Using deep maxout neural networks to improve the accuracy of function prediction from protein interaction networks

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Abstract

Protein-protein interaction network data provides valuable information that infers direct links between genes and their biological roles. This information brings a fundamental hypothesis for protein function prediction that interacting proteins tend to have similar functions. With the help of recently-developed network embedding feature generation methods and deep maxout neural networks, it is possible to extract functional representations that encode direct links between protein-protein interactions information and protein function. Our novel method, STRING2GO, successfully adopts deep maxout neural networks to learn functional representations simultaneously encoding both protein-protein interactions and functional predictive information. The experimental results show that STRING2GO outperforms other network embedding-based prediction methods and one benchmark method adopted in a recent large scale protein function prediction competition.

Introduction

The realisation of the complex relationships between genotypes and phenotypes has been fostering the collection and analysis of genome-wide datasets of molecular interactions detected from patterns of physical binding, transcript co-expression, mutant phenotypes, etc. Many specialised databases exist to store and integrate such heterogeneous data at different levels of biological complexity. At one end of the scale, the IMEx consortium gathers non-redundant protein-protein interactions (PPIs) from peer-reviewed scientific publications, and provides manually curated details about the experimental conditions [1]. At the opposite end, several resources extend these primary data with indirect or predicted associations to paint a more complete picture for whole 10 organisms [2-5]. For instance, STRING [5] considers experimentally detected PPIs, 11 conserved mRNA co-expression, co-mention in abstracts and papers, interactions from 12 curated databases, conserved gene proximity, gene co-occurrence/co-absence and gene 13 fusion events. Interactions in such databases are typically assigned confidence scores, 14 which can be used for integration purposes [2,6,7]. Not only these data provide valuable 15 direct links between genes and their biological roles, but also form the basis for protein 16 function prediction methods that do not rely on traditional annotation transfers from 17 sequence. Omics data have long offered a suitable opportunity by lending themselves to 18 network representations, where genes or protein products are nodes and edges represent 19

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molecular interactions. This modelling approach can be easily exploited using the "guilt-by-association" principle: if the edges reflect biological facts reliably, adjacent nodes have more similar functions than those further away in the network – e.g. because they form a macromolecular complex, or their activities are coordinated in a specific biological process.

The earliest methods therefore transfer annotations from nodes that are either adjacent or within close distance, possibly taking into account the enrichment of the functional labels [8]. Because the network topology is far from uniform and different functions arise from unevenly sized gene sets, using one particular distance or number of neighbours inevitably affects prediction accuracy. More sophisticated algorithms therefore try to group the nodes into functional modules or communities – each associated with a given function – and then make annotation transfers within them [9-14]. The preliminary identification of functionally coherent subgraphs, however, poses additional challenges, which can make module-assisted predictors less accurate than those based on neighbour counting [15]. More recently, network propagation methods have become increasingly popular to address a wide range of problems [16]. They broadcast annotations from labelled proteins to others by running random walks, which visit the nodes in the network randomly until stopping criteria are met [17-19]. If the edges are weighted, this information controls the probability of traversing them; otherwise equal probabilities are used. Because the propagation is affected by node degree and edge weights, this approach reduces the chance of erroneous predictions from highly multifunctional hub proteins to adjacent nodes, which perform fewer functions. Alternatively, the transition probabilities can be used to encode directly the nodes as multi-dimensional features, and thus to make functional annotations with nearest neighbour strategies [20,21]. Cho et al. (2016) [22] and Gligorijević et al. (2018) [23] have instead used them to embed the STRING networks jointly – that is to map nodes to continuous features, which best explain the transition probabilities and the graph topology. The usefulness of the resulting features has been demonstrated for the task of protein function prediction.

This study proposed a novel PPI network-based protein function predicting method, STRING2GO. It adopts deep maxout neural networks to learn a novel type of functional biological network feature representations simultaneously encapsulating both node neighborhoods and co-occurrence functions information. These higher-level representations are learnt in a supervised way by training deep maxout neural networks to output all the terms in biological process domain associated with an input protein – an approach that has led to higher predictive accuracy in the past [24,25]. The experimental results show that STRING2GO significantly outperforms other PPI network embedding-based protein function prediction methods.

Materials and methods

Data Collection

Firstly, human proteins were retrieved from the UniProtKB/SwissProt release 2017_05 [26], while the corresponding protein-protein interactions information was retrieved from STRING v10.0 [27] that includes seven component networks from heterogeneous data sources and one integrated network. The mapping between UniProtKB/SwissProt accession numbers and Ensembl protein identifiers adopted in STRING was obtained by using the Biomart tool [28].

Experimentally supported Gene Ontology (GO) term annotations – identified with evidence code EXP, IDA, IPI, IMP, IGI or IEP – were collated from the UniProtKB/SwissProt release 2017_05 and UniProt-GOA release 168 [29], and

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propagated over "is a" relationships in the Gene Ontology database [30] - GO obo file release 2017-04-28. To assure the feasibility of the following machine-learning experiments, only biological process (BP) annotating at least 100 proteins were initially considered. To guarantee that the predictions are sufficiently specific and informative, this list was subsequently filtered so that only the deepest terms in the ontology were retained - i.e. the terms a and b were kept if and only if there are no "is a" paths from a to b and from b to a. These steps yielded a vocabulary consisting of 204 BP terms (detailed information is included in Table S1).

The set of human proteins was split into a large subset for GO term-specific classifier training and a small subset for held-out evaluation. 10,667 proteins with at least one cellular component term were initially selected from the whole set. Out of these, 1,000 proteins were randomly selected for held-out evaluation from the subset of well-annotated entries - i.e. those with at least 28, 5 and 14 experimental or electronic biological process, molecular function and cellular component terms respectively. After removing electronic annotations, the held-out set for BP terms contains 982 proteins, while the large set contains 5,000 proteins. In addition, we also create a separated protein-set for a temporal annotation validation by selecting 428 proteins who had no experimental annotation by any 204 BP terms but received at least one after 6 months. The source files were collected from UniProtKB/SwissProt release 2017 11, UniProt-GOA release 174 and GO obo file 2017-10-30.

