A Biologically Plausible Model for Continual Learning using Synaptic Weight Attractors

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Abstract

The human brain readily learns tasks in sequence without forgetting previous ones. Artificial neural networks (ANNs), on the other hand, need to be modified to achieve similar performance. While effective, many algorithms that accomplish this are based on weight importance methods that do not correspond to biological mechanisms. Here we introduce a simple, biologically plausible method for enabling effective continual learning in ANNs. We show that it is possible to learn a weight-dependent plasticity function that prevents catastrophic forgetting over multiple tasks. We highlight the effectiveness of our method by evaluating it on a set of MNIST classification tasks. We further find that the use of our method promotes synaptic multi-modality, similar to that seen in biology.

Keywords: Continual Learning, Machine Learning, Computational Neuroscience, Synaptic Plasticity, Multi-task Learning

1. Introduction

Deep learning models such as artificial neural networks have become a staple for researchers and engineers alike – facilitating advances in a variety of supervised learning tasks. The most impressive models in this area tend to follow the traditional supervised learning format. Specifically, given

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a dataset $D_{tr} = \{(x_i, y_i)|i \in N\}$ such that x_i denotes a feature vector sampled from the space X and y_i denotes a corresponding target vector from the space Y, learn a mapping $f_\theta : X \mapsto Y$ that minimizes a loss, e.g. $L(f_\theta, D_{tr}) = \frac{1}{N}\sum_{i=1}^{N}(f_\theta(x_i) - y_i)^2$ such that θ is the set of parameters for a neural network. Deep learning models parameterizing f have been incredibly successful in domains ranging from facial recognition to automatic handwriting identification. (Taigman et al., 2014; Graves and Schmidhuber, 2009)

An important question to ask is whether this problem formulation readily lends itself to learning multiple tasks in sequence. That is, given a set of tasks $\mathcal{T}$ and datasets $D_{\mathcal{T}}$, is minimizing the loss $L(f_\theta, D_{\mathcal{T}_i})$ for each task in sequence the same as minimizing the joint loss $L(f_\theta, D_{\mathcal{T}})$? Following the work of Goodfellow et al. (2015) it became clear this is not the case. Indeed, modern artificial neural networks struggle to learn multiple tasks in sequence without forgetting previous knowledge, a phenomenon termed *catastrophic forgetting* (Goodfellow et al., 2015).

Weight importance methods have been proven to be particularly adept at increasing retention across a large number of tasks (Kirkpatrick et al., 2017; Zenke et al., 2017). As the name suggests, weight importance methods aim to penalize changes to the weights that contribute most to task accuracy on previous tasks – thereby preventing weight changes that would result in the loss of previously learned information. To date, the most successful weight importance methods are Synaptic Intelligence(SI) and Elastic Weight Consolidation (EWC). EWC imposes a quadratic penalty on the distance from the previous weights proportional to the corresponding location on the diagonal of the Fisher information matrix near the previous minimum. In doing so, EWC attempts to estimate the importance of each parameter as inversely proportional to the Laplace approximation of its expected posterior variance. While effective, the method is expensive and requires frequent recomputation of this diagonal, a process whose cost is directly proportional to the number of outputs for a given task (Kirkpatrick et al., 2017). Synaptic Intelligence aims to compute a per-parameter regularization strength based on a discrete approximation of the parameter's previous contribution to decreases in loss (Zenke et al., 2017).

Despite their success, neither SI nor EWC represent compelling biological mechanisms for continual learning. Both the EWC and SI papers correctly make the argument that there is biological evidence for synaptic protection (Yang et al., 2009). However, their methods for computing weight impor-

tance either (1) require the computation of a complicated information matrix near a previous task minimum (as in EWC) or (2) require the addition of extra synaptic dimensions in which parameters importance is stored (as in SI)(Kirkpatrick et al., 2017; Zenke et al., 2017).

Unlike their artificial counterparts, humans are excellent at learning tasks in sequence. We routinely learn multiple languages and our proficiency in one does not imply that we have forgotten the others. It is well known that humans have a mass of genomic and eventually connectomic priors that can intelligently guide learning dynamics (Zador, 2019). We propose a simple prior – a fixed function that maps synapse strength to learning rate. In contrast to SI and EWC, we show that it is not necessary to compute weight importance explicitly to alleviate catastrophic forgetting. Instead, we find that it is possible to meta-learn parameters for this weight-dependent learning rate function. We further show that this prior mapping can effectively prevent catastrophic forgetting in ANNs on sequences of supervised learning tasks. Finally, we find that this method allows the network to naturally induce a form of the synaptic consolidation approximated by SI and EWC (Kirkpatrick et al., 2017; Zenke et al., 2017).

