

Concurrent Multisensory Integration and Segregation with Complementary Congruent and Opposite Neurons

Wen-Hao Zhang^{1,3}, He Wang¹, Aihua Chen⁴, Yong Gu⁵, Tai Sing Lee³,
K. Y. Michael Wong^{1*}, Si Wu^{2*}

¹Department of Physics, Hong Kong University of Science and Technology, Hong Kong.

²School of Electronics Engineering and Computer Science,

IDG/McGovern Institute for Brain Research, Peking University, Beijing 100871, China.

³The Center of the Neural Basis of Cognition, Carnegie Mellon University, Pittsburgh, PA 15213.

⁴Laboratory of Brain Functional Genomics, Primate Research Center,
East China Normal University, Shanghai, China.

⁵Institute of Neuroscience, Chinese Academy of Sciences, Shanghai, China.

*Correspondence should be addressed to either K. Y. Michael Wong or Si Wu.

Email: phkywong@ust.hk or siwu@pku.edu.cn

Abstract

Our brain perceives the world by exploiting multiple sensory modalities to extract information about various aspects of external stimuli. If these sensory cues are from the same stimulus of interest, they should be integrated to improve perception; otherwise, they should be segregated to distinguish different stimuli. In reality, however, the brain faces the challenge of recognizing stimuli without knowing in advance whether sensory cues come from the same or different stimuli. To address this challenge and to recognize stimuli rapidly, we argue that the brain should carry out multisensory integration and segregation concurrently with complementary neuron groups. Studying an example of inferring heading-direction via visual and vestibular cues, we develop a concurrent multisensory processing neural model which consists of two reciprocally connected modules, the dorsal medial superior temporal area (MSTd) and the ventral intraparietal area (VIP), and that at each module, there exists two distinguishing groups of neurons, congruent and opposite neurons. Specifically, congruent neurons implement cue integration, while opposite neurons compute the cue disparity, both optimally as described by Bayesian inference. The two groups of neurons provide complementary information which enables the neural system to assess the validity of cue integration and, if necessary, to recover the lost information associated with individual cues without re-gathering new inputs. Through this process, the brain achieves rapid stimulus perception if the cues come from the same stimulus of interest, and differentiates and recognizes stimuli based on individual cues with little time delay if the cues come from different stimuli of interest. Our study unveils the indispensable role of opposite neurons in multisensory processing and sheds light on our understanding of how the brain achieves multisensory processing efficiently and rapidly.

Keywords: Opposite neuron, Multisensory integration, Concurrent integration and segregation, Decentralized architecture, Continuous attractor neural network.

25 Significance Statement

26 Our brain perceives the world by exploiting multiple sensory cues. These cues need to be integrated
27 to improve perception if they come from the same stimulus and otherwise be segregated. To
28 address the challenge of recognizing whether sensory cues come from the same or different stimuli
29 that are unknown in advance, we propose that the brain should carry out multisensory integration
30 and segregation concurrently with two different neuron groups. Specifically, congruent neurons
31 implement cue integration, while opposite neurons compute the cue disparity, and the interplay
32 between them achieves rapid stimulus recognition without information loss. We apply our model
33 to the example of inferring heading-direction based on visual and vestibular cues and reproduce
34 the experimental data successfully.

35 Introduction

36 To survive as an animal is to face the daily challenge of perceiving and responding fast to a
37 constantly changing world. The brain carries out this task by gathering as much as possible
38 information about external environments via adopting multiple sensory modalities including vision,
39 audition, olfaction, tactile, vestibular perception, etc. These sensory modalities provide different
40 types of information about various aspects of the external world, and serve as complementary
41 cues to improve perception in ambiguous conditions. For instance, while walking, both the visual
42 input (optic flow) and the vestibular signal (body movement) convey useful information about
43 heading-direction, and when integrated together, they give a more reliable estimate of heading-
44 direction than either of the sensory modalities could deliver on its own. Indeed, experimental data
45 has shown that the brain does integrate visual and vestibular cues to infer heading-direction and
46 furthermore the brain does it in an optimal way as predicted by Bayesian inference¹. Over the past
47 years, experimental and theoretical studies verified that optimal information integration were found
48 among many sensory modalities, for example, integration of visual and auditory cues for inferring
49 object location², motion and texture cues for depth perception³, visual and proprioceptive cues for
50 hand position⁴, and visual and haptic cues for object height⁵.

51 However, multisensory integration is only a part of multisensory information processing. While
52 it is appropriate to integrate sensory cues from the same stimulus of interest (Fig. 1A left), sensory
53 cues from different stimuli need to be segregated rather than integrated in order to distinguish
54 and recognize individual stimuli (Fig. 1A right). In reality, the brain does not know in advance
55 whether the cues are from the same or different objects. To recognize stimuli rapidly, we argue
56 that the brain should carry out multisensory integration and segregation concurrently: a group of
57 neurons integrates sensory cues, while the other computes the disparity between cues. The interplay
58 between the two groups of neurons determines the final choice of integration versus segregation.

59 An accompanying consequence of multisensory integration is, however, that it inevitably incurs
60 information loss of individual cues (Fig. 1, also see SI and Fig. S1). Consider the example of
61 integrating the visual and vestibular cues to infer heading-direction, and suppose that both cues
62 have equal reliability. Given that one cue gives an estimate of θ degree and the other an estimate
63 of $-\theta$ degree, the integrated result is always 0 degree, irrespective to the value of θ (Fig. 1B).
64 Once the cues are integrated, the information associated with each individual cue (the value of
65 θ) is lost, and the amount of loss information increases with the extent of integration (see SI).
66 Thus, if only multisensory integration is performed, the brain faces a chicken and egg dilemma
67 in stimulus perception: without integrating cues, it may be unable to recognize stimuli reliably
68 in an ambiguous environment; but once cues are integrated, the information from individual cues
69 is lost. Concurrent multisensory integration and segregation is able to disentangle this dilemma.

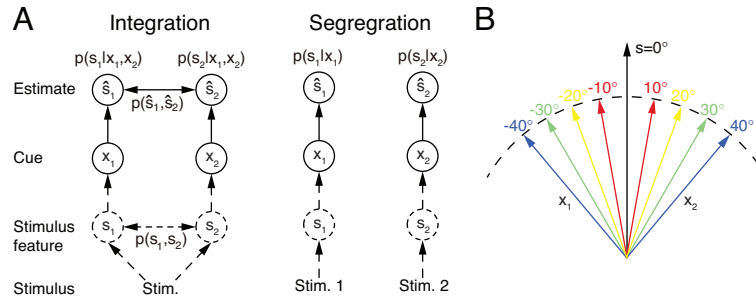


Figure 1: Multisensory integration and segregation. (A) Multisensory integration versus segregation. Two underlying stimulus features s_1 and s_2 independently generate two noisy cues x_1 and x_2 , respectively. If the two cues are from the same stimulus, they should be integrated, and in the Bayesian framework, the stimulus estimation is obtained by computing the posterior $p(s_1|x_1, x_2)$ (or $p(s_2|x_1, x_2)$) utilizing the prior knowledge $p(s_1, s_2)$ (left). If two cues are from different stimuli, they should be segregated, and the stimulus estimation is obtained by computing the posterior $p(s_1|x_1)$ (or $p(s_2|x_2)$) using the single cues (right). (B) Information of single cues is lost after integration. The same integrated result $\hat{s} = 0^\circ$ is obtained after integrating two cues of opposite values (θ and $-\theta$) with equal reliability. Therefore, from the integrated result, the values of single cues are unknown.

70 The information of individual cues can be recovered by using the preserved disparity information if
 71 necessary, instead of re-gathering new inputs from the external world. While there are other brain
 72 regions processing unisensory information, concurrent multisensory integration and segregation
 73 provides an additional way to achieve rapid stimulus perception if the cues come from the same
 74 stimulus of interest, and differentiate and recognize stimuli based on individual cues with little time
 75 delay if the cues come from different stimuli of interest. This processing scheme is consistent with
 76 an experimental finding which showed that the brain can still sense the difference between cues in
 77 multisensory integration^{6,7}.

78 What are the neural substrates for implementing concurrent multisensory integration and segre-
 79 gation? Previous studies investigating the integration of visual and vestibular cues to infer heading-
 80 direction found that in each of two brain areas, namely, the dorsal medial superior temporal area
 81 (MSTd) and the ventral intraparietal area (VIP), there are two types of neurons with comparable
 82 number displaying different multisensory behaviors: congruent and opposite cells (Fig. 2)^{8,9}. The
 83 tuning curves of a congruent cell in response to visual and vestibular cues are similar (Fig. 2A),
 84 whereas the tuning curve of an opposite cell in response to a visual cue is shifted by 180 degrees
 85 (half of the period) compared to that in response to a vestibular cue (Fig. 2B). Data analysis
 86 and modeling studies suggested that congruent neurons are responsible for cue integration^{8,10-12}.
 87 However, the computational role of opposite neurons remains largely unknown. They do not inte-
 88 grate cues as their responses hardly change when a single cue is replaced by two cues with similar
 89 directions. Interestingly, however, their responses vary significantly when the disparity between
 90 visual and vestibular cues is enlarged¹³, indicating that opposite neurons are associated with the
 91 disparity information between cues.

92 In the present study, we explore whether opposite neurons are responsible for cue segregation
 93 in multisensory information processing. Experimental findings showed that many, rather than a
 94 single, brain areas exhibit multisensory processing behaviors and that these areas are intensively
 95 and reciprocally connected with each other^{8,9,14-16}. The architecture of these multisensory areas
 96 is consistent with the structure of a decentralized model¹¹, which successfully reproduces almost
 97 all known phenomena observed in the multisensory integration experiments^{1,17}. Thus we also
 98 consider a decentralized multisensory processing model¹¹ in which each local processor receives a

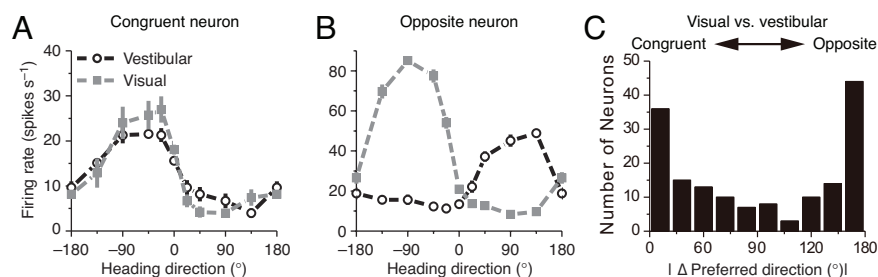


Figure 2: Congruent and opposite neurons in MSTd. Similar results were found in VIP¹⁸. (A-B) Tuning curves of a congruent neuron (A) and an opposite neuron (B). The preferred visual and vestibular directions are similar in (A) but are nearly opposite by 180° in (B). (C) The histogram of neurons according to their difference between preferred visual and vestibular directions. Congruent and opposite neurons are comparable in numbers. (A-B) adapted from ref. 8, (C) from ref. 19.

99 direct cue through feedforward inputs from the connected sensory modality and meanwhile, accesses
 100 information of other indirect cues via reciprocal connections between processors.

