

1 **Radial-axial transport coordination enhances sugar**
2 **translocation in the phloem vasculature of plants**

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8 **Summary: The overall speed of sap increased by including a concentration-**
9 **dependent viscosity in axial and radial directions.**

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10 **Abstract**

11 Understanding the controls of mass transport of photosynthates in the phloem of plants
12 is necessary for describing plant carbon allocation, productivity, and responses to wa-
13 ter and thermal stress. While several hypotheses about optimization of phloem struc-
14 ture and function, and limitations of phloem transport under drought have been tested
15 both with models and anatomical data, the true impact of radial water exchange of phloem
16 conduits with their surroundings on mass transport of photosynthates has not been ad-
17 dressed. Here the physics of the Münch mechanism of sugar transport is re-evaluated to
18 include local variations in viscosity resulting from the radial water exchange in two di-
19 mensions (axial and radial). Model results show that radial water exchange pushes su-
20 crose away from conduit walls thereby reducing wall frictional stress due to a decrease
21 in sap viscosity and increasing sugar concentration in the central region of the conduit.
22 This leads to increased sugar front speed and axial mass transport for a wide range of
23 phloem conduit lengths and allows sugar transport to operate more efficiently than pre-
24 dicted by previous models. A faster front speed leads to higher phloem resiliency under
25 drought because more sugar can be transported with a smaller pressure gradient.

26 **1 Introduction**

27 The efficiency of photosynthate transport from the production sites (sources; usu-
28 ally leaves) to areas of consumption or storage (sinks) within the vascular tissue known
29 as the phloem is drawing significant attention in plant physiology. The implications of
30 efficient photosynthate transport range from local impacts on tissue or plant health and
31 growth to ecosystem-scale effects on carbon and water cycling because of the potential
32 link between phloem transport and stomatal control of photosynthesis (Nikinmaa et al.
33 2013), and a possible link between phloem transport failure and plant mortality under
34 drought (Sevanto et al. 2014). Consequently, several models for phloem transport and
35 the connection between phloem structure and function, as well as for the potential weak
36 points in the transport system have been formulated (Münch 1930, Phillips & Dungan
37 1993, Thompson & Holbrook 2003, Jensen et al. 2009, 2012, Sevanto 2014). The most
38 commonly accepted concept under which all these models operate is that phloem vas-
39 culature is optimized for efficient transport of soluble organic compounds (mostly sug-
40 ars) produced during photosynthesis approximately as described by the pressure-flow hy-
41 pothesis or Münch mechanism (Münch 1930). In the pressure-flow hypothesis, transport

42 is initiated in leaves when sugars and other metabolic products are loaded into the phloem.
43 Once in the phloem, sugars and water molecules are driven to move through the phloem's
44 complex network of narrow but elongated, interconnected, cylindrical living cells (sieve
45 tubes) spanning the length of the plant following pressure gradients. High sugar concen-
46 tration at the loading site (leaves) draws water from the xylem, the tissue supplying wa-
47 ter to the leaves or other surrounding tissues by osmosis towards the phloem. At the sinks,
48 sugars are unloaded from the phloem to growing or storage cells, and water is released
49 back to the xylem or other surrounding tissues. The loading and unloading at sources
50 and sinks build a pressure gradient in the phloem creating a system where plant water
51 and photosynthate transport over long distances occurs without active pumping. This
52 cycle of pressure buildup and transport without active pumping endowed the phloem sys-
53 tem with the label *miracle of ingenuity* (van Bel 2003).

54 The simplicity, plausibility, and intuitive appeal of the Münch mechanism lead to
55 its proliferation in mathematical models (Nikinmaa et al. 2013, Jensen et al. 2016). It
56 is routinely used to connect plant carbon sources and sinks, and their concomitant con-
57 trols in a future CO₂ enriched climate (Mencuccini & Hölttä 2010, Fatichi et al. 2019),
58 and it has been used to explain aspects of plant hydraulic failure during drought (Sav-
59 age et al. 2017, Konrad et al. 2018, Huang et al. 2018, Sevanto 2018, Salmon et al. 2019)
60 and extreme cold temperatures (Swanson & Geiger 1967, Wardlaw 1968). The direct con-
61 sequence of those two abiotic stresses should be a decrease in the overall phloem flow rate
62 because the viscosity of a sucrose solution increases significantly with the drought-induced
63 increase in sugar concentration required for osmoregulation (Hölttä et al. 2009) and with
64 decreasing temperature.