2. Methods

We wanted to develop a continual learning method that is both simple and reasonably effective. Moreover, we hoped to develop an algorithm for which there are already potential biological mechanisms. A weight-dependent plasticity function fit all of these requirements. Neuronal plasticity has already been shown to be activity dependent (Cingolani et al., 2008). As such, a weight-dependent plasticity function would be a simple and biologically plausible analog. Specifically, we propose a set of algorithms in which we optimize hyperparameters $\phi = \{a_1, \mu, \sigma, \gamma, \zeta, \lambda\}$ over an arbitrary number of tasks for a learning rate function $f(W)$ which takes weights as input and outputs a learning rate multiplier.

To design the function f, we draw inspiration from biology. Past work suggests that long-term memories are stored in stable synaptic networks (Yang et al., 2009). To induce this stability, the learning rate function needs to approach zero or dip for some values of the network parameters θ. In this way, the function can trap weights in an important region; this is illustrated in *Figure 1b*. It is important that base learning rate be parameterized as well, so the network is allowed to learn more quickly in certain weight ranges. To

Algorithm 1 Sticky Gradient

Requires: Set of tasks $\mathcal{T}$
Requires: Hyperparameters: $a_1, \mu, \sigma, \gamma, \zeta, \alpha, \lambda, E$
$f(x; a_1, \lambda, \gamma) = -\lambda e^{-\gamma(x-a_1)^2} + \lambda$
Sample θ from $N(\mu, \sigma^2)$
for $D_{\mathcal{T}_i} \in D_{\mathcal{T}}$ **do**
    for epochs $\in$ E **do**
        for $\mathcal{B}_j \in D_{\mathcal{T}_i}$ **do**
            $g = \nabla_\theta(L_\theta(\mathcal{B}_j) + \zeta|\theta|)$
            $g' = g \odot (1[sgn(\theta - a_1) \neq sgn(g)] \odot f(\theta; a_1, \lambda, \gamma) + 1[sgn(\theta - a_1) = sgn(g)])$
            $\theta' = \theta - \alpha g'$

these ends, we chose f to be a negative Gaussian function with an added multiplier/offset parameter. Specifically,

$$f(x; a_1, \lambda, \gamma) = -\lambda e^{-\gamma(x-a_1)^2} + \lambda \tag{1}$$

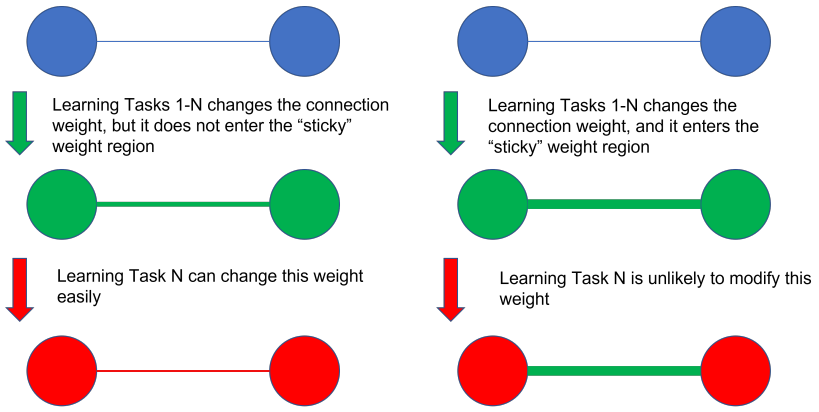
such that $a_1, \lambda,$ and γ are hyperparameters describing the location, depth, and width of the dip – respectively. To encourage weights to fall into these dips, we only apply f to parameters with gradients going away from a_1. Crucially, if $\lambda > 1$, weights with gradients leading away from the dip may be pushed away more quickly than they otherwise would have been. This allows for the creation of a second group of strongly inhibitory weights that can help sparsen the heavy activity induced by the pooling of positive weights at a_1.

It is important that this type of regularization not override the ability of the network to learn. In essence, a good learning rate function f traps as few parameters as is necessary to remember the previous task but leaves the rest free to learn representations for new tasks. To affect this dynamic, we add L1 regularization to the system – decreasing gradients towards the well. We provide a visualization of the function in *Figure 1b*.