101 As a working example, we focus on studying the inference of heading-direction based on visual
 102 and vestibular cues. The network model consists of interconnected MSTd and VIP modules, where
 103 congruent and opposite neurons are widely found^{8,9}. Specifically, we propose that congruent neu-
 104 rons in the two brain areas are reciprocally connected with each other in the congruent manner:
 105 the closer between the preferred directions of a pair of neurons in their respective brain areas, the
 106 stronger their connection is, and this connection profile encodes effectively the prior knowledge
 107 about the two cues coming from the same stimulus. On the other hand, opposite neurons in the
 108 two brain areas are reciprocally connected in the opposite manner: the further away between the
 109 preferred directions of a pair of neurons in their respective brain areas (the maximal difference is
 110 180 degree), the stronger their connection is. Our model reproduces the tuning properties of op-
 111 posite neurons, and verifies that opposite neurons encode the disparity information between cues.
 112 Furthermore, we demonstrate that this disparity information, in coordination with the integration
 113 result of congruent neurons, enables the neural system to assess the validity of cue integration
 114 and to recover the lost information of individual cues if necessary. Our study sheds light on our
 115 understanding of how the brain achieves multisensory information processing efficiently and rapidly.

116 Results

117 Probabilistic models of multisensory processing

118 The brain infers stimulus information based on ambiguous sensory cues. We therefore formulate
 119 the multisensory processing problem in the framework of probabilistic inference, and as a working
 120 example, we focus on studying the inference of heading-direction based on visual and vestibular
 121 cues.

122 Probabilistic model of multisensory integration

123 To begin with, we introduce the probabilistic model of multisensory integration. Suppose two
 124 stimulus features $\{s_m\}$ generate two sensory cues $\{x_m\}$, for $m = 1, 2$ (the visual and vestibular
 125 cues) respectively (Fig. 1A), and we denote the corresponding likelihood functions as $p(x_m|s_m)$.
 126 The task of multisensory processing is to infer $\{s_m\}$ based on $\{x_m\}$. x_m is referred to as the direct

127 cue of s_m (e.g., the visual cue to MSTd) and x_l ($l \neq m$) the indirect cue of s_m (e.g., the vestibular
128 cue to MSTd).

129 Since heading-direction is a circular variable in the range of $(-\pi, \pi]$, we adopt the von Mises,
130 rather than the Gaussian, distribution to carry out the theoretical analysis. In the form of the von
131 Mises distribution, the likelihood function is given by

$$\begin{aligned} p(x_m|s_m) &= [2\pi I_0(\kappa_m)]^{-1} \exp[\kappa_m \cos(x_m - s_m)] \\ &\equiv \mathcal{M}(x_m; s_m, \kappa_m), \end{aligned} \quad (1)$$

132 where $I_0(\kappa)$ is the modified Bessel function of the first kind and order zero, and acts as the nor-
133 malization factor. s_m is the mean of the von Mises distribution, i.e., the mean value of x_m . κ_m is
134 a positive number characterizing the concentration of the distribution, and controls the reliability
135 of cue x_m .

136 The prior $p(s_1, s_2)$ describes the probability of concurrence of stimulus features (s_1, s_2) coming
137 from the same stimulus, and it determines the extent to which the two stimulus features should
138 be integrated. In this study, we consider a prior which has been used in several multisensory
139 integration studies^{11,20-22}, which is written as

$$\begin{aligned} p(s_1, s_2) &= (2\pi)^{-1} \mathcal{M}(s_1 - s_2; 0, \kappa_s) \\ &= [(2\pi)^2 I_0(\kappa_s)]^{-1} \exp[\kappa_s \cos(s_1 - s_2)]. \end{aligned} \quad (2)$$

140 This prior reflects that the two stimulus features from the same stimulus tend to have similar values.
141 The parameter κ_s specifies the concurrence probability of two stimulus features, and determines
142 the extent to which the two cues should be integrated. In the limit $\kappa_s \rightarrow \infty$, it will lead to full
143 integration (see, e.g., ref. 5). Note that the marginal prior $p(s_m)$ is a uniform distribution according
144 to the definition.

145 It has been revealed that the brain integrates visual and vestibular cues to infer heading-
146 direction in a manner close to Bayesian inference^{8,9}. Following Bayes' theorem, optimal multisensory
147 integration is achieved by computing the posterior of two stimuli according to

$$p(s_1, s_2|x_1, x_2) \propto p(x_1|s_1)p(x_2|s_2)p(s_1, s_2).$$

148 Since the calculations of the two stimuli are exchangeable, hereafter we only present the results
149 for s_1 . The posterior of s_1 is calculated through marginalizing the joint posterior in the above
150 equation,

$$\begin{aligned} p(s_1|x_1, x_2) &\propto p(x_1|s_1) \int_{-\pi}^{\pi} p(x_2|s_2)p(s_1, s_2)ds_2 \\ &\propto p(s_1|x_1)p(s_1|x_2) \\ &\approx \mathcal{M}(s_1; x_1, \kappa_1)\mathcal{M}(s_1; x_2, \kappa_{2s}), \end{aligned} \quad (3)$$

151 where we have used the conditions that the marginal prior distributions of s_m and x_m are uniform,
152 i.e., $p(s_m) = p(x_m) = (2\pi)^{-1}$. Note that $p(s_1|x_2) \propto \int p(x_2|s_2)p(s_1, s_2)ds_2$ is approximated to be
153 $\mathcal{M}(s_1; x_2, \kappa_{2s})$ through equating the mean resultant length of distribution (Eq. 12)²³.

154 The above equation indicates that in multisensory integration, the posterior of a stimulus given
155 combined cues is equal to the product of the posteriors given the individual cues. Notably, although
156 x_1 and x_2 are generated independently by s_1 and s_2 (since the visual and vestibular signal pathways
157 are separated), x_2 also provides information of s_1 due to the correlation between s_1 and s_2 specified
158 in the prior.

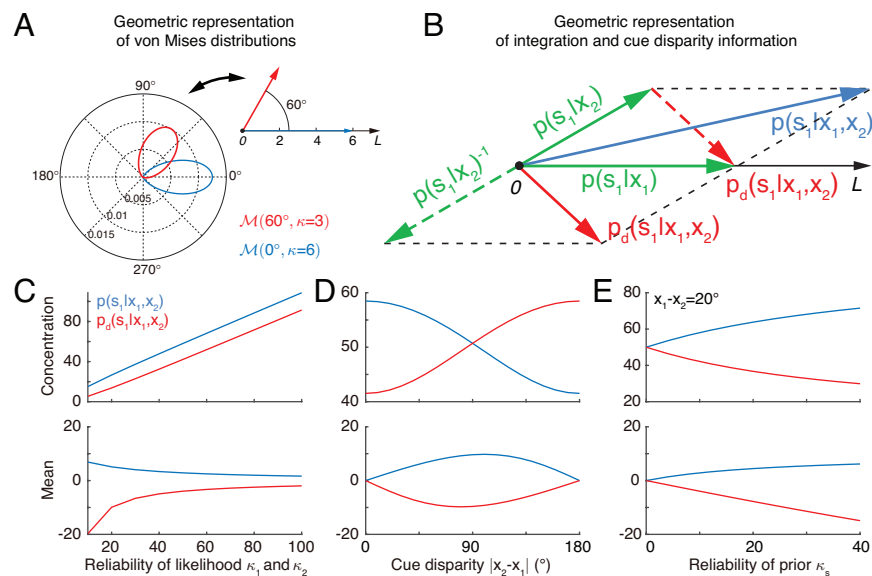


Figure 3: Geometric interpretation of multisensory processing of circular variables. (A) Two von Mises distributions plotted in the polar coordinate (bottom-left) and their corresponding geometric representations (top-right). A von Mises distribution can be represented as a vector, with its mean and concentration corresponding to the angle and length of the vector, respectively. (B) Geometric interpretation of cue integration and the cue disparity information. The posteriors of s_1 given single cues are represented by two vectors (green). Cue integration (blue) is the sum of the two vectors (green), and the cue disparity information (red) is the difference of the two vectors. (C-E) The mean and concentration of the integration (blue) and the cue disparity information (red) as a function of the cue reliability (C), cue disparity (D), and reliability of prior (E). In all plots, $\kappa_s = 50$, $\kappa_1 = \kappa_2 = 50$, $x_1 = 0^\circ$ and $x_2 = 20^\circ$, except that the variables are $\kappa_1 = \kappa_2$ in C, x_2 in D, and κ_s in E.

159 Finally, since the product of two von Mises distributions is again a von Mises distribution,
 160 the posterior distribution is $p(s_1|x_1, x_2) = \mathcal{M}(s_1; \hat{s}_1, \hat{\kappa}_1)$, whose mean and concentration can be
 161 obtained from its moments given by

$$\hat{\kappa}_1 e^{j\hat{s}_1} = \kappa_1 e^{jx_1} + \kappa_2 e^{jx_2}, \quad (4)$$

162 where j is an imaginary number. Eq. 4 is the result of Bayesian optimal integration in the form
 163 of von Mises distributions, and they are the criteria to judge whether optimal cue integration is
 164 achieved in the neural system. A link between the Bayesian criteria for von Mises and Gaussian
 165 distributions are presented in SI.

166 Eq. 4 indicates that the von Mises distribution of a circular variable can be interpreted as
 167 a vector in a two-dimensional space with its mean and concentration representing the angle and
 168 length of the vector, respectively (Fig. 3A). In this interpretation, the product of two von Mises
 169 distributions can be represented by the summation of the corresponding two vectors. Thus, optimal
 170 multisensory integration is equivalent to vector summation (see Eq. 4), with each vector representing
 171 the posterior of the stimulus given each cue (the sum of the two green vectors yields the blue vector
 172 in Fig. 3B).

173 Probabilistic model of multisensory segregation

174 The above probabilistic model for multisensory integration assumes that sensory cues are originated
 175 from the same stimulus. In case they come from different stimuli, the cues need to be segregated,

176 and the neural system needs to infer stimuli based on individual cues. In practice, the brain needs to
 177 differentiate these two situations. In order to achieve reliable and rapid multisensory processing, we
 178 propose that while integrating sensory cues, the neural system simultaneously extracts the disparity
 179 information between cues, so that with this complementary information, the neural system can
 180 assess the validity of cue integration.

181 An accompanying consequence of multisensory integration is that the stimulus information
 182 associated with individual cues is lost once they are integrated (see Supplementary Fig. S1). Hence
 183 besides assessing the validity of integration, extracting both congruent and disparity information
 184 by simultaneous integration and segregation enables the system to recover the lost information of
 185 individual cues if needed.

186 The disparity information of stimulus 1 obtained from the two cues is defined to be

$$p_d(s_1|x_1, x_2) \propto p(s_1|x_1)/p(s_1|x_2), \quad (5)$$

187 which is the ratio between the posterior given two cues and hence measures the discrepancy between
 188 the estimates from different cues. By taking the expectation of $\log p_d$ over the distribution $p(s_1|x_1)$,
 189 it gives rise to the Kullback-Leibler divergence between the two posteriors given each cue. This
 190 disparity measure was also used to discriminate alternative moving directions in ref. 24.