65 However, the Münch mechanism is not free from controversy. The main critique
66 stems from the fact that the sieve tubes seem to have too low of a hydraulic conductance
67 along the phloem to allow sugars to be transported from leaves to roots in the largest
68 and longest of plants (Curtis & Scofield 1933, Lang 1979, Fensom 1981, Knoblauch et al.
69 2016, Liesche & Schulz 2018). These studies also report lower leaf sucrose concentration
70 in tall trees compared to shorter vegetation. When interpreted using simplified trans-
71 port models for hydraulic conductance, this suggests that the Münch mechanism can-
72 not produce effective transport in tall plants because the driving force for sucrose trans-
73 port is lower for a longer path length. To resolve the issues of inadequate conductance
74 and too low-pressure gradient to efficiently drive flow in tall plants, it has been suggested

75 that rather than exchanging water and sugars only at the extreme source and sink ends
76 of the phloem pathway, as suggested by the original Münch mechanism, sugars and wa-
77 ter could be exchanged at different locations along the pathway essentially forming a "re-
78 lay" system to facilitate transport (Lang 1979). While plausible, and based on model-
79 ing studies, also effective in increasing transport capacity (Hölttä et al. 2009), there cur-
80 rently is no clear evidence for unloading and reloading of sugars from and to phloem con-
81 duits along the transport pathway. There is, however, an increasing amount of evidence
82 that water might readily be exchanged between the conduits and their surroundings along
83 the entire length of the pathway (Knoblauch & Oparka 2012, Knoblauch & Peters 2017,
84 Stanfield et al. 2017).

85 Answering the question of whether and how easily phloem conduits exchange wa-
86 ter with their surroundings outside the primary loading and unloading zones at sources
87 and sinks is becoming necessary for evaluating the validity of the Münch mechanism, and
88 because it determines how phloem transport is affected under stress (Sevanto 2014, 2018).
89 Theoretically, if no water exchange occurs, plants run a risk of blocking phloem flow by
90 viscosity increase and reduced pressure gradient under drought conditions because higher
91 amounts of sugar are needed in the phloem conduits at the loading and unloading zones
92 for osmoregulation against the declining water potential of the xylem and the surround-
93 ing tissues. If water exchange occurs readily along the entire transport pathway, the flow
94 may not be restricted by the same constraints that stem from the original interpreta-
95 tion of the Münch mechanism (Phillips & Dungan 1993, Sevanto 2014, Sevanto et al. 2014,
96 Sevanto 2018). In particular, the effects of viscosity buildup can be ameliorated because
97 of the diluting effects of radial water flow velocity, the focus of the work here.

98 Experimental challenges in measuring water and solute fluxes within the phloem
99 (Curtis & Scofield 1933, Housley & Fisher 1977) has led to reliance on mathematical mod-
100 els of simplified phloem transport to understand transport mechanisms in the phloem.
101 As expected when employing such models, values of one or more variables may not be
102 well constrained or are uncertain, possibly by several orders of magnitude. This fact is
103 often used to justify (overly) simplified description of transport physics in models. These
104 simplifications might lead to biased mass fluxes and estimates for total transport. An
105 alternative to the simplification approach is to assess the effects of the simplification on
106 the results, and in the case of phloem transport, test whether the pressure-flow hypoth-
107 esis predicts increases or decreases in sugar mass flux when these simplifications are re-

108 laxed or re-addressed. If increases in mass flux can be demonstrated upon re-addressing
109 key simplifications used with the Münch mechanism, the contradictions with some ob-
110 servations might be explained by the effects of these model simplifications lending fur-
111 ther support to the Münch mechanism.

112 Irrespective of the physics of phloem transport, phloem anatomical structure is as-
113 sumed to have evolved to optimize sugar transport. Within the confines of the Münch
114 mechanism, this optimization arises because of trade-offs between benefits of increasing
115 sugar concentration (c) and its impact on the mass flux J . Increasing c increases flux
116 (J) because the osmotic pressure driving water movement increases approximately lin-
117 early with c (Van't Hopf equation); however, increasing c is accompanied by a nonlin-
118 ear increase in dynamic (and kinematic) viscosity thereby enhancing the viscous forces
119 that oppose movement (drag) thereby reducing J . In most phloem transport models to-
120 day, viscosity is treated either as a constant or it is allowed to vary with loaded sugar
121 concentration assuming that radial water flow does not significantly affect sugar concen-
122 tration or viscosity. Theoretical representation of J along with a number of scaling ar-
123 guments results in a maximal sugar flux J_{max} at around $c = 20\%$ (Jensen et al. 2013)
124 independent of the sieve tube geometric properties. Interestingly, upon averaging across
125 species and experiments, the operating $c = 20\%$ was reasonably confirmed and appears
126 independent of properties of the sieve tube geometry or the loading mechanism (passive
127 versus active) in plants. The scatter in reported values of c around $c = 20\%$, however,
128 was substantial (Jensen et al. 2013) with many species operating at $c < 20\%$. This low
129 loaded sugar concentration value has also been used to argue against the validity of the
130 Münch mechanism, especially in tall trees (Knoblauch & Oparka 2012), since it leads to
131 a decrease in mass flux as predicted by the simplified physics. Therefore, it remains open
132 whether plant sugar transport actually operates sub-optimally, or whether alternatives
133 or modifications to the Münch mechanism are necessary to explain long-distance yet sub-
134 optimal sugar transport.