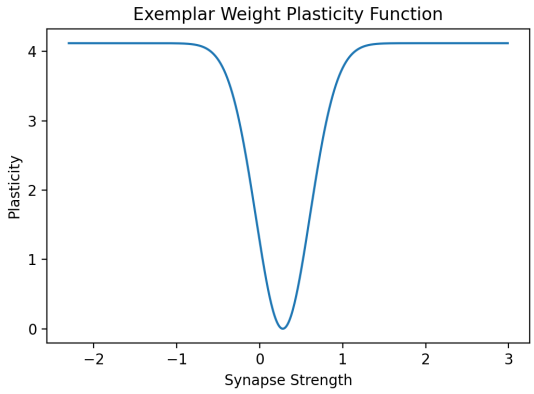
In short, we propose a modified update rule. We compute the learning rate multiplier dictated by the function f and parameterized by ϕ and apply it to the classification gradients going away from a_1.(*Algorithm 1*)

2.1. Meta-learning

To learn the parameters ϕ, we employ a simple Tree-Structured Parzen Estimator (TPE) to intelligently search over parameter space (Bergstra et al.,



(a) Illustration of Method



(b) Example Weight Plasticity Function

Figure 1: (a) Graphical illustration of the effect of our algorithm; our method traps weights close to attractor values and allows movement of weights that are far away (b) Example plasticity function – remains constant at around 4 and dips to zero near 0.279

2011). To prioritize many-task retention, we use a modified meta-objective function in which averages from later tasks are weighted more than those of earlier tasks. Suppose that θ_j are the parameters of a neural network after being trained on task j. Suppose further that $A_{\phi,\theta_j}(i,j)$ denotes validation accuracy on the i^{th} task after being trained on the j^{th} task with learnable hyperparameters ϕ and network state θ_j. Let N be the total number of tasks.

a_1	μ	σ	γ	ζ	λ
0.279	0.00363	0.0204	4.70	0.000574	4.12

Table 1: Learned hyperparameters from 30 trial meta-search. Surprisingly, the meta-optimization yields a large value of λ, implying different learning rate for gradients going away from the attractor is important. The function f with these parameters is visualized in *Figure 1b*.

Then the optimization problem can be formulated as

$$\arg\max_{\phi} \frac{\sum_{j=1}^{N} \frac{log(j)}{j} \sum_{i=1}^{j} A_{\phi,\theta_j}(i,j)}{N \sum_{j=1}^{N} log(j)} \tag{2}$$

We optimize the model on an arbitrary set of ten PMNIST tasks. Remarkably, we find that it takes as few as 30 trials to find an effective set of parameters, given by *Table 1*.

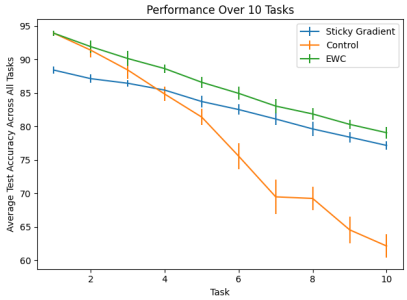
3. Experiments

3.1. Permuted MNIST

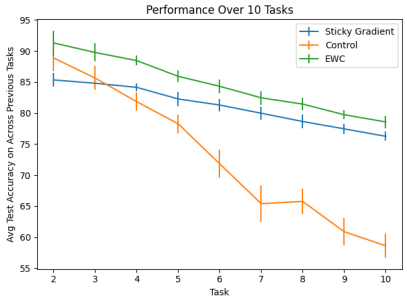
We use permuted MNIST (PMNIST), a simple variant of the MNIST dataset to evaluate our method in a multi-task setting. PMNIST is simply a derivative of the MNIST dataset in which the handwritten images are normalized, flattened, and randomly permuted n-times such that n is the desired number of tasks (Goodfellow et al., 2015).