191 Utilizing the property of the von Mises distribution and the periodicity of heading directions
 192 ($-\cos(s_1 - x_2) = \cos(s_1 - x_2 - \pi)$), Eq. 5 can be re-written as

$$\begin{aligned} p_d(s_1|x_1, x_2) &\propto p(s_1|x_1)p(s_1|x_2 + \pi) \\ &\propto \mathcal{M}(s_1; x_1, \kappa_1)\mathcal{M}(s_1; x_2 + \pi, \kappa_{2s}). \end{aligned} \quad (6)$$

193 Thus, the disparity information between two cues can also be expressed as the product of the
 194 posterior given the direct cue and the posterior given the indirect cue with the cue direction shifted
 195 by π . Indeed, analogous to the derivation of Eq. 3, Eq. 6 can be deduced in the same framework
 196 as multisensory integration but with the stimulus prior $p(s_1, s_2)$ being modified by a shift π in the
 197 angular difference. Similarly, $p_d(s_1|x_1, x_2) = \mathcal{M}(s_1; \Delta\hat{s}_1, \Delta\hat{\kappa}_1)$ whose mean and concentration can
 198 be derived as

$$\Delta\hat{\kappa}_1 e^{\Delta\hat{s}_1} = \kappa_1 e^{jx_1} - \kappa_{2s} e^{jx_2}. \quad (7)$$

199 The above equation is the criteria to judge whether the disparity information between two cues is
 200 encoded in the neural system.

201 Similar to the geometrical interpretation of multisensory integration, multisensory segregation
 202 is interpreted as vector subtraction (the subtraction between two blue vectors yields the red vector
 203 in Fig. 3B). This enables us to assess the validity of multisensory integration. When the two vectors
 204 representing the posteriors given the individual cues have small disparity, i.e., the estimates from
 205 individual cues tend to support each other, the length of the summed vector is long, implying
 206 that the posterior of cue integration has a strong confidence, whereas the length of the subtracted
 207 vector is short, implying that the weak confidence of two cues are disparate (Fig. 3D). If the two
 208 vectors associated with the individual cues have a large disparity, the interpretation becomes the
 209 opposite (Fig. 3D). Thus, by comparing the lengths of the summed and subtracted vectors, the
 210 neural system can assess whether two cues should be integrated or segregated.

211 Figs. 3C and E further describes the integration and segregation behaviors when the model
 212 parameters vary. As shown in Fig. 3C, when the likelihoods have weak reliabilities, the Bayesian
 213 estimate relies more on the prior. Since the prior encourages integration of the two stimuli, the
 214 posterior estimate of stimulus 1 becomes more biased towards cue 2. At the same time, the mean

215 of the disparity information is biased towards the angular difference of the likelihood peaks. On the
216 other hand, when the likelihoods are strong, the Bayesian estimate relies more on the likelihood,
217 and the posterior estimate of stimulus 1 becomes less biased towards cue 2. The behavior when
218 the prior concentration κ_s varies can be explained analogously (Fig. 3E).

219 A notable difference between von Mises distribution and Gaussian distribution is that the con-
220 centration of integration and disparity information changes with cue disparity in von Mises distri-
221 bution (Fig. 3D), while they are fixed in Gaussian distribution²⁵.

222 Neural implementation of cue integration and segregation

223 Before introducing the neural circuit model, we first describe intuitively how opposite neurons
224 encode the cue disparity information and the motivation of the proposed network structure.

225 Optimal multisensory integration computes the posterior of a stimulus given combined cues
226 according to Eq. 3, which is equivalent to solving the equation $\ln p(s_1|x_1, x_2) = \ln p(s_1|x_1) +$
227 $\ln p(s_1|x_2)$. Ma *et al.* found that under the conditions that neurons fire independent Poisson spikes,
228 the optimal integration can be achieved by combining the neuronal responses under single cue
229 conditions, that is $\mathbf{r}_j(x_1, x_2) = \mathbf{r}_j(x_1) + \mathbf{r}_j(x_2)$ (see details in SI), where $\mathbf{r}(x_1, x_2)$ and $\mathbf{r}(x_m)$ are
230 the responses of a population of neurons to the combined and single cues respectively¹². Ma
231 *et al.* further demonstrated that such a response property can be approximately achieved in a
232 biological neural network. Similarly, multisensory segregation computes the disparity information
233 between cues according to $\ln p_d(s_1|x_1, x_2) = \ln p(s_1|x_1) + \ln p(s_1|x_2 + \pi)$ (see Eq. 6). Analogous to
234 multisensory integration, the optimal segregation can be achieved by $\mathbf{r}_j(x_1, x_2) = \mathbf{r}_j(x_1) + \mathbf{r}_{j'}(x_2)$,
235 where the preferred stimulus of neurons satisfying $\theta_{j'} = \theta_j + \pi$ (see details in SI). That is, the
236 neurons combine the responses to the direct cue and the responses to the indirect cue but shifted
237 to opposite direction. This inspires us to consider a network model where the inputs of indirect cue
238 received by opposite neurons are shifted to opposite direction via connections. Below, we present
239 the network model and demonstrate that the opposite neurons emerge from the connectivity and
240 are able to achieve optimal segregation.

241 The decentralized neural network model

242 The neural circuit model we consider has the decentralized structure¹¹, in the sense that it consists
243 of two reciprocally connected modules (local processors), representing MSTd and VIP respectively
244 (Fig. 4A). Each module carries out multisensory processing via cross-talks between modules. This
245 decentralized architecture agrees with the experimental findings that neurons in MSTd and VIP
246 both exhibit multisensory responses and that the two areas are abundantly connected with each
247 other^{15,16}. Below we only describe the key features of the decentralized network model, and its
248 detailed mathematical description is presented in Methods (Eqs. 14-20).

249 At each module, there exist two groups of excitatory neurons: congruent and opposite neurons
250 (blue and red circles in Fig. 4A respectively), and they have the same number of neurons, as sup-
251 ported by experiments (Fig. 2C)^{18,19}. Each group of neurons is modelled as a continuous attractor
252 neural network (CANN), mimicking the encoding of heading-direction in neural systems^{26,27}. In
253 CANN, each neuron is uniquely identified by its preferred heading direction θ with respect to the
254 direct cue conveyed by feedforward inputs. The neurons in the same group are recurrently con-
255 nected, and the recurrent connection strength between neurons θ and θ' is modelled as a von Mises
256 function decaying with the disparity between two neurons's preferred directions $|\theta - \theta'|$ (Fig. 4B
257 black line and Eq. 15). In the model, the recurrent connection strength is not very strong to
258 support persistent activities after switching off external stimuli, because no persistent activity is

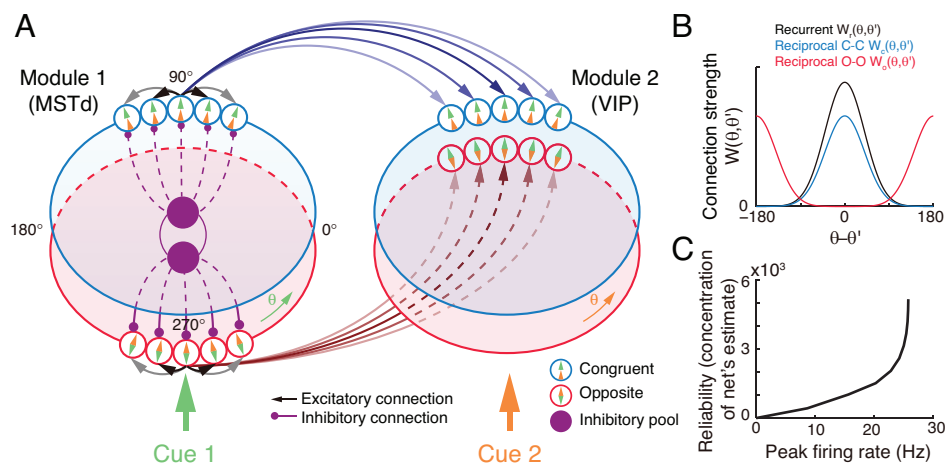


Figure 4: The decentralized neural circuit model for multisensory processing. (A) The network consists of two modules, which can be regarded as MSTd and VIP respectively. Each module has two groups of excitatory neurons, congruent (blue circles) and opposite neurons (red circles). Each group of excitatory neurons are connected recurrently with each other, and they are all connected to an inhibitory neuron pool (purple disk) to form a continuous attractor neural network. Each module receives a direct cue through feedforward inputs. Between modules, congruent neurons are connected in the congruent manner (blue arrows), while opposite neurons are connected in the opposite manner (brown lines). (B) Connection profiles between neurons. Black line is the recurrent connection pattern between neurons of the same type in the same module. Blue and red lines are the reciprocal connection patterns between congruent and opposite neurons across modules respectively. (C) The reliability of the networks estimate of a stimulus is encoded in the peak firing rate of the neuronal population. Typical parameters of network model: $\omega = 3 \times 10^{-4}$, $J_{int} = 0.5$, $J_{rc} = 0.3J_c$, $J_{rp} = 0.5J_{rc}$, I_b and F in Eq. 20 are 1 and 0.5 respectively.

259 observed in multisensory areas. Moreover, neuronal responses in the same group are normalized by
 260 the total activity of the population (Eq. 18), called divisive normalization²⁸, mimicking the effect
 261 of a pool of inhibitory neurons (purple disks in Fig. 4B). Each group of neurons has its individual
 262 inhibitory neuron pool, and the two pools of inhibitory neurons in the same module share their
 263 overall activities (Eq. 19), which intends to introduce mutual inhibition between congruent and
 264 opposite neurons.

265 Between modules, neurons of the same type are reciprocally connected with each other (Figs. 4A-
 266 B). For congruent neurons, they are connected with each other in congruent manner (Eq. 16 and
 267 Fig. 4B blue line), that is, the more similar their preferred directions are, the stronger the neuronal
 268 connection is. For opposite neurons, they are connected in the opposite manner (Eq. 17 and
 269 Fig. 4B red line), that is, the more different their preferred directions are, the stronger the neuronal
 270 connection is. Since the maximum difference between two circular variables is π , an opposite neuron
 271 in one module preferring θ has the strongest connection to the opposite neuron preferring $\theta + \pi$
 272 in the other module. This agrees with our intuitive understanding as described above (as suggested
 273 by Eq. 6): to calculate the disparity information between two cues, the neuronal response to the
 274 combined cues should integrate its responses to the direct cue and its response to the indirect one
 275 but with the cue direction shifted by π (through the offset reciprocal connections). We set the
 276 connection profile between the opposite neurons to be of the same strength and width as that
 277 between the congruent ones (comparing Eqs. 16 and 17), ensuring that the tuning functions of
 278 the opposite neurons have the similar shape as those of the congruent ones, as observed in the
 279 experimental data¹⁸.