135 Motivated by these issues, we ask to what degree refinements and addressing the
136 model simplifications in the description of the transport physics within the Münch mech-
137 anism enhance J above and beyond expectations from earlier theories. To address this
138 question generically, an idealized, unsteady, two-dimensional, osmotically driven pipe flow
139 governed by the physics of the Münch hypothesis is considered. No attempt is made to
140 represent all the complexities of the geometry of the phloem tissue or in the loading and

141 unloading mechanisms of sugars. Instead, the main novelty here stems from the inclu-
142 sion of the simultaneous effects of concentration-dependent viscosity where local changes
143 in viscosity with c (axially and radially) are allowed. It is shown that including such ad-
144 justments to viscous stresses lead to significant enhancement in the magnitude of the mass
145 flux, especially in long tubes, when compared to prior one-dimensional (axial) models
146 (Thompson & Holbrook 2003, Jensen et al. 2016) and globally averaged Poiseuille mod-
147 els (Jensen et al. 2013). Moreover, this enhancement in J is shown to be accompanied
148 by a reduced pressure gradient driving the flow. Thus, the work here adds support to
149 the Münch hypothesis by offering a new perspective regarding the contribution of co-
150 ordination between axial and radial flow to J .

151 Before describing the new representation of viscous stresses within the context of
152 the Münch hypothesis, some comments and clarifications about efficient sugar transport
153 and its relation to c are illustrated through the occurrence of a maximum sugar flux $J =$
154 J_{max} at a well-defined c in globally averaged Poiseuille models. That measured sugar
155 concentration in the leaves of tall trees is lower than this c corresponding to maximum
156 sugar flux was the basis for some critique of the Münch hypothesis (Knoblauch et al. 2016).
157 It is to be noted that the c corresponding to J_{max} in globally averaged Poiseuille mod-
158 els is shown not to be sensitive to the phloem hydraulic properties or even tube geom-
159 etry. Hence, the occurrence of such a c is weakly connected to phloem hydraulics as later
160 discussed.

161 In prior work (Jensen et al. 2013), the sugar mass flux J (kg s^{-1}) was assumed to
162 be only advective and given by

$$163 \quad J(c) = Q(c) \rho(c) c, \quad (1)$$

164 where $Q(c)$ is the volumetric flow rate ($\text{m}^3 \text{s}^{-1}$), $\rho(c)$ is the density of the phloem sap
165 that varies with c , and c is the sucrose concentration inside the sieve tube as before. In
166 this approach, the driving force for Q and the constraints on it are now formulated to
167 be c dependent. The maximal flux J_{max} emerges when solving for c at the critical point
168 $\partial J / \partial c = 0$. For $c > 0$, the existence of this single critical point is virtually guaran-
169 teed provided the water flux $\rho(c)Q(c)$ non-linearly decreases with increasing c . For lam-
170 inar flows in tubes, the Hagen-Poiseuille (HP) equation for Q and the resulting J can
171 be expressed as (Jensen et al. 2013)

$$172 \quad Q = X_f \frac{\Delta P}{\mu(c)} ; J = X_f \frac{\Delta P}{v(c)} c ; v(c) = \frac{\mu(c)}{\rho(c)}, \quad (2)$$

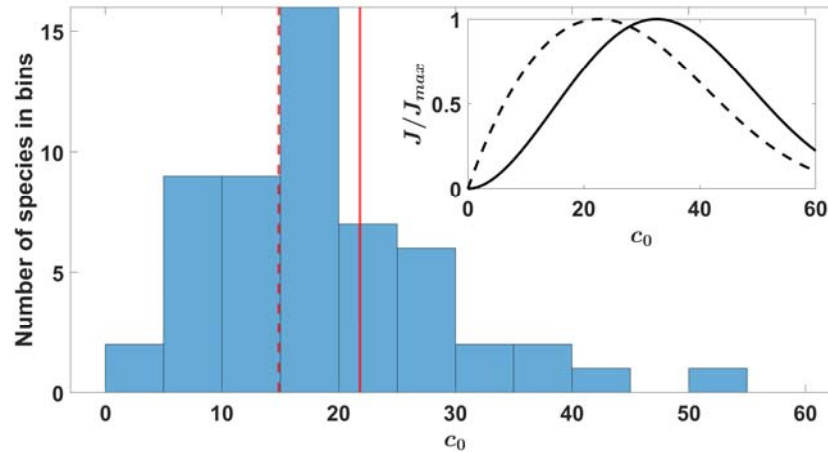


Figure 1: Histogram showing the reported number of species (ordinate) operating at the measured phloem sugar concentration (c_0) values (abscissa) taken from (Jensen et al. 2013). Solid red line ($c_0 \sim 21.8\%$) denotes the average concentration for active loading species and dashed red line ($c_0 \sim 14.8\%$) denotes the average concentration for passive loading species. Inset figure shows the computed normalized flux J/J_{max} for the globally averaged Poiseuille models where the solid black line denotes a concentration dependant pressure gradient (through the osmotic relation) and black dashed line denotes an externally imposed constant pressure gradient.