3.2. EWC Comparison

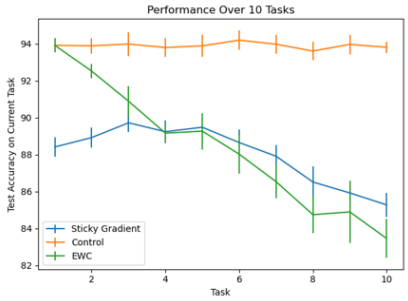
To establish the value of this method, we compare it to EWC on the permuted MNIST benchmark (Kirkpatrick et al., 2017). We use a simple feed-forward architecture with a single ReLU hidden layer consisting of 2000 units and instantiate three networks to be trained using EWC with a penalty multiplier of 1000, the *sticky gradient* method, or a vanilla learning algorithm (control) (Kirkpatrick et al., 2017). All networks are trained for a single epoch on ten tasks, different than those used in the meta-optimization process, and optimized using Adam with a base learning rate of 1e-4. It is important to note that we sample a new set of ten tasks for each random seed



(a) Average test accuracy across all tasks



(b) Average test accuracy on past tasks



(c) Test accuracy on current task

Figure 2: (a) Baseline Comparison Study detailing the test accuracy of trials of trials under 10 different random seeds. Accuracy on all tasks is recomputed after every task on a held-out test set and averaged; error bars denote 95% confidence intervals. EWC and the *sticky gradient* method outperform the control by an average margin of around 19%, but EWC outperforms our method by an average margin of 2.5 % at ten tasks. (b) Average test accuracy on all tasks, not including the current one. Accuracy is computed on all previous tasks after every task except the first and averaged; error bars are 95% confidence intervals. Our method lags further behind EWC in this metric – at around a 2.7% performance penalty. (c) Test accuracy on current task; error bars denote a 95% confidence interval. Unsurprisingly, neither our method nor EWC retain a strong ability to learn new tasks, both lag the control's current task performance by around 9% at ten tasks. Interestingly, our network seems to be significantly better at learning new tasks at 10 tasks than EWC by a margin of 2.2%.

utilized but do not alter ϕ. That is, the meta-parameters remain constant across different samples of 10 tasks from PMNIST.

Both the *sticky gradient* method and EWC enjoy a hefty benefit over the control at ten tasks. While EWC outperforms our method by an average accuracy of 2.5% at ten tasks, we observe that this difference seems to decrease across tasks (*Fig. 2a*). However, our method does seem to better retain the

ability to learn new tasks, enjoying a 3% benefit on the most recent task at 10 tasks (*Fig. 2c*). It is unsurprising, then, that our method underperforms EWC on a test set containing all previous tasks by about 2.7%. *Sticky gradient* method seems to penalize movement more heavily than EWC, resulting better adaptability but inferior retention (*Fig. 2b*).

3.3. Ablation Studies

Given that this method is an amalgamation of other methods – we find it necessary to disentangle the effects of each of these sub-procedures from the whole. Controlling for ϕ, we test every combination of L1 regularization, the *sticky gradient* method, and initialization distribution. We denote network initialization from a learned normal distribution parameterized by μ and σ as *smart initialization*. All networks are simple feed-forward MLPs with a single hidden layer of 2000 units coupled with ReLU activations, trained for a single epoch, and optimized by Adam with a base learning rate of 1e-4.

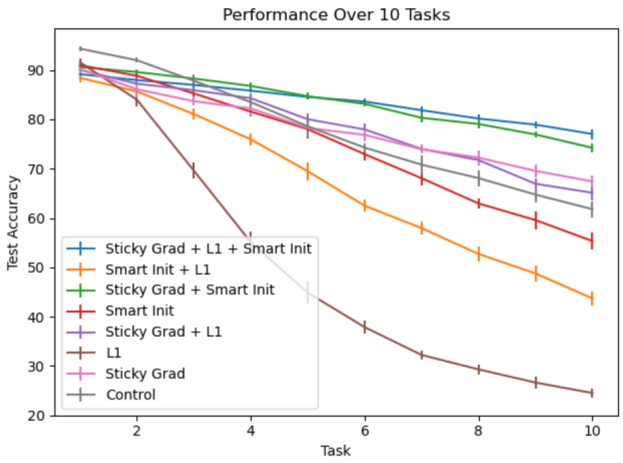


Figure 3: Ablation Study detailing the test accuracy of trials of trials under 10 different random seeds. Accuracy on all previous tasks is recomputed after every task on a held-out test set and averaged; error bars denote 95% confidence intervals. The sticky gradient method is present in both of the two most accurate graphs. The control simply refers to a network initialized with a standard Kaiming initialization and no L1 regularization (He et al., 2015).

We find that the *sticky gradient* method, coupled with both the L1 regularization and *smart initialization* has the best average accuracy at 10 tasks – around 3% better than its closest competitor, a combination of the *sticky*

gradient and *smart initialization*, and about 6% better than its second closest competitor – simple *smart initialization*. Methods with the *sticky gradient* and *smart initialization* retain a heavy benefit over the control throughout the training process, starting at around four tasks (*Fig. 3*). For brevity, we now refer to the combination of the *sticky gradient*, L1 regularization, and smart initialization as the *sticky gradient* method.