Predictive performance evaluation

Predictive performance was evaluated on the ability to annotate both individual labels (GO term-centric) and protein function (protein-centric), following the methodology adopted in [31]. For the GO term-centric evaluation, we calculate the F1 score for evaluating the GO term-specific classifier training quality over 10-fold cross validation on the large training protein-set and the predictive performance on the held-out protein-set. In details, the GO term-centric F1 (i.e. $F1_{GO}$) score is used for evaluating the performance of methods when predicting protein annotations for individual GO terms. As shown in Equation 1, the F1 score is obtained by calculating the harmonic mean of precision and recall values. The precision value (Equation 2) is calculated by dividing the number of true positive (TP) predictions over the summation of true positive and false positive (FP) predictions, while the recall value (Equation 3) is 100 calculated by dividing the number of true positive (TP) predictions over the summation 101 of true positive and false negative (FN) predictions. 102

$$F1 = 2 * \frac{Precision * Recall}{Precision + Recall}$$
(1)

$$Precision = \frac{TP}{TP + FP}$$
(2)

$$\text{Recall} = \frac{\text{TP}}{\text{TP} + \text{FN}} \tag{3}$$

For the protein-centric evaluation, we calculate the F_{max} score by predicting the GO 104 term annotations for the held-out protein-set using the trained GO term-specific 105 classifiers. The F_{max} score is used by CAFA experiments [31] for evaluating the 106 performance of methods when predicting GO term annotations for all protein samples. 107 As shown in Equation 4, the F_{max} score is obtained by choosing the maximum averaged 108 F1 score over all protein samples' GO term annotation prediction, according to the 109 varied decision threshold. The averaged F1 score for threshold τ is calculated by the 110 averaged precision $Precision_{\tau}$ (Equation 5) and recall $Recall_{\tau}$ (Equation 6) values. The 111

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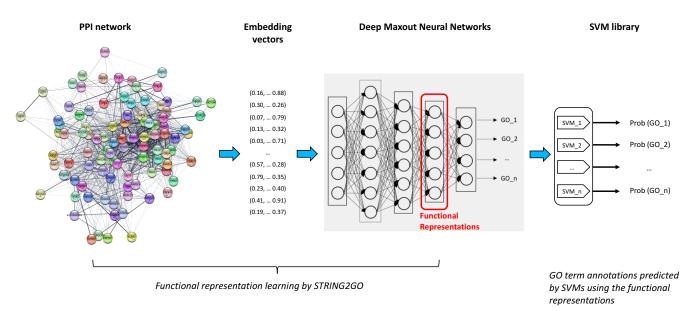


Fig 1. Flow-chart of STRING2GO-based protein function prediction method

Precision τ value is calculated by the total amount of precision values for the GO term 112 annotation predictions of all protein sequences S, over the number of protein sequence 113 m with at least one GO term annotation predictive posterior probability being equal or 114 greater than the value of threshold τ . Analogously, the $\overline{\text{Recall}_{\tau}}$ value is calculated by 115 the total amount of recall values for the GO term annotation predictions of all protein 116 sequences S, over the total number of protein sequences n. Then the corresponding τ to 117 F_{max} score is used as the prior knowledge to calculate the other type of protein-centric 118 averaged F1 score, i.e. F_{τ} , for the temporal annotation validation. 119

$$F_{\max} = \max_{\tau} \{ 2 * \frac{\overline{\operatorname{Precision}_{\tau}} * \overline{\operatorname{Recall}_{\tau}}}{\overline{\operatorname{Precision}_{\tau}} + \overline{\operatorname{Recall}_{\tau}}} \}$$
(4)

$$\overline{\text{Precision}_{\tau}} = \frac{1}{\text{m}} \sum_{s} \frac{\text{TP}_{s,\tau}}{\text{TP}_{s,\tau} + \text{FP}_{s,\tau}}$$
(5)

$$\overline{\text{Recall}_{\tau}} = \frac{1}{n} \sum_{s} \frac{\text{TP}_{s,\tau}}{\text{TP}_{s,\tau} + \text{FN}_{s,\tau}}$$
(6)

STRING2GO - a novel protein function prediction method based on learning representations simultaneously encoding the protein-protein interaction and functional annotation information

In general, the STRING2GO method is composed of a three-stage machine learning 124 procedure. As shown in the flow-chart of Fig 1, at the first stage, it adopts the network 125 embedding representation generation methods (e.g. Mashup and Node2vec discussed in 126 this work) to generate the vector representations for individual proteins based on the 127 protein-protein interaction network. Then the Deep Maxout Neural Networks (DMNNs) 128 feed-forward those generated representations as the inputs to a set of GO term 129 annotations of individual proteins as the outputs. The new type of functional 130 representations (denoted as $STRING2GO_{Embedding}$) that simultaneously encode the PPI 131

and protein functional annotation information are extracted from the outputs of the 3^{rd} 132 hidden layer of DMNNs after finishing the backward propagation optimisation. Finally, 133 STRING2GO trains a library of Support Vector Machines (SVMs) to predict the 134 posterior probability of annotating individual GO terms to the target proteins. Here, we 135 denote this type of STRING2GO method as STRING2GO_{Embedding+SVM} for clarity. In 136 addition, due to the natural functionality of DMNNs, we also propose another type of 137 STRING2GO method, denoted as STRING2GO_{Embedding+Sigmoid}, which directly adopts 138 the sigmoid function in the last layer of DMNNs to make predictions. 139

In this work, we evaluate the predictive performance of our two types of STRING2GO method on predicting the BP terms located in the deep positions in the GO-DAG, benchmarking with the conventional raw network embedding representations-based method, i.e. Embedding+SVM, that merely adopts the raw network embedding representations to train the SVMs for making predictions.