173 where X_f is a geometric factor that varies with L and a , ΔP is the pressure difference
 174 inside the tube that drives the flow (and need not be only osmotic), $\mu(c)$ is the concentration-
 175 dependent dynamic fluid viscosity that increases exponentially with c (Bouchard & Granjean
 176 1995), and $\nu(c)$ is the kinematic viscosity that also increases with c . With increasing c ,
 177 the rise in $\mu(c)$ far exceeds any increase in $\rho(c)$ so that the functional relations of $\mu(c)$
 178 and $\nu(c)$ with c are assumed to be the same (within a constant ρ). For an order of mag-
 179 nitude illustration, increasing c from 10% to 50% increases ρ by a factor of 1.2 whereas
 180 $\mu(c)$ increases by a factor of 4. Because $\nu(c)$ increases non-linearly with c as discussed
 181 before, a maximum $J = J_{max}$ must exist at a corresponding optimal c value that is in-
 182 dependent of X_f . Moreover, the existence of this maximum is not predicated based on
 183 the precise details of the osmotic controls on ΔP . Returning to J_{max} , for a preset X_f ,
 184 the hydraulic conductance of the tube K_t can be related to the inverse of viscosity us-
 185 ing $K_t = X_f/\mu(c)$. Independent of whether osmotic effects on ΔP are fully represented,

186 a J_{max} associated with an optimal c can be derived (numerically here) and graphically
187 shown in figure 1, and it can vary significantly depending on the model presentation within
188 the range of observed values (Jensen et al. 2013).

189 The Van't Hopf relation approximating the ΔP solely from osmotic potential (solid
190 line in figure 1 inset) is given by

$$191 \quad \Delta P \propto \frac{R_g T \rho(c)}{M_s} c, \quad (3)$$

192 where R_g is the gas constant, T is the absolute temperature and M_s is the molar mass
193 of sugar. Similar results with a single J_{max} but at lower optimum loading sugar concen-
194 tration (dashed line in figure 1 inset) are produced using an externally supplied constant
195 pressure difference that varies from 1 to 2 MPa with no concentration dependency. This
196 analysis demonstrates that the existence of a J_{max} is not tightly connected with the Münch
197 hypothesis in the following sense: the precise functional dependence of ΔP on c is not
198 necessary for the existence of a J_{max} . Hence, the fact that a J_{max} exists for a certain
199 c is not particularly informative about how axial-radial coordination in the phloem op-
200 erates and what the role of radial viscosity in this coordination is.

201 **2 Results**

202 This section discusses the effect of including viscosity variations in the flow equa-
203 tions on mass flux J . First, a comparison between the constant viscosity model in 2-D
204 (called the Poiseuille model hereafter) and the globally averaged Poiseuille model used
205 to generate the results in figure 1 is presented. This comparison shows the effect of prob-
206 lem set-up and dispersion effects (molecular and Taylor dispersion), which are included
207 in the 2-D Poiseuille (and the 2-D model with variable viscosity) model. Second, the ef-
208 fect of viscosity is discussed by comparing the 2-D model with variable viscosity (called
209 generalized model hereafter) and the Poiseuille model in 2-D (i.e. the two end-member
210 cases discussed in section 4.1). Finally, the enhancement of mass transport due to local
211 coordination between axial and radial movement is presented using the generalized model
212 simulations.

213 **2.1 Effect of dispersion and problem set-up on J variations with c_0**

214 A comparison between the globally averaged Poiseuille model (Jensen et al. 2013)
215 and the Poiseuille model in 2-D that excludes local viscosity effects by using a constant

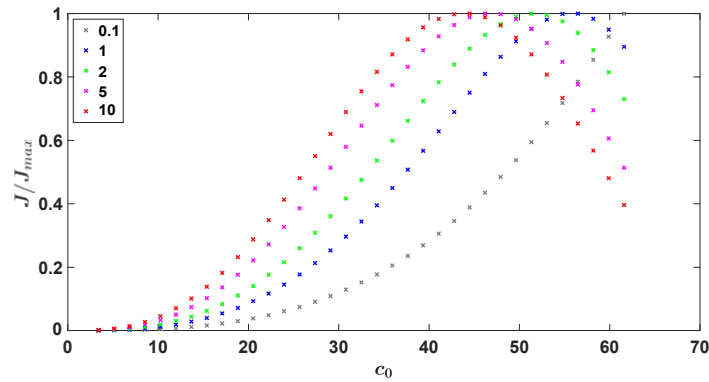


Figure 2: Normalized flux J/J_{max} as a function of the initial concentration c_0 for the Poiseuille model in 2-D in a closed tube. The five-tube lengths (L in m) are shown using different colors.