3.4. Shared Information

Sets of real world image-processing tasks almost never have uncorrelated inputs. As such, we evaluate the method on variations of the PMNIST dataset for which a proportion of pixels are not permuted. We compare a baseline 2000 ReLU ANN with Kaiming initialization against a network of the same architecture equipped with the *sticky gradient* method for different levels of shared information (He et al., 2015). Batch size was set to 50 and each network was trained for a single epoch on every task and optimized via Adam.

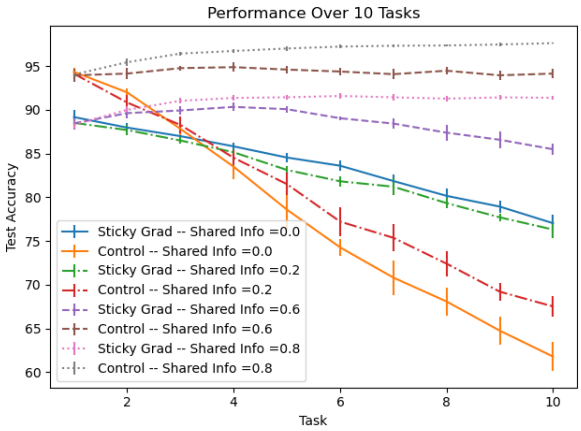


Figure 4: Accuracy profiles from runs for 10 random seeds for different levels of shared information; error bars denote 95% confidence intervals. Our method retains a benefit over the control until the amount of shared information reaches 60%. After this point, the *sticky gradient's* inability to tweak frozen weights for similar tasks outweighs its ability to retain previous knowledge.

We find that the *sticky gradient* method does not lose a benefit until the proportion of shared information reaches 0.6 (Figure. 4). This illustrates the effectiveness of our method on correlated tasks akin to real-world scenarios

in which agents may be asked to complete tasks which leverage knowledge gained from previous tasks. However, it also points out a trade-off that our method makes between expressivity and retention. As the *sticky gradient* network trains, more weights get caught in the dip of the weight-plasticity function and are unable to shift in response to new examples that may improve performance.

3.5. Weight Dynamics

It is also instructive to look at the dynamics of the weights during training. To visualize only important connections, we remove weights which are within a small tolerance of zero (1e-5). We expect the *sticky gradient* weights to behave quite differently than those of the control over the course of training. We discover that this is indeed the case – the eventual weights for the *sticky gradient* method are essentially trimodal. There is a group of weights near zero, another near the dip of the weight-plasticity function, and yet another contingent of strongly inhibitory weights. In contrast, both the control and EWC exhibit clear unimodality around 0 (*Figure 5*) .

In essence, the *sticky gradient* method seems to encourage the freezing of information dense synapses near the attractor, and the masked gradient multiplier induces a sort of symmetry by increasing the rate at which far-away weights move away from the attractor. This interplay allows for a sort of balancing in which a contingent of inhibitory neurons arises to combat the excitatory effect of their positive counterparts near the attractor. We hypothesize that it is this balanced concentration of information in these parameters that enables superior retention of knowledge from previous tasks.

4. Discussion

We show that groups of synapses endowed with even the simplest prior measures of weight-dependent plasticity can be used to enable continual learning in ANNs. Specifically, we show that it is possible to meta-learn parameters for a flipped gaussian function mapping weights to plasticity that aids in continual learning.

Our method bears similarity to Zenke et al.'s Synaptic Intelligence and Kirkpatrick et al.'s Elastic Weight Consolidation in that we aim to regulate plasticity in response to some measure of weight importance. However, both of these methods do this explicitly by keeping track of this measure of weight importance and penalizing it in the loss (Kirkpatrick et al., 2017; Zenke et al.,