Network embedding representation generation

In this work, we adopt two types of network embedding representation generation 146 methods, i.e. Mashup [22] and Node2vec [32], to derive representations from STRING 147 networks. Mashup firstly evaluates the diffusion states of nodes in the network by 148 random walks with a restart approach. Then the truncated singular value 149 decomposition is applied to the diffusion state matrix in order to learn a lower 150 dimensional representation space that optimally approximates the original diffusion 151 states information. The usefulness of the resulting network embedding representations 152 has been demonstrated for a range of functional classification tasks, including function 153 and genetic interaction prediction. As suggested, the best-performing Mashup-derived 154 representations are 800 dimensional and generated by the random-walk sampling 155 strategy with the restart probability of 0.5. 156

Analogously, Node2vec firstly obtains the node neighborhood information by truncated random walks. Then a Skip-gram [33,34] shallow neural network is used to generate a representation space, where the nodes contain the maximum likelihood of preserving corresponding node neighborhood information. In this work, the neighborhood information was sampled through random walks of length ten, which were biased towards close neighbors by setting the parameter q to 2. We also evaluate the performance of representations in different dimensions, i.e. 32, 64, 128, 256 and 512, generated from all different STRING networks [20,21].

Deep maxout neural networks training

Deep Maxout Neural Networks (DMNNs) are used for learning the more abstract 166 representations simultaneously encoding the PPI network information and the patterns 167 of term co-occurrence in the biological process functional domain. The network 168 architecture was implemented using the Keras package with Theano backend and 169 consisted of three fully connected hidden layers, followed by an output layer with as 170 many neurons as the numbers of terms selected for the biological process functional domain. Each hidden layer had batch-normalized inputs [35], which were combined 172 through maxout units [36], and were subject to dropout [37] in the course of training. A 173 sigmoid function was used to activate the output neurons. 174

To limit the computational requirements for model optimization, the initial 10-fold cross validation (with random split of instances) experiments were run in order to identify the best combination of optimizer (AdaGrad), number of maxout units (3), learning rate (0.05), batch size (100 elements), and number of epochs (150), keeping fixed the weight initialisation (Glorot uniform method) and the number of units in all hidden layers, by considering the highest F1_{GO} scores for predicting all 204 BP terms.

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> Subsequent training stages were aimed at selecting the optimal dimensions of hidden layers that lead to the highest median $F1_{GO}$ scores (here rounded to two decimal places), from a limited set of options (300, 500, 700 and 1,000). In addition, we also evaluate the predictive performance when using the same dimensions for both input features and the 3^{rd} hidden layer outputs. Note that, due to the well-known curse of dimensionality issue [38], if more than two different dimensions of the 3^{rd} hidden layer outputs obtain the same median $F1_{GO}$ scores, we only choose the lowest ones as the optimal dimensions.

Support vector machine training

Scikit-learn [39] was used to train a set of GO term-specific Support Vector Machines 190 (SVMs) with a radial basis function (RBF) kernel, the parameters of which were 191 identified through a grid search as those maximising the $F1_{GO}$ score across the stratified 192 10-fold cross validation experiments. To train each classifier, the set of positive 193 instances consisted of the proteins annotated with the target GO term t or its 194 descendants, while the set of negative instances are all remaining proteins not annotated 195 with the target GO term or its descendants. Finally, the well-known Platt scaling 196 method [40] was used to transform the predictive scores of individual SVMs into a 197 probability distribution of binary classes. The data and code can be accessed via 198 https://github.com/psipred/STRING2GO 199

Results

We firstly report the experimental results about evaluating the predictive information 201 included in different STRING networks that are used for generating the raw network 202 embedding representations by two different methods, i.e. Mashup and Node2vec. Then 203 we evaluate the predictive performance of the STRING2GO-learnt functional 204 representation (i.e. $STRING2GO_{Mashup}$ and $STRING2GO_{Node2vec}$) by comparing with 205 their corresponding raw network embedding representations. We also compare the 206 performance of Mashup and Node2vec methods when they are used to generate the raw 207 network embedding representations or be the component methods of STRING2GO to 208 learn the functional representations. Finally, we further compare all prediction methods 209 involved in this work, also benchmarking with the Naïve method [31]. 210

Predictive power included in different STRING networks

To begin with, we compare the predictive power of different STRING networks by 212 adopting the Mashup or Node2vec-generated network embedding representations as the 213 inputs of DMNNs for predicting protein function (i.e. STRING2GO_{Mashup+Sigmoid} and 214 STRING2GO_{Node2vec+Sigmoid}). Overall, the Combinedscore network-derived embedding 215 representations show the best predictive performance among all different STRING 216 networks-derived ones when using either Mashup or Node2vec methods, while the 217 Textmining network-derived representations also obtain the competitive predictive 218 accuracy. As shown in the 4th and 7th columns of Table 1, the Combinedscore 219 network-derived representations obtain the highest median $F1_{GO}$ (hereafter, denoted by 220 $F1_{GO}$ scores (0.23 and 0.17) using Mashup and Node2vec respectively. The 221 Combinedscore network also contains the largest number of proteins, interactions and 222 the highest coverage (as shown in the columns 8-10 of Table 1), when mapping the 223 STRING network-included proteins to the training protein-set. The Textmining 224 network-derived representations obtain the second highest $F1_{GO}$ score (0.22) using the 225 Mashup method, while also obtain the same highest $F1_{GO}$ score (0.17) using the 226