216 viscosity that depends on c_0 (Jensen et al. 2016) while keeping molecular diffusion is pre-
 217 sented in figure 2. The relation between J/J_{max} and c_0 appears to be similar in shape
 218 to the one obtained from the globally averaged Poiseuille model shown in figure 1. De-
 219 spite the qualitative agreement, three differences are observed. First, the optimal sugar
 220 concentration at which $J/J_{max} = 1$ is larger for the Poiseuille model than the globally
 221 averaged Poiseuille model for any conduit length (shown in figure 1). Second, there is
 222 an increase in the J/J_{max} versus c_0 curvature with increasing c_0 at low c_0 (i.e. slow rise
 223 in the normalized flux for low initial concentrations), and third, the optimal point $J/J_{max} =$
 224 1 is different for different tube lengths in each case with short tubes reaching $J/J_{max} =$
 225 1 at higher c_0 than long tubes (figure 2). Recall that the optimal point for the globally
 226 averaged Poiseuille model was not affected by the tube length L (not shown). The first
 227 two differences between the models are due to the problem set-up where the closed tube
 228 assumption requires higher c_0 to drive the flow as expected. Optimal point difference is
 229 attributed to dispersion and molecular diffusion not explicitly resolved in the simplified
 230 analysis.

2.2 Effect of a concentration-dependent viscosity on $J(c_0)$

231
 232 Typical phloem conditions are used to generate the results in both models: $a =$
 233 $10\mu\text{m}$, $k = 5 \times 10^{-14}\text{msPa}^{-1}$ and $D = 4 \times 10^{-10}\text{m}^2\text{s}^{-1}$. The tube length L was varied
 234 from 0.1 to 10 m to describe small and large plants or trees, respectively. The initial sugar

235 concentration inside the tube c_0 was varied from 100 to 1800 mol m⁻³, which does not
236 significantly affect the applicability of the Van't Hopf relation and the Newtonian fluid
237 approximation (to detect the largest signature of viscosity effects on J/J_{max} see supple-
238 mental materials and methods S3 for results on larger variation in c_0 (up to 2450 mol
239 m³) where the viscosity effects may be overestimated). In this range of values, the nor-
240 malized flux for the generalized model (not shown) exhibits a similar behavior for the
241 $L = 0.1$ m as the Poiseuille case here. However, with increasing L , the optimal point
242 where $J/J_{max} = 1$ was not reached for the range of c_0 studied (not shown). Interest-
243 ingly, the normalized flux over a wider range of c_0 for $L = 2$ m (selected for illustration)
244 shows that an optimal point does exist for the generalized model as well but c_0 must op-
245 erate well outside the range of the Van't Hopf approximation (see supplemental figure
246 S1B). The fact that $J/J_{max} = 1$ was not attained in the generalized model for the range
247 of c_0 selected here may lead to the erroneous conclusion that the inclusion of local vis-
248 cosity changes retards sugar transport. The normalization by J_{max} hides some facts about
249 the magnitude of J , which is much higher for the generalized model for the same c_0 .

250 Resolving the local changes in viscosity results in an increase in the overall con-
251 ductivity of the tube above and beyond the 2-D Poiseuille model (figure 3A). The gen-
252 eralized model appears to have far higher J than the Poiseuille model at a given c_0 for
253 all tube lengths except for $L = 0.1$ m where the two models are almost indistinguish-
254 able. The effect of the tube length is present in both models where an increase in L leads
255 to an increase in the flux J until a certain value is reached after which J starts to de-
256 crease with increasing L (for example, in the generalized model, J when $L = 10$ m is
257 lower than J when $L = 5$ m for the same c_0 as discussed next). An interesting obser-
258 vation is that the value of L for which there is a loss of conductivity (the sugar flux de-
259 creased for the same initial concentration c_0) is different for both models, $L = 5$ m for
260 the generalized model and $L = 2$ m for the Poiseuille model. The importance of vari-
261 able viscosity can be evaluated by the relative difference in sucrose fluxes

$$262 \quad e = \frac{J_g - J_p}{J_g + J_p}, \quad (4)$$

263 where the subscripts 'g' and 'p' denote generalized and Poiseuille, respectively (figure
264 3B). The results show that e increases with L and c_0 as expected. This is because the
265 c_0 affects the overall viscosity value itself and L affects the development of the velocity
266 profile over which viscosity gradients are allowed to buildup and increase with increas-

267 ing L . The increase in actual mass flux magnitude due to the inclusion of a variable vis-