280 When sensory cues are applied, the neurons combine the feedforward, recurrent, and reciprocal

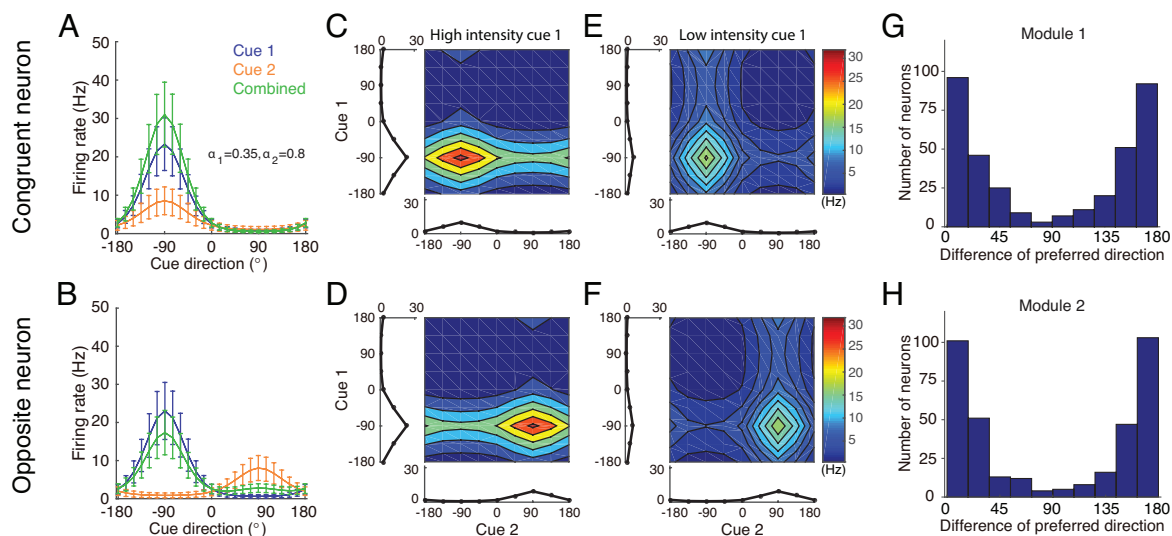


Figure 5: Tuning properties of congruent and opposite neurons in the network model. (A-B) The tuning curves of an example congruent neuron (A) and an example opposite neuron (B) in module 1 under three cueing conditions. (C-D) The bimodal tuning properties of the example congruent (C) and the example opposite (D) neurons when cue 1 has relatively higher reliability than cue 2 in driving neurons in module 1, with $\alpha_1 = 0.58\alpha_2$, where α_m is the amplitude of cue m given by Eq. 20. The two marginal curves around each contour plot are the unimodal tuning curves. (E-F) Same as (C-D), but cue 1 has a reduced reliability with $\alpha_1 = 0.12\alpha_2$. (G-H) The histogram of the differences of neuronal preferred directions with respect to two cues in module 1 (G) and module 2 (H), when the reciprocal connections across network modules contain random components of roughly the same order as the connections. Parameters: (A-B) $\alpha_1 = 0.35U_0$, and $\alpha_2 = 0.8U_0$; (C-F) $\alpha_2 = 1.5U_0$. $\alpha_1 = 0.35U_0$ in (C-D) while $\alpha_1 = 0.1U_0$ in (E-F). Other parameters are the same as those in Fig. 4.

281 inputs to update their activities (Eq. 14), and the multisensory integration and segregation will be
 282 accomplished by the reciprocal connections between network modules. The results are presented
 283 below.

284 Tuning properties of congruent and opposite neurons

285 Simulating the neural circuit model, we first checked the tuning properties of neurons. The sim-
 286 ulation results for an example congruent neuron and an example opposite neuron in module 1
 287 responding to single cues are presented in Fig. 5. It shows that the congruent neuron, in response
 288 to either cue 1 or cue 2, prefers the same direction (-90°) (Fig. 5A), whereas the opposite neuron,
 289 while preferring -90° for cue 1, prefers 90° for cue 2 (Fig. 5B). Thus, the tuning properties of
 290 congruent and opposite neurons naturally emerge through the network dynamics.

291 We further checked the responses of neurons to combined cues, and found that when there
 292 is no disparity between the two cues, the response of a congruent neuron is enhanced compared
 293 to the single cue conditions (green line in Fig. 5A), whereas the response of an opposite neuron is
 294 suppressed compared to its response to the direct cue (green line in Fig. 5B). These properties agree
 295 with the experimental data^{8,9} and is also consistent with the interpretation that the integrated and
 296 segregated amplitudes are respectively proportional to the vector sum and difference in Fig. 3.
 297 Following the experimental protocol¹³, we also plotted the bimodal tuning curves of the example
 298 neurons in response to the combined cues of varying reliability, and observed that when cue 1 has a
 299 relatively high reliability, the bimodal responses of both neurons are dominated by cue 1 (Fig. 5C-

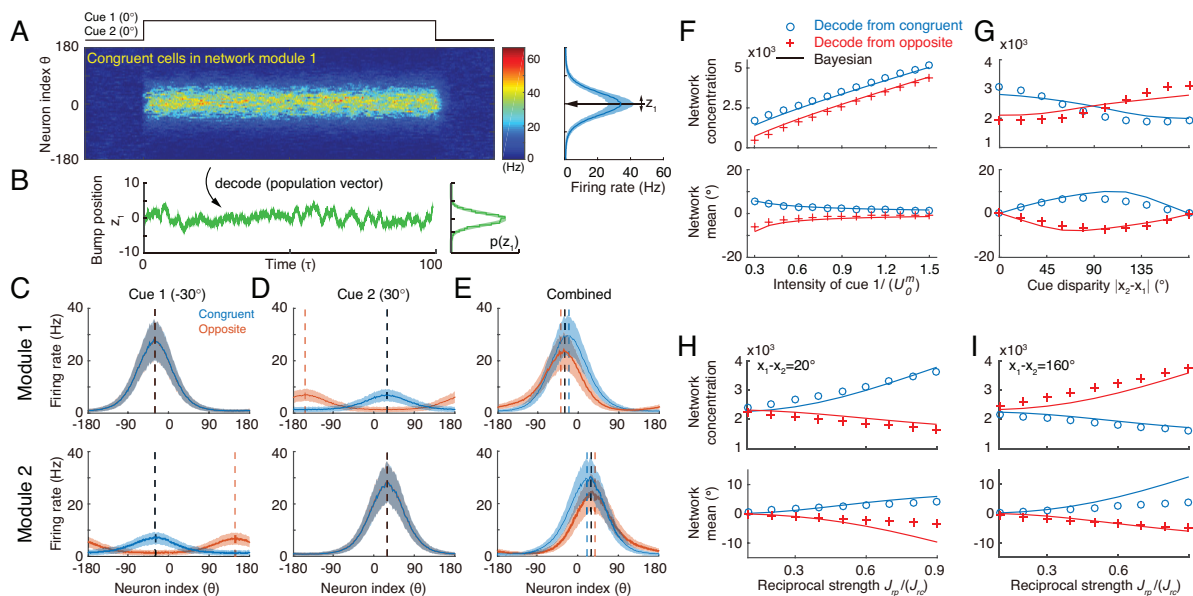


Figure 6: Optimal cue integration and segregation collectively emerge in the neural population activities in the network model. (A) Illustration of the population response of congruent neurons in module 1 when both cues are presented. Color indicates firing rate. Right panel is the temporal average firing rates of the neural population during cue presentation, with shaded region indicating the standard deviation (SD). (B) The position of the population activity bump at each instance is interpreted as the network’s estimate of the stimulus, referred to as z_1 , which is decoded by using population vector. Right panel is the distribution of the decoded network’s estimate during cue presentation. (C-E) The temporal average population activities of congruent (blue) and opposite (red) neurons in module 1 (top row) and module 2 (bottom row) under three cueing conditions: only cue 1 is presented (C), only cue 2 is presented (D), and both cues are simultaneously presented (E). (F-I) Comparing the estimates from congruent and opposite neurons in module 1 with the Bayesian predictions, with varying cue intensity (F), with varying cue disparity (G), and with varying reciprocal connection strength between modules (H&I). Symbols: network results; lines: Bayesian prediction. The Bayesian predictions for the estimates of congruent and opposite neurons are obtained by Eq. 4 and Eq. 7. Parameters: (A-E) $\alpha_1 = \alpha_2 = 0.35U_0$; (F) $\alpha_2 = 0.7U_0$; (G-I) $\alpha_1 = \alpha_2 = 0.7U_0$, and others are the same as those in Fig. 4. In (F-H), $x_1 = 0^\circ$, $x_2 = 20^\circ$ and in (I), $x_1 = 0^\circ$, $x_2 = 160^\circ$.

300 D), indicating that the neuronal firing rates are affected more significantly by varying the angle of
 301 cue 1 than by that of cue 2, whereas when the reliability of cue 1 is reduced, the result becomes
 302 the opposite (Fig. 5E-F). These behaviors agree with the experimental observations¹³.

303 Apart from the congruent and opposite neurons, the experiments also found that there exist
 304 a portion of neurons, called intermediate neurons, whose preferred directions to different cues are
 305 neither exactly the same nor the opposite, but rather have differences in between 0° and 180° ^{18,19}.
 306 We found that by considering the realistic imperfectness of neuronal reciprocal connections (e.g.,
 307 adding random components in the reciprocal connections in Eqs. (16 and 17), see Methods), our
 308 model reproduced the distribution of intermediate neurons as observed in the experiment (Fig. 5G-
 309 H)^{18,19}.

310 Optimal cue integration and segregation via congruent and opposite neurons

311 In response to the noisy inputs in a cueing condition, the population activity of the same group of
 312 neurons in a module exhibits a bump-shape (Fig. 6A), and the position of the bump is interpreted as
 313 the network’s estimate of the stimulus (Fig. 6B)^{27,29,30}. In a single instance, we used the population

314 vector to read out the stimulus value (Eq. 21)³¹. The statistics of the bump position sampled from
315 a collection of instances reflects the posterior distribution of the stimulus estimated by the neural
316 population under the given cueing condition. Note that in this probabilistic population coding
317 scheme, the concentration of the decoded posterior distribution is independent of the widths of the
318 bumps at individual instances.

319 To validate the hypothesis that congruent and opposite neurons are responsible for optimal
320 cue integration and segregation respectively, we carried out simulations following the protocol in
321 multisensory experiments¹, that is, we first applied individual cues to the network and decoded the
322 network's estimate of the stimulus through population vector (see details in Methods). With these
323 results, the Bayesian predictions for optimal integration and segregation were calculated according
324 to Eq. 4 and Eq. 7 respectively; we then applied the combined cues to the network, decoded the
325 network's estimate, and compared them with the Bayesian predictions.

326 Let us first look at the network's estimate under single cue conditions. Consider the case that
327 only cue 1 is presented to module 1 at -30° . The population activities of congruent and opposite
328 neurons at module 1 are similar, both centered at -30° (Fig. 6C top), since both types of neurons
329 receive the same feedforward input. On the other hand, in module 2, congruent neurons' responses
330 are centered at -30° , while opposite neurons' responses are centered at 150° due to the offset
331 reciprocal connections (Fig. 6C bottom). Similar population activities exist under cue 2 condition
332 (Fig. 6D).

333 We further look at the the network's estimate under the combined cue condition. Consider the
334 case that cues 1 and 2 are simultaneously presented to the network at the directions -30° and 30°
335 respectively. Then the disparity between the two cues is 60° , which is less than 90° . Compared
336 with single cue conditions, the responses of congruent neurons are enhanced (comparing Fig. 6E
337 with 6C-D), reflecting the increased reliability of the estimate after cue integration. Indeed, the
338 decoded distribution from congruent neurons sharpens in the combined cue condition and moves to
339 a location between cue 1 and cue 2 (Fig. S2 green), which is a typical phenomenon associated with
340 cue integration. In contrast, with combined cues, the responses of opposite neurons are suppressed
341 compared with those of the direct cue (comparing Fig. 6E with 6C-D). Certainly, the distribution
342 of cue disparity information decoded from opposite neurons in combined cue condition is wider
343 than that that under the direct cue condition (Fig. S2 purple). Note that when the cue disparity
344 is larger than 90° , the relative response of congruent and opposite neurons will be reversed (results
345 are not shown here).