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STRING Networks		Mashup			Node2vec		No. Proteins	No. Interactions	Coverage on Training set	
	Input	3^{rd} Hidden	$\tilde{F1}_{GO}$	Input	3^{rd} Hidden	$\tilde{F1}_{GO}$	No. Proteins	No. Interactions		
Combinedscore	800	800	0.23	128	500	0.17	19247	8548002	93.4%	
Textmining	800	700	0.22	128	1000	0.17	19088	7632934	93.3%	
Experimental	800	700	0.19	128	1000	0.13	16858	3473862	90.4%	
Coexpression	800	700	0.14	256	700	0.09	12774	1537924	72.0%	
Database	800	700	0.11	128	700	0.04	7937	424860	56.9%	
Neighborhood*	800	300	0.00	32	32	0.00	3514	152248	20.9%	
Cooccurrence*	800	300	0.00	32	32	0.00	2754	47478	16.6%	
Fusion*	800	300	0.00	32	32	0.00	1495	4120	9.7%	

Table 1. The optimal dimensions of raw network embedding representations and the corresponding 3^{rd} hidden layer outputs (a.k.a. the STRING2GO-learnt functional representations) with their corresponding predictive power for biological process terms prediction, and the main characteristics of different STRING networks

* : Note that those STRING networks obtain 0.00 of $\tilde{F1}_{GO}$ scores with all different dimensions, only the lowest dimensions are reported.

Node2vec method. Moreover, in terms of the predictive information included in other component networks, the Experimental network-derived embedding representations 228 show the third highest predictive accuracy, since they obtain sequentially higher $F1_{GO}$ 229 scores than the ones derived by the Database and Coexpression networks respectively. 230 Note that, the embedding representations derived from Neighbourhood, Cooccurrence 231 and Fusion networks show poor predictive performance, since their $F1_{GO}$ scores are all 232 equal to zero, and the mapping coverages are all lower than 21.0%. Hereafter, we 233 consider learning the functional representations by STRING2GO only from those 5 234 networks including relatively rich PPI information and high coverage. 235

We then report the optimal dimensions of network embedding representations 236 derived by Mashup and Node2vec methods from those 5 STRING networks. According 237 to the suggestion in [22], we define 800 as the optimal dimensions for the input network 238 embedding representations derived by Mashup. In terms of the Node2vec-derived 239 network embedding representations, as shown in the 5th column of Table 1, 128 are the 240 overall optimal dimensions, since 4 out of 5 network-derived embedding representations 241 in 128 dimensions obtain the highest $F1_{GO}$ scores for predicting 204 biological process 242 terms. We then report the optimal dimensions of the STRING2GO-learnt functional 243 representations (a.k.a. the 3^{rd} hidden layer outputs of DMNNs) w.r.t. the 244 corresponding optimal dimensions of raw network embedding representation inputs. 245 Generally, STRING2GO encodes the functional predictive information in a high 246 dimensional representation space (ranging from 500 - 1000 dimensions), when using 247 either Mashup or Node2vec as the raw network embedding representation generation 248 method. As shown in the 3^{rd} and 6^{th} columns of Table 1, the optimal dimensions of the 249 3^{rd} hidden layer outputs vary between 500 to 1000. Recall that we also evaluate the 250 cases when the dimensions of the 3^{rd} hidden layer outputs are the same to the 251 dimensions of raw network embedding representation inputs. None of the functional 252 representations based on Node2vec-derived network embedding representations obtain 253 higher $F1_{GO}$ scores when using the same dimensions of inputs as the dimensions of 3^{rd} 254 hidden layer outputs, e.g. using 128 as the dimensions of both representation inputs and 255 the 3^{rd} hidden layer outputs. 256

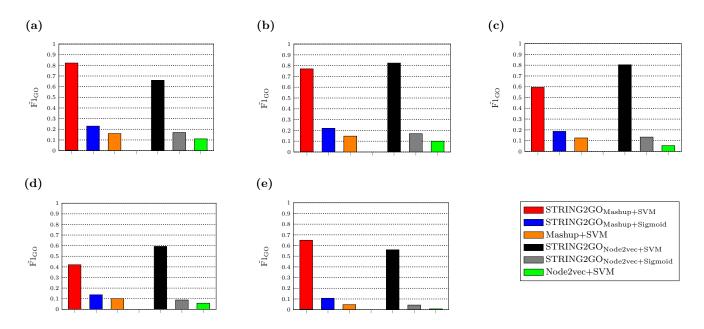


Fig 2. $\tilde{F1}_{GO}$ scores obtained by network embedding representations and the corresponding STRING2GO-learnt functional representations based on (a) Combinedscore, (b) Textmining, (c) Experimental, (d) Database and (e) Coexpression networks by using SVM or Sigmoid function over the 10-fold cross validation during the GO term-specific classifiers training stage

The functional representations learnt by STRING2GO encode higher predictive power than the corresponding raw network embedding representations

We evaluate the predictive performance of STRING2GO-learnt functional 261 representations by conducting pairwise comparisons with the corresponding raw network 262 embedding representations respectively. Generally, in terms of GO term and 263 protein-centric metrics, both STRING2GO_{Mashup} and STRING2GO_{Node2vec} functional 264 representations obtain higher predictive accuracy than Mashup and Node2vec-derived 265 raw network embedding representations. In detail, during the GO term-specific classifier 266 training stage, as shown in Fig 2.a-2.e, both orange and green bars are lower than other 267 ones. This fact indicates better classifier training quality by using 268 $STRING2GO_{Mashup+SVM}$, $STRING2GO_{Node2vec+SVM}$, $STRING2GO_{Mashup+Sigmoid}$ and 269 $STRING2GO_{Node2vec+Sigmoid}$ than the ones obtained by Mashup+SVM and 270 Node2vec+SVM, when using all five different STRING networks to generate embedding 271 representations. 272