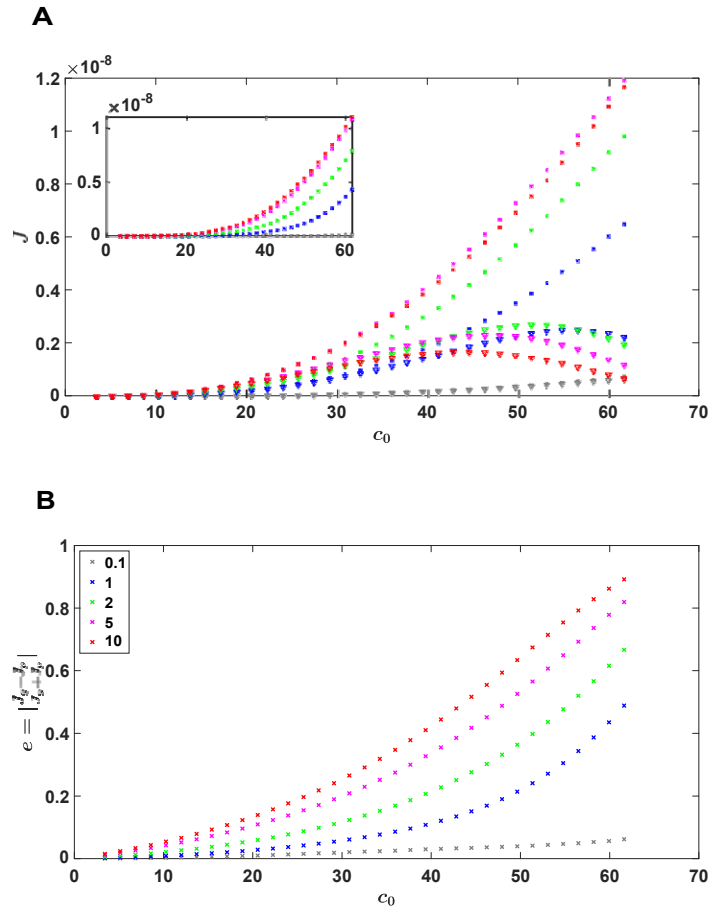


Figure 3: (a) Sugar mass flux J (Kg s^{-1}) as a function of the initial phloem sugar concentration c_0 for the generalized model (cross markers) and the Poiseuille model (triangleremarkers). Inset plot shows the flux for the viscosity effect (difference between both models). (b) Relative difference in sucrose fluxes $e = |J_g - J_p| / (J_g + J_p)$ as a function of the initial concentration c_0 . Different tube lengths are presented using different colors.

268 cosity can be approximated by subtracting the flux of the Poiseuille model from the gen-
 269 eralized model (Fig 3A inset). As expected, this effect increases with increasing L .

270 **2.3 Two-dimensional flow results**

271 To understand why the increase in mass flux occurs for the generalized model (vis-

272 a-vis the Poiseuille), the 2-D simulations are used to illustrate the radial-axial flow and

273 their coordination. These simulations show the local effect of concentration gradients on
274 flow velocity components affected by viscosity and its gradients. Model results show that
275 the computed axial and radial velocities are higher in magnitude because of a lower vis-
276 cosity near the conduit walls (figures 4A, 4B, 4C). Additionally, the pressure gradient
277 driving the flow is lower compared to the constant viscosity case (Fig 4D). The results
278 presented here are for initial concentration $c_0 = 800 \text{ mol m}^{-3}$ and $L = 2\text{m}$ for the time
279 when the sugar front is located at about 30% of the conduit length, chosen for illustra-
280 tion only. The time τ it took for the front to reach this location was different for the mod-
281 els: $\tau = 170.5$ for the generalized model and $\tau = 204.2$ for the Poiseuille model (i.e.
282 in the Poiseuille model the flow is about 1.2 times slower). Despite this difference in flow
283 velocity, the sugar front at this location appears similar in the models (4A). The high-
284 est difference is near the front location. After this position, the difference appears to be
285 enlarged partly due to concentrations being near zero after the fronts. The velocity pro-
286 file, on the other hand, appears to be wider, and the difference between the models is
287 higher after the front position (Fig 4B). The axial velocity is also higher in the gener-
288 alized model compared to the Poiseuille model.

289 The effect of local viscosity gradients generated by concentration gradients is more
290 apparent in the radial velocity than the axial velocity as expected (figure 4C). This find-
291 ing can be anticipated from equation (11) since the radial velocity profile is directly re-
292 lated to viscosity gradients in the axial direction. These gradients are generated based
293 on concentration gradients from the axial direction that are large, due to the wave na-
294 ture of the problem (because the advection-diffusion equation has a wave shape espe-
295 cially when it is advection-dominated). The relative difference e between the models is
296 high near the front location. Moreover, the location of the maximum radial velocity is
297 shifted further away from the membrane for the generalized model since the tube con-
298 ductance has a new term that depends on non-local changes in the radial direction (i.e.
299 $\langle K_t \rangle_r$ in equation (10)). Similar to the axial velocity profile, this non-local effect is also
300 apparent in the radial velocity profile for the generalized model that has a wider veloc-
301 ity range, when compared to the Poiseuille model. Moreover, due to a lower sugar con-
302 centration near the membrane, the viscosity of the sap decreases leading to less resistance
303 to the radial inflow of water (that is the driving force for osmotically driven flows) in the
304 generalized model. This can be conceptually understood as a decrease in wall-friction
305 when area-averaging the equations.