346 To demonstrate that the network implements optimal cue integration and segregation and how
347 the network encodes the probabilistic model (Eqs. 1 and 2), we changed a parameter at a time,
348 and then compared the decoded results from congruent and opposite neurons with the Bayesian
349 prediction. Fig. 6F-I indicates that the network indeed implements optimal integration and seg-
350regation. Moreover, comparing the network results with the results of the probabilistic model,
351 we could find the analogy that the input intensity encodes the reliability of the likelihood (Eq. 1,
352 comparing Fig. 6F with Fig. 3C), and the reciprocal connection strength effectively represents the
353 reliability of the prior (Eq. 2, comparing Fig. 6H with Fig. 3E), which is consistent with a pre-
354-vious study¹¹. We further systematically changed the network and input parameters over a large
355 parameter region and compare the network results with Bayesian prediction. Our results indicated
356 that the network model achieves optimal integration and segregation robustly over a large range
357 of parameters (Fig. S3), as long as the connection strengths are not so large that winner-take-all
358 happens in the network model.

359 **Concurrent multisensory processing**

360 The above results elucidate that congruent neurons integrate cues, whereas opposite neurons com-
361 pute the disparity between cues. Based on these complementary information, the brain can access
362 the validity of cue integration and can also recover the stimulus information associated with single
363 cues lost due to integration. Below, rather than exploring the detailed neural circuit models, we
364 demonstrate that the brain has resources to implement these two operations based on the activities
365 of congruent and opposite neurons.

366 **Assessing integration vs. segregation**

367 The competition between congruent and opposite neurons can determine whether the brain should
368 integrate or segregate two cues. Fig. 7A displays how the mean firing rates of two types of neurons
369 change with the cue disparity, which shows that the activity of congruent neurons decreases with
370 the disparity, whereas the activity of opposite neurons increases with the disparity, and they are
371 equal at the disparity value of 90° . The brain can judge the validity of integration based on the
372 competition between these two groups of neurons (see more remarks in Conclusions and Discus-
373 sions). Specifically, the group of congruent neurons wins when the cue disparity is small, indicating
374 the choice of integration, and the group of opposite neurons wins when the cue disparity is large,
375 indicating the choice of segregation. The decision boundary is at the disparity of 90° , if the activi-
376 ties of congruent and opposite neurons have equal weights in decision-making. In reality, however,
377 the brain may assign different weights to congruent and opposite neurons and realize a decision
378 boundary at the position satisfying the statistics of inputs (Fig. 7B).

379 **Recovering the single cue information**

380 Once the decision for cue segregation is reached, the neural system at each module needs to decode
381 the stimulus based purely on the direct cue, and ignores the irrelevant indirect one. Through
382 combining the complementary information from congruent and opposite neurons, the neural system
383 can recover the stimulus estimates lost in integration, without re-gathering new inputs from lower
384 brain areas if needed (see more remarks in Conclusions and Discussions).

385 According to Eqs. 3 and 6, the posterior distribution of the stimulus given the direct cue can
386 be recovered by

$$\ln p(s_1|x_1) = [\ln p(s_1|x_1, x_2) + \ln p_d(s_1|x_1, x_2)] / 2. \quad (8)$$

387 As suggested in refs. 12,24, the above operation can be realized by considering neurons receiving
388 the activities of congruent neurons (representing $\ln p(s_1|x_1, x_2)$, Fig. 7C blue) and opposite neurons
389 (representing $\ln p_d(s_1|x_1, x_2)$, Fig. 7C red) as inputs and generate Poisson spikes, such that the
390 location of population responses and the summed activity encode respectively the mean and variance
391 of the posterior $p(s_1|x_1)$ (Fig. 7C green).

392 Without actually building a neural circuit model, we decoded the stimulus by utilizing the
393 activities of congruent and opposite neurons according to Eq. 8, and compared the recovered result
394 with the estimate of a module when only the direct cue is presented (see the detail in Methods).
395 Fig. 7D further shows that the recovering agrees with actual distribution and is robust against
396 a variety of parameters ($R^2 = 0.985$). Thus, through combining the activities of congruent and
397 opposite neurons, the neural system can recover the lost stimulus information from direct cues if
398 necessary.

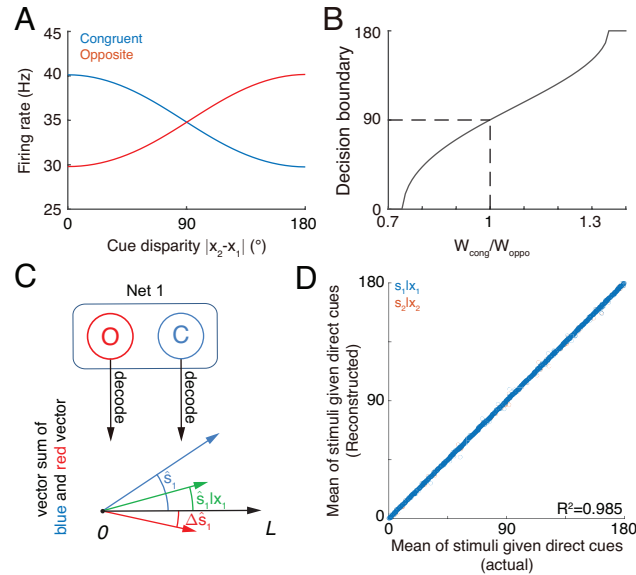


Figure 7: Concurrent multisensory processing with congruent and opposite neurons. (A-B) Accessing integration versus segregation through the joint activity of congruent and opposite neurons. (A) The firing rate of congruent and opposite neurons exhibit complementary changes with cue disparity $x_1 - x_2$. (B) The decision boundary of the competition between congruent and opposite neurons changes with read out weight from congruent W_{cong} and opposite neurons W_{oppo} . It is given by the value of $x_1 - x_2$ at which $W_{cong}r_m^c = W_{oppo}r_m^o$. Dashed line is when $W_{cong} = W_{oppo}$, the decision boundary is at 90° . (C-D) Recovering single cue information from two types of neurons. (C) Illustration of recovering through the joint activities of congruent (blue) and opposite (red) neurons under the combined cue condition. We decoded the estimate from congruent and opposite neurons respectively, and then vector sum the decoded results recovering the single cue information. (D) Comparing the recovered mean of the stimulus given the direct cue with the actual value. Parameters: those in (A-B) are the same as those in Fig. 6A, and those in D are the same as those in Fig. S3.

399 Conclusions and Discussions

400 Animals face challenges of processing information fast in order to survive in natural environments,
 401 and over millions of years of evolution, the brain has developed efficient strategies to handle these
 402 challenges. In multisensory processing, such a challenge is to integrate/segregate multisensory sen-
 403 sory cues rapidly without knowing in advance whether these cues are from the same or different
 404 stimuli. To resolve this challenge, we argue that the brain should carry out multisensory process-
 405 ing concurrently by employing congruent and opposite cells to realize complementary functions.
 406 Specifically, congruent neurons perform cue integration with opposite neurons computing the cue
 407 disparity simultaneously, so that they generate complementary information, based on which the
 408 neural system can assess the validity of integration and recover the lost information associated
 409 with single cues if necessary. Through this process, the brain can, on one hand, achieve rapid
 410 stimulus perception if the cues are from the same stimulus of interest, and on the other hand, dif-
 411 ferentiate and recognize stimuli based on individual cues with little time delay if the cues are from
 412 different stimuli of interest. We built a biologically plausible network model to validate this pro-
 413 cessing strategy. The model consists of two reciprocally connected modules representing MSTd and
 414 VIP, respectively, and it carries out heading-direction inference based on visual and vestibular cues.
 415 Our model successfully reproduces the tuning properties of opposite neurons, verifying that oppo-
 416 site neurons encode the disparity information between cues, and demonstrates that the interplay

417 between congruent and opposite neurons can implement concurrent multisensory processing.

418 Opposite neurons have been found in experiments for years^{8,9}, but their functional role remains
419 a mystery. There have been few studies investigating this issue, and two computational works were
420 reported^{32,33}, where the authors explored the contribution of opposite neurons in a computational
421 task of inferring self-motion direction by eliminating the confound information of object motion.
422 They showed that opposite neurons are essential, as they provide complementary information to
423 congruent neurons necessary to accomplish the required computation. This result is consistent with
424 our idea that opposite neurons are indispensable in multisensory processing, but our study goes
425 one step further by theoretically proposing that opposite neurons encode the disparity information
426 between cues and that congruent and opposite neurons jointly realize concurrent multisensory
427 processing.

428 Our hypothesis on the computational role of opposite neurons can be tested in experiments.
429 Through recording the activities of individual congruent neurons in awake monkeys when the mon-
430 keys are performing heading-direction discrimination, previous studies demonstrated that congru-
431 ent neurons implement optimal cue integration^{8,9}. We can carry out a similar experiment to check
432 whether opposite neurons encode the cue disparity information. The task is to discriminate whether
433 the disparity from two cues, $x_1 - x_2$, is either smaller or larger than 0° . To rule out the influence
434 of the change of integrated direction to the activities of neurons, we fix the center of two cues, for
435 example, the center is fixed at 0° , i.e., $x_1 + x_2 = 0^\circ$, but the disparity between cues $x_1 - x_2$ varies
436 over trials. Fig. 8A plots the responses of an example opposite neuron and an example congruent
437 neuron respectively in our model with respect to the cue disparity $x_1 - x_2$. It shows that the firing
438 rate of the opposite neurons changes much more significantly with the cue disparity than that of the
439 congruent neuron, suggesting that the opposite neuron's response might be more informative to the
440 change of cue disparity compared with a congruent neuron. To quantify how the activity of a single
441 neuron can be used to discriminate the cue disparity, we apply receiver-operating-characteristics
442 (ROC) analysis to construct the neurometric function (Fig. 8B), which measures the fraction of
443 correct discrimination (see Methods). Indeed, the opposite neurons can discriminate the cue dis-
444 parity much finer than congruent neurons (Fig. 8C). In addition, our model also reproduces the
445 same discrimination task studied in refs. 8,9, i.e., to discriminate whether the heading-direction is
446 on the left or right hand side of a reference direction under different cueing conditions (Fig. S4).

447 The present study only investigated integration and segregation of two sensory cues, but our
448 model can be generalized to the cases of processing more than two cues that may happen in reality³⁴.
449 In such situations, the network model consists of $N > 2$ modules, and in module m , the received
450 sensory cues can be differentiated as the direct one and the integrated results through combining
451 all cues,

$$p_d(s_m|x_1, \dots, x_N) \propto \frac{p(s_m|x_m)}{\left[\prod_{j=1}^N p(s_m|x_j)\right]^{1/N}}. \quad (9)$$

452 Congruent neurons can be reciprocally connected with each other between modules in the congru-
453 ent manner as described above, so that they integrate the direct and all indirect cues optimally
454 in the distributed manner. Opposite neurons could receive the direct cue from feedforward in-
455 puts (numerator in Eq. 9), and receive the activities of congruent neurons in the opposite manner
456 (denominator in Eq. 9) through offset connection by 180° . The interplay between congruent and
457 opposite neurons determines whether the direct cue should be integrated with all other cues at each
458 module, and their joint activities can recover the stimulus information based only on the direct cue
459 if necessary. This encoding strategy is similar with the norm-based encoding of face found in IT
460 neurons³⁵.