The held-out evaluation results further confirm that the STRING2GO-learnt 273 functional representations contain higher predictive information. As shown in Table 2, 274 the $F1_{GO}$ scores obtained by STRING2GO_{Mashup+SVM} and STRING2GO_{Node2vec+SVM} 275 reach to 0.270 and 0.182 respectively, whereas the $F1_{GO}$ scores obtained by 276 Mashup+SVM and Node2vec+SVM are both equal to 0.000. This pattern is consistent 277 when adopting all other types of STRING component networks, except 278 STRING2GO_{Node2vec+SVM} and Node2vec+SVM both obtain zero F1_{GO} scores when 279 using the Coexpression network to generate the raw embedding representations (as 280 shown in Table 2). STRING2GO_{Mashup+Sigmoid} and STRING2GO_{Node2vec+Sigmoid} also 281 respectively obtain higher $\overline{F1}_{GO}$ scores than Mashup+SVM and Node2vec+SVM based 282 on all five different STRING networks. The scatter-plots in Fig 3 show the pairwise 283

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Table 2. Summary on experimental results obtained by different network embedding representations and corresponding									
functional representations based on Combinedscore, Textmining, Experimental, Database and Coexpression networks working									
with different classification algorithms during held-out evaluation and temporal annotation validation									

	-		0					-						
Combinedscore			Textmining		Experimental			Database			Coexpression			
Held-out Tem		Temporal	mporal Held-out		Temporal	Held-out		Temporal	Held-out		Temporal	Held-out		Temporal
$\tilde{\rm F1}_{\rm GO}$	F _{max}	F_{τ}	$\tilde{\rm F1}_{\rm GO}$	F _{max}	F_{τ}	$\tilde{F1}_{GO}$	$\mathbf{F}_{\mathrm{max}}$	F_{τ}	$\tilde{\rm F1}_{\rm GO}$	\mathbf{F}_{\max}	F_{τ}	$\tilde{\rm F1}_{\rm GO}$	F _{max}	F_{τ}
					Mashu	p-based								
0.270	0.497	0.309	0.275	0.483	0.296	0.146	0.450	0.263	0.130	0.412	0.225	0.116	0.392	0.258
0.237	0.495	0.312	0.239	0.478	0.290	0.183	0.442	0.247	0.131	0.427	0.144	0.121	0.392	0.247
0.000	0.470	0.290	0.000	0.463	0.287	0.000	0.420	0.229	0.000	0.392	0.238	0.000	0.371	0.242
					Node2v	ec-based								
0.182	0.458	0.319	0.115	0.446	0.290	0.124	0.422	0.256	0.087	0.353	0.169	0.000	0.349	0.236
0.187	0.471	0.312	0.188	0.472	0.314	0.143	0.440	0.258	0.111	0.408	0.238	0.043	0.381	0.246
0.000	0.444	0.293	0.000	0.437	0.278	0.000	0.418	0.249	0.000	0.386	0.221	0.000	0.360	0.219
N/A	0.363	0.254												
	Held F1 _{G0} 0.270 0.237 0.000 0.182 0.187 0.000	$\begin{tabular}{ c c c c } \hline Held-out \\ \hline F1_{GO} & F_{max} \\ \hline 0.270 & 0.497 \\ 0.237 & 0.495 \\ 0.000 & 0.470 \\ \hline 0.182 & 0.458 \\ 0.187 & 0.471 \\ 0.000 & 0.444 \\ \hline \end{tabular}$	$\begin{tabular}{ c c c c c c } \hline Held-out & Temporal \\ \hline F_{1GO} & F_{max} & F_{\tau} \\ \hline \\ \hline 0.270 & 0.497 & 0.309 \\ 0.237 & 0.495 & 0.312 \\ 0.000 & 0.470 & 0.290 \\ \hline \\ \hline \\ 0.182 & 0.458 & 0.319 \\ 0.187 & 0.471 & 0.312 \\ 0.000 & 0.444 & 0.293 \\ \hline \end{tabular}$	$\begin{tabular}{ c c c c } \hline Combinedscore & Held-out & Temporal & Held \\ \hline \hline Held-out & Temporal & F_{1} & F_{1} \\ \hline \hline F1_{GO} & F_{max} & F_{\tau} & F_{1} \\ \hline 0.270 & 0.497 & 0.309 & 0.275 \\ 0.237 & 0.495 & 0.312 & 0.239 \\ 0.000 & 0.470 & 0.290 & 0.000 \\ \hline & & & & & \\ 0.182 & 0.458 & 0.319 & 0.115 \\ 0.187 & 0.471 & 0.312 & 0.188 \\ 0.000 & 0.444 & 0.293 & 0.000 \\ \hline \end{tabular}$	$\begin{tabular}{ c c c c c c } \hline Combinedscore & Textmin \\ \hline Held-out & Temporal & Held-out \\ \hline F1_{GO} & F_{max} & F_{\tau} & F_{1GO} & F_{max} \\ \hline \hline 0.270 & 0.497 & 0.309 & 0.275 & 0.483 \\ 0.237 & 0.495 & 0.312 & 0.239 & 0.478 \\ 0.000 & 0.470 & 0.290 & 0.000 & 0.463 \\ \hline \hline 0.182 & 0.458 & 0.319 & 0.115 & 0.446 \\ 0.187 & 0.471 & 0.312 & 0.188 & 0.472 \\ 0.000 & 0.444 & 0.293 & 0.000 & 0.437 \\ \hline \end{tabular}$	$\begin{tabular}{ c c c c c } \hline C ombinedscore & T extmining $$ T extmining $$ $$ T extmining $$ $$ $$ $$ T extmining $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$$	$\begin{tabular}{ c c c c c } \hline C onbinedscore $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$$	$\begin{tabular}{ c c c c c c } \hline C ombineds: c or $$ C max$ $$ $Textmining $$ $Textmining $$ $$ $Textmining $$ $$ $$ $Textmining $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$$	$\begin{tabular}{ c c c c c c } \hline $$ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $ $$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{ c c c c c c c c c } \hline Combinedscore & Textming & Experimental & Database \\ \hline Held-out & Temporal & Held-out & Temporal & Held-out & Temporal & Held-out & Held-$	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$