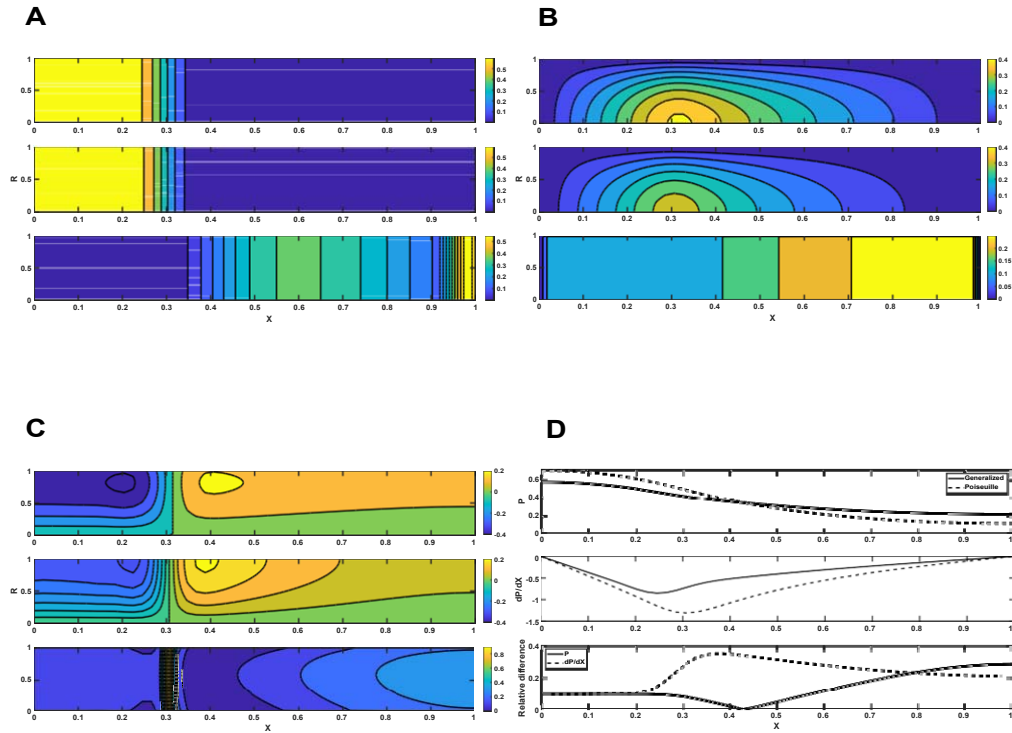


Figure 4: Results for a tube length $L = 2\text{m}$ and sugar concentration $c_0 = 800\text{ mol m}^{-3}$ when the sugar front is located at approximately 30% of the domain. Due to cylindrical symmetry, only half of the domain in the radial direction R is shown. (a) concentration profiles, (b) axial velocity profiles, and (c) radial velocity profiles for the generalized model (top), Poiseuille model (middle), and the relative difference (bottom), respectively. (d) pressure (top figure), pressure gradient (middle figure), and their relative difference (bottom figure) for the generalized and Poiseuille models. The relative difference between the models is calculated by $e_c = \left| \frac{C_g - C_p}{C_g} \right|$ where the subscripts 'g' and 'p' denote generalized and Poiseuille models, respectively.

306 The generalized model also has a smoother pressure field compared to the Poiseuille
307 model (figure 4D). An interesting result in this figure is that the global pressure gradi-
308 ent over the domain is smaller for the generalized model and yet the front travels at a
309 higher speed. This paradoxical result can only be explained by the increase in conduc-
310 tivity of the tube because of local viscosity effects being coordinated in radial and ax-
311 ial directions.

312 **3 Discussion and concluding remarks**

313 The concentration-dependent viscosity has two effects on J - both leading to an
314 increase in its magnitude. The first is that the smaller viscosity in the vicinity of the mem-
315 brane wall (c at the membrane is set by clear water in the xylem) results in an increase
316 in v into the pipe wall region. This increase has the effect of radially advecting sugar molecules
317 away from the pipe walls and towards the pipe center. Because the axial velocity pro-
318 file peaks in the pipe center yet the radial viscous stress is zero there, the overall mass
319 flux is also increased. The second is that the increase in radial velocity near the pipe walls,
320 when coupled to a zero radial velocity at the pipe center (as required by symmetry), must
321 be accompanied by an increase in axial velocity gradients due to the incompressibility
322 approximation. Thus, a speeding up of u is expected. The analysis of the axial pressure
323 distribution further suggests that this effect is sensed over a broader region of L . This
324 speeding up yields a faster front advancement of sugar away from the loading zone. Both
325 mechanisms are operating in concert to increase J above and beyond Poiseuille's model.
326 That those two effects act together to enhance tube permeability affecting J , not the driv-
327 ing force for water (i.e. pressure gradients) is also supported by the analysis here. For
328 this reason, the J_{max} analysis and its dependence on c_0 leading to $c_0 \approx 20\%$ in Figure
329 1 being not sensitive to X_f (as earlier shown) also cannot detect this coordination be-
330 tween axial and radial flow.