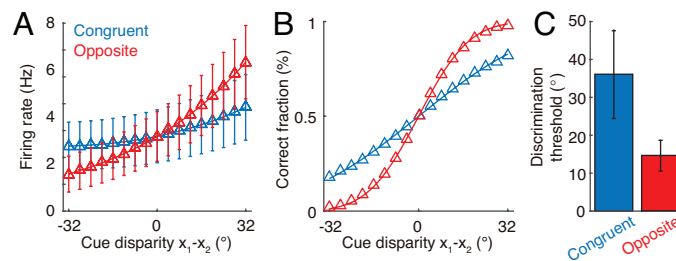


Figure 8: Discrimination of cue disparity by single neurons. (A) The tuning curve of an example congruent (green) and opposite (red) neuron with respect to cue disparity $x_1 - x_2$. In the tuning with respect to cue disparity, the mean of two cues was always at 0° , i.e., $x_1 + x_2 = 0$, while their disparity $x_1 - x_2$ was varied from -32° to 32° with a step of 4° . The two example neurons are in network module 1, and both prefer 90° with respect to cue 1. However, the congruent neuron prefers 90° of cue 2, while the opposite neuron prefers -90° with respect to cue 2. Error bar indicates the SD of firing rate across trials. (B) The neurometric function of the example congruent and opposite neuron in a discrimination task to determine whether the cue disparity $x_1 - x_2$ is larger than 0° or not. Lines are the cumulative Gaussian fit of the neurometric function. (C) Averaged neuronal discrimination thresholds of the example congruent and opposite neurons. Parameters: $\alpha_1 = 0.25U_0$, $\alpha_2 = 0.8U_0$, and others are the same as those in Fig. 4.

461 In the present study, we only demonstrated by analysis that the neural system can utilize the
 462 joint activities of congruent and opposite neurons to assess the validity of cue integration and to
 463 recover the information of direct cues in cue integration, but we did not go into the detail of how
 464 the brain actually carries out these operations. For assessing the validity of cue integration, essen-
 465 tially it is to compare the activities of congruent and opposite neurons and the winner indicates
 466 the choice. This competition process can be implemented easily in neural circuitry. For instance,
 467 it can be implemented by considering that congruent and opposite neurons are connected to the
 468 same inhibitory neuron pool which induces competition between them, such that only one group of
 469 neurons will sustain active responses after competition to represent the choice; alternatively, the ac-
 470 tivities of congruent and opposite neurons provide competing inputs to a decision-making network,
 471 and the latter generates the choice by accumulating evidence over time^{36,37}. Both mechanisms are
 472 feasible but further experiments are needed to clarify which one is used in practice. For recovering
 473 the stimulus information from direct cues by using the activities of congruent and opposite neurons,
 474 this study has shown that it can be done in a biologically plausible neural network, since the op-
 475 eration is expressed as solving the linear equation given by Eq. 8. A concern is, however, whether
 476 recovering is really needed in practice, since at each module, the neural system may employ an
 477 additional group of neurons to retain the stimulus information estimated from the direct cue. An
 478 advantage of recovering the lost stimulus information by utilizing congruent and opposite neurons
 479 is saving the computational resource, but this needs to be verified by experiments.

480 The present study focused on investigating the role of opposite neurons in heading-direction
 481 inference with visual and vestibular cues as an example. In essence, the contribution of opposite
 482 neurons is to retain the disparity information between features to be integrated for the purpose
 483 of rapid concurrent processing. We therefore expect that opposite neurons, or their counterparts
 484 of similar functions, is a general characteristic of neural information processing where feature in-
 485 tegration and segregation are involved. Indeed, it has been found that in the middle temporal
 486 cortex (MT), two types of neurons exhibit congruent and opposite tuning properties with respect
 487 to moving directions at the center and surrounding of their receptive fields, respectively, and their
 488 numbers are comparable³⁸. Moreover, MT neurons also exhibit congruent and opposite tunings
 489 with respect to binocular disparity and motion parallax, respectively³⁹. We hope that this study

490 gives us insight into understanding the general principle of how the brain integrates/segregates
491 multiple sources of information efficiently and rapidly.

492 Methods

493 Probabilistic model and its inference

494 The probabilistic model used in this study is widely adopted in multisensory research^{20–22,25}. Sup-
495 pose that two sensory cues x_1 and x_2 are independently generated by two underlying stimuli s_1 and
496 s_2 respectively. In the example of visual-vestibular cue integration¹, s_1 and s_2 refer to the underly-
497 ing visual and vestibular moving direction, while x_1 and x_2 are internal representations of moving
498 direction in the visual and vestibular cortices. Because moving direction is a circular variable, we
499 also assume that both s_m and x_m ($m = 1, 2$) are circular variables distributed in the range $(-\pi, \pi]$.
500 Because each cue is independently generated by the corresponding underlying stimulus, the joint
501 likelihood function can be factorized

$$p(x_1, x_2 | s_1, s_2) = p(x_1 | s_1)p(x_2 | s_2).$$

502 In this study, each likelihood function $p(x_m | s_m)$ ($m = 1, 2$) is modelled by the von Mises distribu-
503 tion, which is a variant of circular Gaussian distribution^{23,40}, given by Eq. 1. Note that in Eq. 1,
504 κ_m is a positive number characterizing the concentration of the distribution, which is analogous
505 to the inverse of the variance (σ^{-2}) of Gaussian distribution. In the limit of large κ_m , a von
506 Mises distribution $\mathcal{M}(x_m; s_m, \kappa_m)$ approaches to a Gaussian distribution with variance of κ_m^{-1} ²³.
507 $I_0(\kappa_m) = (2\pi)^{-1} \int_{-\pi}^{\pi} e^{\kappa \cos \theta} d\theta$ is the modified Bessel function of the first kind and order zero, which
508 acts as the normalization factor of the von Mises distribution.

509 The prior $p(s_1, s_2)$ specifies the probability of occurrence of s_1 and s_2 , and is set as a von Mises
510 distribution of the discrepancy between two stimuli^{11,20,21}, given by Eq. 2. Note that the marginal
511 prior of either stimulus, e.g., $p(s_1) = \int_{-\pi}^{\pi} p(s_1, s_2) ds_2 = 1/2\pi$ is a uniform distribution.

512 Inference

513 The inference of underlying stimuli can be conducted by using Bayes' theorem to derive the posterior

$$p(s_1, s_2 | x_1, x_2) \propto p(x_1 | s_1)p(x_2 | s_2)p(s_1, s_2), \quad (10)$$

514 The posterior of either stimuli, e.g., stimulus s_1 , can be obtained by marginalizing the joint posterior
515 (Eq. 10) as follows (the posterior of can be similarly obtained by interchanging indices 1 and 2)

$$\begin{aligned} p(s_1 | x_1, x_2) &= \int_{-\pi}^{\pi} p(s_1, s_2 | x_1, x_2) ds_2 \\ &\propto p(x_1 | s_1) \int_{-\pi}^{\pi} p(x_2 | s_2) p(s_1, s_2) ds_2 \\ &\propto p(s_1 | x_1) p(s_1 | x_2), \end{aligned} \quad (11)$$

516 where we used the fact that both marginal distributions $p(s_m)$ and $p(x_m)$ are uniform and then
517 interchanged the role of x_m and s_1 in their conditional distributions. It indicates that the posterior
518 of s_1 given two cues corresponds to a product of posterior of s_1 when each x_m is individually
519 presented, which could effectively accumulate the information of s_1 from both cues. $p(s_1 | x_2)$ can

520 be calculated as (see details in SI),

$$p(s_1|x_2) \propto \int_{-\pi}^{\pi} p(x_2|s_2)p(s_1, s_2)ds_2 \simeq \mathcal{M}(s_1; x_2, \kappa_{2s}),$$

where $A(\kappa_{2s}) = A(\kappa_2)A(\kappa_s).$ (12)

521 $A(\kappa) = \int_{-\pi}^{\pi} \cos \theta e^{\kappa \cos \theta} d\theta / \int_{-\pi}^{\pi} e^{\kappa \cos \theta} d\theta$ calculates the mean resultant length (first order trigono-
 522 metric statistics), measuring the dispersion of a von Mises distribution. An approximation was used
 523 in the calculation through equating the mean resultant length of the integral with that of a von
 524 Mises distribution²³, because the integral of the product of two von Mises distributions is no longer
 525 a von Mises distribution. The meaning of $A(\kappa_{2s})$ can be understood by considering the Gaussian
 526 equivalent of von Mises distribution, where the inverse of concentration κ^{-1} can approximate the
 527 variance of Gaussian distribution, yielding $\kappa_{2s}^{-1} \approx \kappa_2^{-1} + \kappa_s^{-1}$.

528 Finally, substituting the detailed expression into Eq. 11,

$$\begin{aligned} p(s_1|x_1, x_2) &\propto \exp[\kappa_1 \cos(s_1 - x_1) + \kappa_{2s} \cos(s_1 - x_2)] \\ &\propto \exp[(\kappa_1 \cos x_1 + \kappa_{2s} \cos x_2) \cos s_1 \\ &\quad + (\kappa_1 \sin x_1 + \kappa_{2s} \sin x_2) \sin s_1] \\ &\propto \exp[\hat{\kappa}_1 \cos(s_1 - \hat{s}_1)]. \end{aligned}$$
 (13)

529 The expressions of the mean \hat{s}_1 and concentration $\hat{\kappa}_1$ can be found in Eq. 4. The expressions of
 530 $\Delta\hat{s}_1$ and $\Delta\hat{\kappa}_1$ in the disparity information can be similarly calculated and is shown in Eq. 7.

531 Dynamics of decentralized network model

532 We adopted a decentralized network model in this study¹¹. The network model contains two
 533 network modules, with each module consisting of two groups of neurons with the same number:
 534 one is intended to model congruent neurons and another is for opposite neurons. Each neuronal
 535 group is modelled as a continuous attractor neural network^{27,41,42}, which has been widely used
 536 to model the coding of continuous stimuli in the brain^{31,43,44} and it can optimally implement
 537 maximal likelihood inference^{29,30}. Denote $u_m^n(\theta, t)$ and $r_m^n(\theta, t)$ as the synaptic input and firing
 538 rate at time t respectively for an n -type neuron ($n = c, o$ represents the congruent and opposite
 539 neurons respectively) in module m ($m = 1, 2$) whose preferred heading direction with respect to
 540 the feedforward cue m is θ . It is worthwhile to emphasize that θ is the preferred direction only
 541 to the feedforward cue, e.g., the feedforward cue to network module 1 is cue 1, but θ does not
 542 refer to the preferred direction given another cue, because the preferred direction of an opposite
 543 neuron given each cue is different. In the network model, the network module $m = 1, 2$ can be
 544 regarded as the brain areas MSTd and VIP respectively. For simplicity, we assume that the two
 545 network modules are symmetric, and only present the dynamical equations for network module 1.
 546 The dynamical equations for network module 2 can be obtained by interchanging the indices 1 and
 547 2 in the following dynamical equations.