comparisons of $F1_{GO}$ scores obtained by different methods, and the dashed-lines indicate the median values of difference between pairs of $F1_{GO}$ scores. In detail, Fig 3.a-3.d show that almost all dots (in blue) drop in the area above the diagonal, indicating higher $F1_{GO}$ scores for predicting the majority of BP terms by using the functional representations learnt by STRING2GO based on the Combinedscore network by using either SVM or Sigmoid function as the classification algorithm. As shown in Fig 3.e-3.t This pattern is consistently observed when applying on almost all other four different STRING networks, except the Coexpression network that leads to competitive performance between STRING2GO_{Node2vec} and Node2vec, since the dashed-lines in Fig 3.s and Fig 3.t are almost overlapping on the diagonal. The Wilcoxon signed-rank test results in Table S3 further confirm that the STRING2GO-learnt functional representations obtain significantly higher GO term-centric F1_{GO} scores than the raw network embedding representations.

From the perspective of protein-centric evaluation (i.e. considering the F_{max} and F_{τ} metrics), the STRING2GO-learnt functional representations also obtain higher predictive accuracy based on the Combinedscore network. As shown in Table 2, the functional representations STRING2GO_{Mashup} and STRING2GO_{Node2vec} both obtain higher F_{max} scores (i.e. 0.497 and 0.458 obtained by using SVM, 0.495 and 0.471 obtained by using Sigmoid function) than the network embedding representations generated by Mashup and Node2vec (i.e. 0.470 and 0.444 obtained by using SVM). The precision-recall curves in Fig 4.a also show that the STRING2GO-learnt functional representations obtain higher precision and recall values simultaneously, since the middle parts of red and blue curves locate in higher position than the green one. As shown in Table 2 and Fig 4.b-4.e, this pattern is consistent when adopting the other four types of STRING component networks to generate representations, except STRING2GO_{Node2vec+SVM} obtaining lower F_{max} scores than Node2vec+SVM based on the Database and Coexpression networks.

Analogously, the functional representations STRING2GO_{Mashup} and 312 STRING2GO_{Node2vec} obtain higher F_{τ} scores based on the Combinedscroe network (0.309 and 0.319 obtained by SVM, while 0.312 obtained by Sigmoid function) than the raw network embedding representations generated by Mashup and Node2vec (0.290 and 0.293 by using SVM). This pattern is consistent when using all other STRING networks, except the Database network which only leads to higher F_{τ} score obtained by StrRING2GO_{Node2vec+Sigmoid} than the one obtained by Node2vec+SVM. 318

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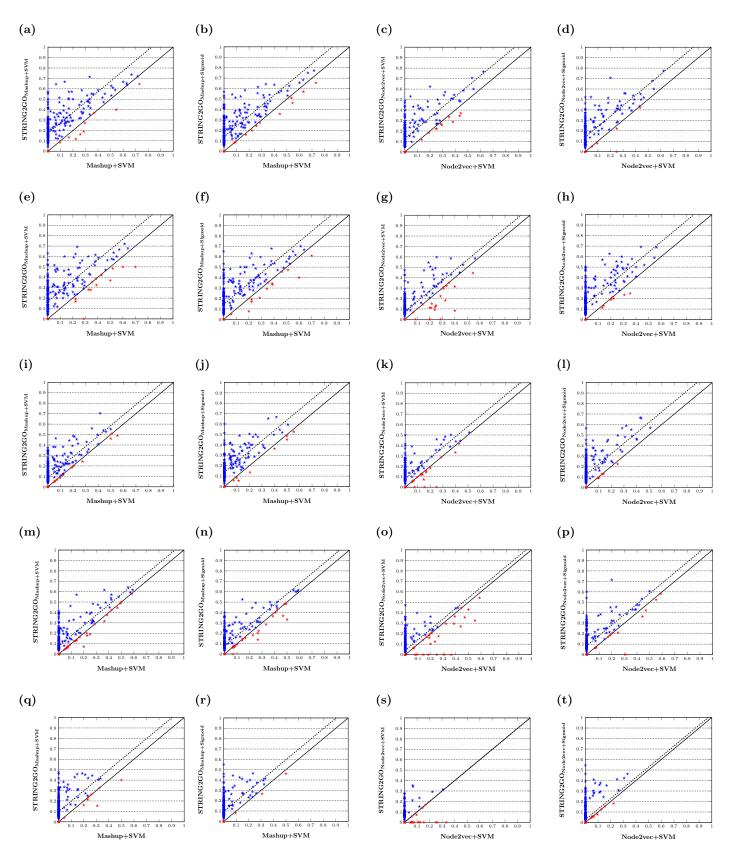


Fig 3. $F1_{GO}$ scores obtained by different network embedding representations and the corresponding STRING2GO-learnt functional representations based on (a-d) Combinedscore, (e-h) Textmining, (i-l) Experimental, (m-p) Database and (q-t) Coexpression networks by using SVM or Sigmoid function for classification

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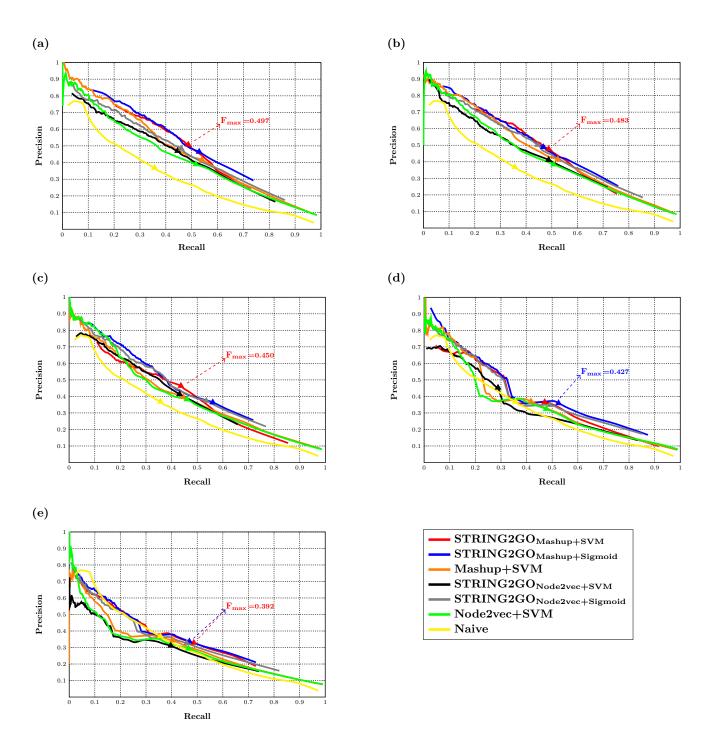


