds to a single pose (Monkey L: N = 24; Monkey F: N = 32). **(A)** Right eye vs. left eye biases. Biases clustered near zero. The average difference between right and left eye biases was  $-2.12^\circ$  for Monkey L and  $-1.87^\circ$  for Monkey F. The difference was not significant for Monkey L (circular median test for multiple samples,  $p = 0.25$ ), but was significant for Monkey F ( $p = 4.65 \times 10^{-4}$ ). Although the difference was significant for Monkey F, it was less than for Monkey L and much smaller than the  $45^\circ$  tilt sampling interval. Thus, biases were small and comparable for the two eyes. **(B)** Right eye vs. left eye precisions. Across the two monkeys, the right and left eye precisions were highly correlated ( $r = 0.94$ ,  $p = 5.85 \times 10^{-27}$ ). The intercept ( $-0.22$ ) and slope ( $1.06$ ) of the type-II regression line (yellow) nearly specified the identity line (black diagonal). The precisions for the two eyes were not significantly different (Wilcoxon signed-rank test; Monkey L:  $p = 0.65$ ; Monkey F:  $p = 0.16$ ). The four clusters correspond to the four slants. Thus, precisions were comparable for the two eyes.



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930 **S6 Fig. Cue integration at cardinal and oblique tilts.** Each point shows the optimal vs.  
931 observed combined-cue precision for a single pose (Monkey L: N = 24; Monkey F: N = 32). Type-  
932 II regression lines are shown in yellow ( $\kappa = 18$  excluded). **(A)** Cue integration at cardinal tilts.  
933 **(B)** Cue integration at oblique tilts.