548 The dynamics of the synaptic input of n -type neurons in network module m , $u_m^n(\theta, t)$, is governed
 549 by

$$\begin{aligned} \tau \frac{\partial u_m^n(\theta, t)}{\partial t} &= -u_m^n(\theta, t) + \sum_{\theta'=-\pi}^{\pi} W_{rc}(\theta, \theta') r_m^n(\theta', t) + \\ &\quad \sum_{\theta'=-\pi}^{\pi} W_{rp}^n(\theta, \theta') r_{k \neq m}^n(\theta', t) + I_m^n(\theta, t), \end{aligned}$$
 (14)

550 where $I_m^n(\theta, t)$ is the feedforward inputs from unisensory brain areas conveying cue information.
 551 $W_{rc}(\theta, \theta')$ is the recurrent connections from neuron θ' to neuron θ within the same group of neurons
 552 and in the same network module, which is set to be

$$W_{rc}(\theta, \theta') = \frac{J_{rc}}{2\pi I_0(a)} \exp [a \cos(\theta - \theta')], \quad (15)$$

553 where a is the connection width and effectively controls the width of neuronal tuning curves.
 554 $W_{rp}^n(\theta, \theta')$ denotes the reciprocal connections between congruent neurons across network modules
 555 ($n = c$), or between opposite neurons across network modules ($n = o$). $W_{rp}^c(\theta, \theta')$ is the reciprocal
 556 connections between congruent cells across two modules (the superscript c denotes the connections
 557 are in a congruent manner, i.e., a 0° neuron will have the strongest connection with a 0° neuron),

$$W_{rp}^c(\theta, \theta') = \frac{J_{rp}}{2\pi I_0(a)} \exp [a \cos(\theta - \theta')]. \quad (16)$$

558 For simplicity, $W_{rp}^c(\theta, \theta')$ and $W_{rc}(\theta, \theta')$ have the same connection width a . This simplification
 559 does not change the basic conclusion substantially. A previous study indicates that the reciprocal
 560 connection strength J_{rp} determines the extent of cue integration, and effectively represents the
 561 correlation of two underlying stimuli in the prior $p(s_1, s_2)$ ¹¹. Moreover, the opposite neurons from
 562 different network modules are connected in an opposite manner with an offset of π ,

$$W_{rp}^o(\theta, \theta') = \frac{J_{rp}}{2\pi I_0(a)} \exp [a \cos(\theta - \theta' + \pi)]. \quad (17)$$

563 Hence, an opposite neurons preferring 0° of cue 1 in network module 1 will have the strongest
 564 connection with the opposite neurons preferring of 180° of cue 2 in network module 2. It is
 565 worthwhile to note that the strength and width of $W_{rp}^c(\theta, \theta')$ and $W_{rp}^o(\theta, \theta')$ are the same, in order
 566 to convey the same information from the indirect cue. This is also supported by the fact that the
 567 tuning curves of the congruent and opposite neurons have similar tuning strengths and widths¹⁸.

568 Each neuronal group contains an inhibitory neuron pool which sums all excitatory neurons'
 569 activities and then divisively normalize the response of the excitatory neurons,

$$r_m^n(\theta, t) = \frac{[u_m^n(\theta, t)]_+^2}{1 + \omega D_m^n(t)}, \quad (18)$$

570 where ω controls the magnitude of divisive normalization, and $[x]_+ = \max(x, 0)$ is the negative
 571 rectified function. $D_m^n(t)$ denotes the response of the inhibitory neuron pool associated with neurons
 572 of type n in network module m at time t , which sums up the synaptic inputs of the same type of
 573 excitatory neurons $u_m^n(\theta, t)$ and also receives the inputs from the other type of neurons $u_m^{n'}(\theta, t)$,

$$D_m^n(t) = \sum_{\theta} [u_m^c(\theta, t)]_+^2 + J_{int} \sum_{\theta} [u_m^{n'}(\theta, t)]_+^2. \quad (19)$$

574 J_{int} is a positive coefficient not larger than 1, which effectively controls the sharing between the
 575 inhibitory neuron pool associated with the congruent and opposite neurons in the same network
 576 module. The partial share of the two inhibitory neuron pools inside the same network module
 577 introduces competition between two types of neurons, improving the robustness of network.

578 The feedforward inputs convey the direct cue information from the unisensory brain area to a
 579 network module, e.g., the feedforward inputs received by MSTd neurons is from MT which extracts
 580 the heading direction from optic flow,

$$I_m^n(\theta, t) = I_m^{ff}(\theta) + \sqrt{F I_m^{ff}(\theta)} \xi_m(\theta, t) + I_b + \sqrt{F I_b} \epsilon_m^n(\theta, t),$$

where $I_m^{ff}(\theta) = \alpha_m \exp[a \cos(\theta - x_m)/2 - a/2].$ (20)

581 The feedforward inputs contain two parts: one conveys the cue information (the first two terms in
582 above equation), and another the background inputs (the last two terms in the above equation)
583 which are always present no matter whether a cue is presented or not. The variance of the noise
584 in the feedforward inputs $FI_m^{ff}(\theta)$ is proportional to their mean, and F characterizes the Fano
585 factor. The multiplicative noise is in accordance with the Poisson variability of the cortical neurons'
586 response. α_m is the intensity of the feedforward input and effectively controls the reliability of cue m .
587 x_m is the direction of cue m . I_b is the mean of background input. $\xi_m(\theta, t)$ and $\epsilon_m^n(\theta, t)$ are mutually
588 independent Gaussian white noises of zero mean with variances satisfying $\langle \xi_m(\theta, t)\xi_{m'}(\theta', t) \rangle =$
589 $\delta_{mm'}\delta(\theta-\theta')\delta(t-t')$, and $\langle \epsilon_m^n(\theta, t)\epsilon_{m'}^n(\theta', t) \rangle = \delta_{mm'}\delta_{nn'}\delta(\theta-\theta')\delta(t-t')$. Note that the cue-associated
590 noise $\xi_m(\theta, t)$ to congruent and opposite neurons are exactly the same, while the background noise
591 $\epsilon_m^n(\theta, t)$ to congruent and opposite neurons are independent of each other. Previous works indicated
592 that the exact form of the feedforward inputs is not crucial, as long as they have a uni-modal
593 shape⁴².

594 Network simulation and parameters

595 Each network module contains 180 congruent and opposite neurons respectively, whose preferred di-
596 rection with respect to the feedforward cue is uniformly distributed in the feature space $(-180^\circ, 180^\circ]$.
597 For simplicity, the parameters of the two network modules were chosen symmetric with each other,
598 i.e., all structural parameters of the two modules have the same value. The synaptic time constant
599 τ was rescaled to 1 as a dimensionless number and the time step size was 0.01τ in simulation. All
600 connections have the same width $a = 3$, which is equivalent to a value of about 40° for the width
601 of tuning curves of the neurons. The dynamical equations are solved by using Euler method.

602 The range of parameters was listed in the following if not mentioned otherwise. The detailed
603 parameters for each figure can be found in figure captions. The strength of divisive normalization
604 was $\omega = 3 \times 10^{-4}$, and $J_{int} = 0.5$ which controls the proportion of share between the inhibition pools
605 affiliated with congruent and opposite neurons in the same module (Eq. 19). The absolute values
606 of ω and J_{int} did not affect our basic results substantially, and they only determine the maximal
607 firing rate the neurons can reach. Of the particular values we chose, the firing rate of the neurons
608 saturates at around 50 Hz. The recurrent connection strength between neurons of the same type and
609 in the same network module was $J_{rc} = [0.3, 0.4]J_c$, where J_c is the minimal recurrent strength for a
610 network module to hold persistent activity after switching off feedforward inputs. The expression
611 of J_c can be found in SI. The strength of the reciprocal connections between the network modules
612 is $J_{rp} = [0.1, 0.9]J_{rc}$, and is always smaller than the recurrent connection strength within the same
613 network module. The sum of the recurrent strength J_{rc} and reciprocal strength J_{rp} cannot be too
614 large, since otherwise the congruent and opposite neurons in the same network module will have
615 strong competition resulting in the emergence of winner-take-all behavior. However, the winner-
616 take-all behavior was not observed in experiments. The input intensity α was scaled relative to
617 $U_0 = J_c e^{a/2} / [2\pi\omega(1 + J_{int})I_0(a/2)]$, and is distributed in $[0.3, 1.5]U_0$, where U_0 is the value of the
618 synaptic bump height that a group of neurons can hold without receiving feedforward input and
619 reciprocal inputs when $J_{rc} = J_c$. The range of the input intensity was chosen to be wide enough to
620 cover the super-linear to nearly saturated regions of the input-firing rate curve of the neurons. The
621 strength of the background input was $I_b = 1$, and the Fano factors of feedforward and background
622 inputs were set to 0.5, which led to the Fano factor of single neuron responses taking values of the
623 order 1. In simulations, the position of the population activity bump was read out by calculating
624 the population vector^{31,45}. For example, the position of the population activities of the congruent
625 neurons in module 1 at time t was estimated as

$$z_1^c(t) = \arg \left[\sum_{\theta} r_1^c(\theta, t) e^{j\theta} \right], \quad (21)$$

626 where j is the imaginary unit, and the function $\arg[\cdot]$ outputs the angle of a vector.

627 **Demo tasks of network model**

628 **Testing network's performance of integration and segregation**

629 We firstly applied each single cue to the network model individually. Under each cueing condition,
630 we recorded the population activities in equilibrium state across time during cue presentation.
631 In equilibrium state, the statistics of neuronal activities across time is equivalent to across trial.
632 For each group of neurons in a module, e.g., the congruent neurons in network module 1, the
633 instantaneous firing activities at an instance are fed into the population vector decoder (Eq. 21)
634 to get the instantaneous stimulus estimate z_1 made by neurons. When the direct cue (cue 1) is
635 presented, the estimates z_1 of a collection of instances are then substituted into Eqs. (S59) and
636 (S61) to calculate the mean and concentration of the activities of the congruent neurons. In the
637 single cue condition, the mean angle of the bump position is effectively the same as x_1 . Hence this
638 decoded mean and concentration can be substituted into the first term on the right hand side of
639 Eq. 4. Similarly, when only the indirect cue (cue 2) is presented, the estimates of a collection of
640 instances of neural activities contribute to the second term. The sum of the two terms yields the
641 Bayesian prediction of the optimal integration in combined cue condition. For opposite neurons,
642 we substituted the decoded means and concentrations into Eq. (7) to get the prediction of optimal
643 segregation in combined cue condition.

644 **Reconstructing stimulus estimate under direct cue from congruent and opposite neurons' ac-** 645 **tivity**

646 The stimulus estimate from its direct cue can be recovered from the joint activities of congruent
647 and opposite neurons in real-time when two cues are simultaneously presented. Eq. 8 indicates that
648 the reconstruction of the posterior distribution of the direct cue can be achieved by multiplying
649 the decoded distribution from congruent and opposite neurons in a network module. Thus, for
650 example, the reconstructed estimate of stimulus 1 at time t given its direct cue can be obtained by

$$\hat{s}_1(t)|x_1 = \arg \left[(\sum_{\theta} r_1^c(\theta, t)) e^{jz_1^c(t)} + (\sum_{\theta} r_1^o(\theta, t)) e^{jz_1^o(t)} \right], \quad (22)$$

651 where $z_1^c(t)$ and $z_1^o(t)$ are the positions of the population activities of the congruent and opposite
652 neurons in network module 1 respectively, which were decoded by using population vector (Eq. 21).
653 In real-time reconstruction, the sum of firing rate represents the concentration of the distribution.
654 This is supported by the finding that the reliability of the distribution is encoded by the summed
655 firing rate in probabilistic population code^{11,12}.