Fig 4. Precision-recall curves of different methods and the F_{max} scores obtained by the best-performing methods based on (a) Combinedscore, (b) Textmining, (c) Experimental, (d) Database and (e) Coexpression networks

The raw network embedding representations derived by Mashup show higher predictive power

We also compare the predictive performance of Mashup and Node2vec-derived network embedding representations and the corresponding STRING2GO-learnt functional

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representations respectively. Generally, the raw network embedding representations 323 derived by Mashup and Node2vec methods obtain competitive predictive accuracy by 324 using SVM as the classification algorithm. To begin with, during the training stage, the 325 $\tilde{F1}_{GO}$ score obtained Mashup+SVM is higher than the one obtained by Node2vec+SVM 326 based on the Combinedscore network, since the orange bar is higher than the green one 327 in Fig 2.a. However, both Mashup+SVM and Node2vec+SVM obtain poor predictive 328 performance on the held-out evaluation, due to the zero $F1_{CO}$ scores. But the statistical 329 significance test results (see Table S2) show that the former still outperforms the latter. 330 Those patterns are consistent when using all other 4 types of STRING networks to 331 generate the raw embedding representations, as reported in Fig 2.b-2.e, Tables 2 and S1. 332 In terms of the protein-centric evaluation, Mashup+SVM obtains a higher F_{max} score 333 (0.470) than Node2vec+SVM (0.444). The Combinedscore network-based 334 precision-recall curves in Fig 4.a confirm that the orange curve locates in higher position 335 than the green one. Those patterns are also consistent in cases when using other four 336 different STRING component networks to generate representations, as shown in Fig 337 4.b-4.e. However, Node2vec+SVM outperforms Mashup+SVM on the temporal 338 annotation validation. As reported in Table 2, although the latter obtains higher F_{τ} 339 score based on three STRING component networks (i.e. Textmining, Database and 340 Coexpression), the former obtains the highest F_{τ} score (0.293) based on the 341 Combinedscore network. 342

We then further conduct comparisons on predictive performance of two different STRING2GO-learnt functional representations respectively based on Mashup and 344 Node2vec-derived raw network embedding representations. During the GO term-specific 345 classifiers training stage, $STRING2GO_{Mashup}$ obtains higher $F1_{GO}$ scores than 346 STRING2GO_{Node2vec} by using either SVM or Sigmoid function as the classification 347 algorithm, based on the Combinedscore and Coexpression networks. As shown in Fig 2.a 348 and 2.e, where red and blue bars are higher than the black and grey ones respectively. 349 When using the other 3 STRING component networks, STRING2GO_{Node2vec} obtains 350 higher $F1_{GO}$ scores by using SVMs, whereas STRING2GO_{Mashup} still outperforms the former by using Sigmoid function as the classification algorithm. 352

The held-out evaluation results in Table 2 show a consistent pattern that 353 STRING2GO_{Mashup} obtains higher F1_{GO} scores (statistically significant according to 354 Table S2) and F_{max} scores than STRING2GO_{Node2vec} based on the Combinedscore 355 network by using either SVM or Sigmoid function, respectively. As shown in Fig 4.a, 356 the majority parts of the red and blue curves clearly locate in higher position than the 357 black and grey ones. Those patterns are consistent when using the other 4 STRING 358 networks, as shown in Table 2 and Fig 4.b-4.e. However, STRING2GO_{Node2vec} obtains 359 better predictive performance during the temporal annotation validation, since the former obtains the highest F_{τ} score (0.319) by using SVM (based on the Combinedscore 361 network) among all methods when adopting all different STRING networks. 362

The STRING2GO-learnt functional representations with support vector machines obtain the highest accuracy on predicting 204 BP terms

We then compare all prediction methods discussed in previous sections, i.e. two types of 366 STRING2GO methods (i.e. STRING2GO_{Embedding+SVM} and 367 STRING2GO_{Embedding+Sigmoid}) adopting two types of raw network embedding 368 representations (i.e. the ones generated by Mashup and Node2vec respectively), and the 369 methods that only exploit the raw network embedding representations to make 370 predictions by using SVM as the classification algorithm. We also compare those 371 methods with the Naïve prediction method [31], which makes predictions by considering 372

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the annotation frequency in the database as the prior knowledge. Overall, 373 $STRING2GO_{Embedding+SVM}$ is the best-performing method according to both the GO 374 term and protein-centric metrics. During the GO term-specific classifiers training stage, 375 STRING2GO_{Mashup+SVM} and STRING2GO_{Node2vec+SVM} obtain almost the same 376 highest $F1_{GO}$ scores among all prediction methods by using all different STRING 377 networks. As shown in Fig 2, the latter obtains the highest GO score (0.824) based on 378 the Textmining network, while the former obtained almost the same highest $F1_{GO}$ score 379 (0.822) based on the Combinedscore network. The held-out evaluation results also 380 confirm that STRING2GO_{Mashup+SVM} obtains the highest $F1_{GO}$ score (0.275) by using 381 the Textmining network, while also obtains the significantly higher $F1_{GO}$ scores than 382 other methods basing on the Combinescore network (see Friedman test with Holm 383 post-hoc correction results in Table S3). STRING2GO_{Mashup+SVM} obtains the highest 384 F_{max} score (0.497) based on the Combinedscore network and higher F_{max} scores than 385 all other methods based on all other STRING networks except the Database network. 386 In terms of the F_{τ} score metric, STRING2GO_{Node2vec+SVM} obtains the highest F_{τ} score 387 (0.319) by using the Combinedscore network among all methods based on all different 388 STRING networks. 389