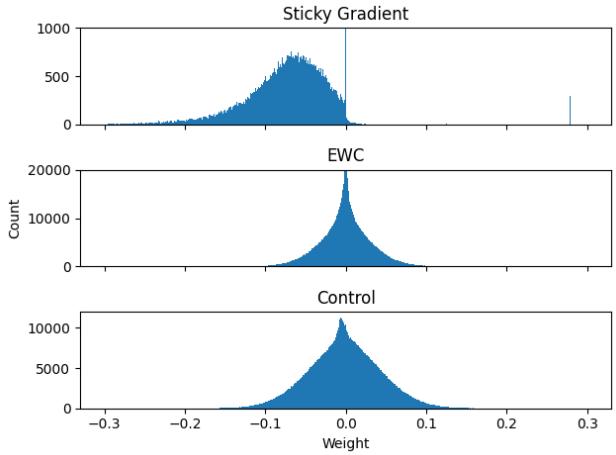


Figure 5: Weight distributions from Kaiming initialized control, EWC, and *sticky gradient* network. We truncate the y-axis for the sticky gradient histogram to ensure that the non-zero peaks remain visible. The EWC and control networks seem to exhibit similar patterns of unimodality around 0, differing only in width. The *sticky gradient* network produces an interesting distribution of weights with local modes near -0.06, 0, and 0.279 (the attractor value).

2017). In contrast, we allow the meta-learner to implicitly come up with a region of a high weight importance based on training dynamics by meta-learning parameters for the weight-plasticity function.

Of all the plasticity-related methods for continual learning, ours is potentially the easiest to justify biologically. While SI requires complicated handwaving about the plausibility of a separate store of importance in neurons, we do not make use of extra neuronal dimensions (Montgomery and Madison, 2002). Indeed, we show that it is possible to effectively alleviate catastrophic forgetting simply by creating a global mapping of synapse strength to importance. This mapping is eminently biologically plausible – there is already evidence that plasticity is activity dependent (Cingolani et al., 2008). This almost certainly implies that plasticity is also weight dependent and therefore that our method is biologically grounded.

The weight distributions created by the *sticky gradient* method are actually startlingly reminiscent of recent work concerning distributions of synaptic weights in mouse connectomes. In this study, the authors find that, controlling for cell type, synapse strength can be described as a bimodal mixture of log-normal distributions (Dorkenwald et al., 2019). Indeed, the

weight distributions produced by the *sticky gradient* method show marked multi-modality due to pooling near the attractor. While this bimodality does not exactly match that described in the study, the similarity that exists even under these radically different conditions undeniably adds to the biological plausibility of our method (Dorkenwald et al., 2019).

There are a number of straightforward extensions to this work that we plan to pursue. First, we would like to add an extra parameter to *Eq. 1* such that the minimum learning rate is not zero. In doing so, the meta-optimization procedure would shed some light on how much freedom frozen synapses should be given to allow for movement towards joint optima. We also plan to test wider and deeper networks with the *sticky gradient* method and see if the expressivity constraints imposed by the method can be overcome simply with bigger networks.

Our work points to the effectiveness of this particular brand of biological prior on enabling continual learning in artificial neural networks. More broadly, our research highlights the ability of artificial neural networks to benefit from even incredibly simple plasticity-related priors. This opens a number of avenues for further work in which a broader class of system-wide plasticity-related priors can be leveraged to effectively manipulate training dynamics for a wide variety of learning environments.

5. Acknowledgments

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Base Learning Rate	$10^{-}4$
Epochs per Task	1
Hidden Layer Width	2000
Depth	1
N. Hidden Layers	1
Optimizer	Adam

Table 2: Hyperparameters for PMNIST Experiment

6. Appendix

6.1. Permuted MNIST Experiments

We use fully-connected feedforward 2000 ReLU multi-layer perceptrons for classification on every task. To enable the control networks to better retain knowledge from previous tasks, we train each model for a single epoch on each dataset. A full list of fixed hyperparameters is given by *Table 2*. All networks trained with the *sticky gradient* method meta-optimize parameters on a validation set, disparate from the training set. All networks are tested on held-out test sets over 10 random seeds. Additionally, we use an Adam optimizer every epoch but reset its state after every task to prevent differing learning rates per task.

6.2. Meta-Learning

We use a simple TPE-mediated search to find ϕ under the following conditions over 30 trials using the search ranges in *Table 3* (Bergstra et al., 2011). Parameters were assumed to have a uniform prior over the log domain of the specified ranges. All networks are trained using hyperparameters ϕ for a single epoch per task for 10 tasks using the Adam optimizer with learning rate $1e-4$.

a_1	μ	σ	γ	ζ	λ
0.01 - 10.0	1e-3 - 8.0	0.01 - 7.0	0.01 - 7.0	1e-7 - 1e-3	4.0 - 10.0

Table 3: Ranges for meta-search

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