656 **Discriminating cue disparity on single neurons**

657 A discrimination task was designed on the responses of single neurons to demonstrate that opposite
658 neurons encode cue disparity information. The task is to discriminate whether the cue disparity,
659 $x_1 - x_2$, is either smaller or larger than 0° . In the discrimination task, the mean direction of two
660 cues, $x_1 + x_2 = 0$, is fixed at 0° , in order to rule out the influence of the change of integrated
661 direction to neuronal activity. Meanwhile, the disparity between two cues, $x_1 - x_2$, is changed from
662 -32° to 32° with a step of 4° . For each combination of cue direction, we applied three cueing
663 conditions (cue 1, cue 2, combined cues) to the network model for 30 trials and the firing rate
664 distributions of the single neurons were obtained (Fig. 8A and B).

665 We chose an example congruent neuron preferring 90° in network module 1, and also an example
666 opposite neuron in network module 1 preferring 90° with respect to cue 1. We used receiver
667 operating characteristic (ROC) analysis⁴⁶ to compute the discriminating ability of the example
668 neurons on cue disparity. The ROC value counts the proportion of instances where the direction of
669 cue 1, x_1 , is larger than the one of cue 2. Neurometric functions (Fig. 8B and E) were constructed
670 from those ROC values and were fitted with cumulative Gaussian functions by least square, and
671 then the standard deviation of the cumulative Gaussian function was interpreted as the neuronal
672 discrimination threshold (Fig. 8C)⁸. A smaller value of the discrimination threshold means that
673 the neuron is more sensitive in the discrimination task. Although we adopted the von Mises
674 distribution in the probabilistic model, the firing rate distribution of single neurons can be well
675 fitted by a Gaussian distribution, justifying the use of the cumulative Gaussian distribution to fit
676 the ROC values.

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681 Author Contributions

682 W.H.Z, K.Y.M.W and S.W. designed research; W.H.Z., H.W., K.Y.M.W. and S.W. performed
683 research; A.C., Y.G., and T.S.L. analyzed data and results; W.H.Z., H.W., K.Y.M.W. and S.W.
684 wrote the paper.

685 References

- 686 1. C. R. Fetsch, G. C. DeAngelis & D. E. Angelaki. Bridging the gap between theories of sensory
687 cue integration and the physiology of multisensory neurons. *Nature Reviews Neuroscience*
688 **14**(6), 429–442 (2013).
- 689 2. D. Alais & D. Burr. The ventriloquist effect results from near-optimal bimodal integration.
690 *Current Biology* **14**(3), 257–262 (2004).
- 691 3. R. A. Jacobs. Optimal integration of texture and motion cues to depth. *Vision Research*
692 **39**(21), 3621–3629 (1999).
- 693 4. R. J. van Beers, A. C. Sittig & J. J. D. van Der Gon. Integration of proprioceptive and visual
694 position-information: An experimentally supported model. *Journal of neurophysiology* **81**(3),
695 1355–1364 (1999).
- 696 5. M. O. Ernst & M. S. Banks. Humans integrate visual and haptic information in a statistically
697 optimal fashion. *Nature* **415**(6870), 429–433 (2002).
- 698 6. M. T. Wallace et al. Unifying multisensory signals across time and space. *Experimental Brain*
699 *Research* **158**(2), 252–258 (2004).
- 700 7. A. R. Girshick & M. S. Banks. Probabilistic combination of slant information: weighted
701 averaging and robustness as optimal percepts. *Journal of Vision* **9**(9), 8–8 (2009).

- 702 8. Y. Gu, D. E. Angelaki & G. C. DeAngelis. Neural correlates of multisensory cue integration in
703 macaque mstd. *Nature Neuroscience* **11**(10), 1201–1210 (2008).
- 704 9. A. Chen, G. C. DeAngelis & D. E. Angelaki. Functional specializations of the ventral intra-
705 parietal area for multisensory heading discrimination. *The Journal of Neuroscience* **33**(8),
706 3567–3581 (2013).
- 707 10. Y. Gu, G. C. DeAngelis & D. E. Angelaki. Causal links between dorsal medial superior temporal
708 area neurons and multisensory heading perception. *The Journal of Neuroscience* **32**(7), 2299–
709 2313 (2012).
- 710 11. W.-H. Zhang, A. Chen, M. J. Rasch & S. Wu. Decentralized multisensory information integra-
711 tion in neural systems. *The Journal of Neuroscience* **36**(2), 532–547 (2016).
- 712 12. W. J. Ma, J. M. Beck, P. E. Latham & A. Pouget. Bayesian inference with probabilistic
713 population codes. *Nature Neuroscience* **9**(11), 1432–1438 (2006).
- 714 13. M. L. Morgan, G. C. DeAngelis & D. E. Angelaki. Multisensory integration in macaque visual
715 cortex depends on cue reliability. *Neuron* **59**(4), 662–673 (2008).
- 716 14. Y. Gu, Z. Cheng, L. Yang, G. C. DeAngelis & D. E. Angelaki. Multisensory convergence of
717 visual and vestibular heading cues in the pursuit area of the frontal eye field. *Cerebral Cortex*
718 **26**(9), 3785–3801 (2016).
- 719 15. D. Boussaoud, L. G. Ungerleider & R. Desimone. Pathways for motion analysis: cortical
720 connections of the medial superior temporal and fundus of the superior temporal visual areas
721 in the macaque. *Journal of Comparative Neurology* **296**(3), 462–495 (1990).
- 722 16. J. S. Baizer, L. G. Ungerleider & R. Desimone. Organization of visual inputs to the inferior
723 temporal and posterior parietal cortex in macaques. *The Journal of Neuroscience* **11**(1), 168–
724 190 (1991).
- 725 17. B. E. Stein & T. R. Stanford. Multisensory integration: current issues from the perspective of
726 the single neuron. *Nature reviews. Neuroscience* **9**(4), 255 (2008).
- 727 18. A. Chen, G. C. DeAngelis & D. E. Angelaki. Representation of vestibular and visual cues to
728 self-motion in ventral intraparietal cortex. *The Journal of Neuroscience* **31**(33), 12036–12052
729 (2011).
- 730 19. Y. Gu, P. V. Watkins, D. E. Angelaki & G. C. DeAngelis. Visual and nonvisual contributions
731 to three-dimensional heading selectivity in the medial superior temporal area. *The Journal of*
732 *Neuroscience* **26**(1), 73–85 (2006).
- 733 20. J.-P. Bresciani, F. Dammeier & M. O. Ernst. Vision and touch are automatically integrated
734 for the perception of sequences of events. *Journal of Vision* **6**(5), 2 (2006).
- 735 21. N. W. Roach, J. Heron & P. V. McGraw. Resolving multisensory conflict: a strategy for
736 balancing the costs and benefits of audio-visual integration. *Proceedings of the Royal Society*
737 *of London B: Biological Sciences* **273**(1598), 2159–2168 (2006).
- 738 22. Y. Sato, T. Toyoizumi & K. Aihara. Bayesian inference explains perception of unity and ventril-
739 oquism aftereffect: identification of common sources of audiovisual stimuli. *Neural Computation*
740 **19**(12), 3335–3355 (2007).

- 741 23. K. V. Mardia & P. E. Jupp. *Directional Statistics*, volume 494 (John Wiley & Sons, 2009).
- 742 24. M. Jazayeri & J. A. Movshon. Optimal representation of sensory information by neural popu-
743 lations. *Nature Neuroscience* **9**(5), 690–696 (2006).
- 744 25. M. O. Ernst. A bayesian view on multimodal cue integration. *Human body perception from the*
745 *inside out* 105–131 (2006).
- 746 26. K. Zhang. Representation of spatial orientation by the intrinsic dynamics of the head-direction
747 cell ensemble: a theory. *The Journal of Neuroscience* **16**(6), 2112–2126 (1996).
- 748 27. S. Wu, K. Hamaguchi & S.-i. Amari. Dynamics and computation of continuous attractors.
749 *Neural Computation* **20**(4), 994–1025 (2008).
- 750 28. M. Carandini & D. J. Heeger. Normalization as a canonical neural computation. *Nature Reviews*
751 *Neuroscience* **13**(1), 51–62 (2012).
- 752 29. S. Deneve, P. E. Latham & A. Pouget. Reading population codes: a neural implementation of
753 ideal observers. *Nature Neuroscience* **2**(8), 740–745 (1999).
- 754 30. S. Wu, S.-i. Amari & H. Nakahara. Population coding and decoding in a neural field: a
755 computational study. *Neural Computation* **14**(5), 999–1026 (2002).
- 756 31. A. P. Georgopoulos, A. B. Schwartz & R. E. Kettner. Neuronal population coding of movement
757 direction. *Science* **233**(4771), 1416–1419 (1986).
- 758 32. H. R. Kim, X. Pitkow, D. E. Angelaki & G. C. DeAngelis. A simple approach to ignoring
759 irrelevant variables by population decoding based on multisensory neurons. *Journal of neuro-*
760 *physiology* **116**(3), 1449–1467 (2016).
- 761 33. R. Sasaki, D. E. Angelaki & G. C. DeAngelis. Dissociation of self-motion and object motion
762 by linear population decoding that approximates marginalization. *Journal of Neuroscience*
763 1177–17 (2017).
- 764 34. D. R. Wozny, U. R. Beierholm & L. Shams. Human trimodal perception follows optimal
765 statistical inference. *Journal of vision* **8**(3), 24–24 (2008).
- 766 35. D. A. Leopold, I. V. Bondar & M. A. Giese. Norm-based face encoding by single neurons in
767 the monkey inferotemporal cortex. *Nature* **442**(7102), 572–575 (2006).
- 768 36. X.-J. Wang. Decision making in recurrent neuronal circuits. *Neuron* **60**(2), 215–234 (2008).
- 769 37. T. A. Engel & X.-J. Wang. Same or different? a neural circuit mechanism of similarity-based
770 pattern match decision making. *The Journal of Neuroscience* **31**(19), 6982–6996 (2011).
- 771 38. R. T. Born. Center-surround interactions in the middle temporal visual area of the owl monkey.
772 *Journal of neurophysiology* **84**(5), 2658–2669 (2000).
- 773 39. J. W. Nadler et al. Joint representation of depth from motion parallax and binocular disparity
774 cues in macaque area mt. *Journal of Neuroscience* **33**(35), 14061–14074 (2013).
- 775 40. R. F. Murray & Y. Morgenstern. Cue combination on the circle and the sphere. *Journal of*
776 *vision* **10**(11), 15–15 (2010).

- 777 41. C. C. A. Fung, K. Y. M. Wong & S. Wu. A moving bump in a continuous manifold: A
778 comprehensive study of the tracking dynamics of continuous attractor neural networks. *Neural*
779 *Computation* **22**(3), 752–792 (2010).
- 780 42. W.-H. Zhang & S. Wu. Neural information processing with feedback modulations. *Neural*
781 *Computation* **24**(7), 1695–1721 (2012).
- 782 43. R. Ben-Yishai, R. L. Bar-Or & H. Sompolinsky. Theory of orientation tuning in visual cortex.
783 *Proceedings of the National Academy of Sciences* **92**(9), 3844–3848 (1995).
- 784 44. A. Samsonovich & B. L. McNaughton. Path integration and cognitive mapping in a continuous
785 attractor neural network model. *The Journal of Neuroscience* **17**(15), 5900–5920 (1997).
- 786 45. P. Dayan & L. F. Abbott. *Theoretical neuroscience*, volume 806 (Cambridge, MA: MIT Press,
787 2001).
- 788 46. K. H. Britten, M. N. Shadlen, W. T. Newsome & J. A. Movshon. The analysis of visual motion:
789 a comparison of neuronal and psychophysical performance. *Journal of Neuroscience* **12**(12),
790 4745–4765 (1992).