The second best performing method is STRING2GO_{Embedding+Sigmoid}. 390 $STRING2GO_{Mashup+Sigmoid}$ obtains higher $F1_{GO}$ scores than either Mashup+SVM or 391 Node2vec+SVM during the classifier training stage. It also obtains the second highest 392 $F1_{GO}$ scores during the held-out evaluation based on 2 out of 5 networks (except the 393 case when $STRING2GO_{Mashup+Sigmoid}$ obtains the highest $F1_{GO}$ score based on the 394 Experimental, Database and Coexpression networks). From the perspective of 395 protein-centric metrics, $STRING2GO_{Mashup+Sigmoid}$ obtains the second highest F_{max} 396 based on 3 out of 5 STRING networks, while $STRING2GO_{Node2vec+Sigmoid}$ obtains the 397 overall second highest F_{τ} score (0.314) based on the Textmining network. 398

In addition, all of those methods discussed above obtains higher F_{max} scores than the Naïve prediction method based on almost all 5 individual STRING networks (as the yellow curves shown in Fig 4.a-4.e), with exception of STRING2GO_{Node2vec+SVM} based on the Database and Coexpression networks and Node2vec+SVM based on the Coexpression network. All those methods also obtain higher F_{τ} scores than the Naïve prediction method based on the Combinedscore and Textmining networks.

Discussion

Overall, as discussed in previous sections, the functional representations learnt by 406 STRING2GO show substantial improvement on the predictive power of the raw network 407 embedding representations. We further investigate the improvement of predictive power 408 of the STRING2GO-learnt functional representations by evaluating the enlarged 409 distances between two classes of training protein samples. We firstly calculate the 410 Euclidean distance between the centroids of two classes by using the Mashup-based 411 representations' values standardized into the range of (0,1) in the same dimensional 412 space, i.e. 800 dimensions for both Mashup and STRING2GO_{Mashup}. Then we calculate 413 the correlation coefficient between the distances and $F1_{GO}$ scores obtained by held-out 414 evaluation. As shown in Fig 5.a, the x axis denotes the distance between two classes 415 calculated by using either the raw Mashup-derived network embedding representations 416 (blue), or the corresponding functional representations (red) $STRING2GO_{Mashup}$, based 417 on the Combinedscore network, while the y axis denotes the corresponding F1_{GO} score 418 obtained by adopting those different representations working with SVMs to predict 419 individual BP terms. It is obvious that the distances between two classes of proteins for 420 individual GO terms are all enlarged by STRING2GO, while the correlation coefficient 421 values between distances and $F1_{GO}$ scores for both types of representations are positive, 422

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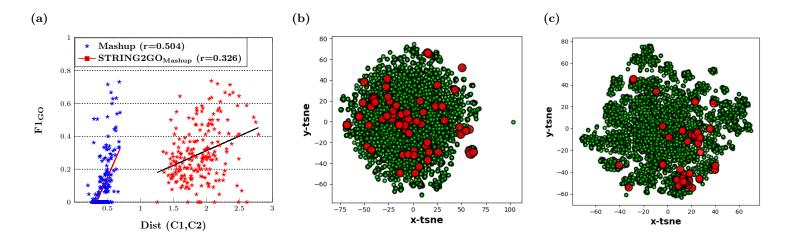


Fig 5. (a) Linear relationship between distances of two classes protein samples and $F1_{GO}$ scores obtained by Mashup-derived network embedding representations and the corresponding functional representations on classifier training stage (c) The 2D space visualization of distribution of protein samples belonging to GO:0090150 using the Mashup-derived network embedding representations and (d) the STRING2GO_{Mashup} functional representations transformed by t-SNE.

indicating that the larger distances lead to higher predictive accuracy.

We also display an example of the increased distance between two classes of proteins 424 when predicting the term GO:0090150, which shows the highest improvement on the 425 classifier training quality obtained by using $STRING2GO_{Mashup+SVM}$, compared by 426 using Mashup+SVM. Fig 5.b-5.c respectively show the 2-D visualization of raw 427 Mashup-derived network embedding representations and the corresponding 428 STRING2GO-learnt functional representations after transforming by t-SNE [41]. The 429 red dots denote the protein samples belonging to class "Annotated", while the green dots 430 denote the protein samples belonging to class "Not-annotated". The red dots are 431 distributed in the similar scale of green dots in Fig 5.b, whereas the most of red dots are 432 clustered in the right side in Fig 5.c. This fact indicates that the functional 433 representations successfully encode higher discriminating power against two classes of 434 protein samples. 435

Conclusion

In this work, we present a novel deep learning-based protein function prediction method STRING2GO, which successfully learns a novel type of functional representations to train the down-stream classifiers for making predictions. STRING2GO shows the highest accuracy when predicting biological process protein functions, compared with other state-of-the-art network embedding representation-based protein function prediction methods. Based on this STRING2GO learning framework, there is potential for further improving the predictive accuracy by integrating representations in a future study.

Supporting information

 Table S1
 List of 204 biological process Gene Ontology terms studied in this work.
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Table S2 Two-tailed Wilcoxon signed-rank tests at 0.05 of significance level on $F1_{GO}$ 447 scores obtained by different pairs of prediction methods over the hold-out evaluation. 448

Table S3Friedman test with Holm post-hoc correction results on $F1_{GO}$ scores449obtained by different prediction methods over the hold-out evaluation.450

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