331 The importance of this finding is also highlighted when comparing typical concen-
332 tration values for crops (that are mainly active sugar loaders) and trees (mainly passive
333 loaders). From figure 1, active loaders have a higher concentration than passive loaders.
334 For these high concentrations (for example, maize has $c_0 \sim 50\%$), the simplified Poiseuille
335 model predicts an optimal length around 2 m where the generalized model predicts an
336 optimal length around 5 m (shown in figure 3A). For low concentrations (for example

337 a pine tree), the generalized model shows more resilience for increasing phloem length
338 when compared to the Poiseuille model (shown in figure 3B).

339 To summarize, the effect of a concentration-dependent viscosity on sucrose trans-
340 port in phloem allowed the flow to have a higher velocity for the same initial concentra-
341 tion, especially for long tubes. In addition, it resulted in a lower pressure gradient driv-
342 ing the flow along the axial direction. This finding contributes to the growing evidence
343 that the pressure-flow hypothesis can provide the necessary mechanism for long-distance
344 sugar transport as long as the complexity in transport physics is accommodated. It is
345 to be emphasized that the work here showed that viscosity adjustments lead to conduc-
346 tivity enhancement for sugar transport instead of pressure gradients for water flow.

347 More negative xylem water tension requires a higher osmotic potential to maintain
348 phloem transport in dry conditions (Sevanto 2014). During drought, the increase in op-
349 timal sugar concentration operating range for J would allow plants to increase their su-
350 crose concentration to potentially overcome those large tensions arising in the xylem with-
351 out a substantial decrease in flow rate. Future work will focus on the effects of nonlin-
352 ear xylem water potential on phloem transport by including sugar sources and sinks along
353 the phloem path.

354 **4 Materials and methods**

355 To isolate the effect of a concentration-dependent viscosity on radial-axial flow co-
356 ordination, many simplifications must still be invoked when representing the physics of
357 translocation in a cylindrical tube. In all formulations considered here, it is assumed that
358 i) the phloem vasculature can be approximated by a long slender tube of length L and
359 radius a ($\epsilon = a/L \ll 1$) with rigid semipermeable walls characterized by a constant
360 permeability k that allows the exchange of water molecules but not sugars with the sur-
361 roundings, ii) sieve plates have minimal effect on the flow and can be modeled as either
362 an 'extra' drag force uniformly acting along with L or ignored altogether, (iii) the bulk
363 flow is at very low Reynolds number $Re \sim \rho a u \mu^{-1} \ll 1$ where u is a characteristic
364 longitudinal velocity, μ the dynamic viscosity, and ρ the density, so that creeping flow
365 is maintained throughout, iv) sugar sources and sinks are modeled as boundary condi-
366 tions at the entry and exit end of the tube. Hence, water can be exchanged with the sur-
367 roundings but not sugars thereby suppressing any enhancement due to a relay effect.

368 **4.1 Variable Viscosity Model**

369 To derive the general model that includes concentration and viscosity variations
 370 in axial and radial directions, the governing equations under certain assumptions and
 371 simplifications are first analyzed in Cartesian coordinates. Then, the non-dimensional
 372 form of these equations, which are necessary for analyzing the numerical model results,
 373 are presented in cylindrical coordinates. The model to be presented in this section is con-
 374 sidered one 'end-member' case in including a concentration-dependent viscosity. The other
 375 'end-member' case this model is compared to is the Poiseuille model that assumes a con-
 376 stant viscosity in the domain set by the initial loading concentration. An example of this
 377 model in a globally-averaged case is the model discussed by Jensen et al. (2013). Thomp-
 378 son & Holbrook (2003) present a different model that is between these 'end-member' cases.
 379 In their work, they included local variations in viscosity inside the domain but only us-
 380 ing radially averaged equations (i.e. variations in viscosity along radial directions ignored).
 381 Due to the nonlinear relationship in the viscous stress between velocity and viscosity, area-
 382 averaging the equations leads to a simplified model that excludes the effect of this non-
 383 linearity. This issue can be resolved at the expense of solving the equations in axial and
 384 radial directions and frames the main approach here.

385 **4.1.1 Governing equations**

386 In a three-dimensional Cartesian coordinate system defined by longitudinal ($x_1 =$
 387 x), lateral ($x_2 = y$), and vertical ($x_3 = z$) directions, water flow within the phloem
 388 satisfies the continuity equation

$$389 \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0, \quad (5)$$

390 where t is time, $i = 1, 2, 3$ describe direction x_i , and u_i is the instantaneous velocity
 391 along direction x_i . Here, index notation is used with repeated indices implying summa-
 392 tion unless otherwise stated. The flow must also satisfy the conservation of momentum,
 393 which describes the force balance along direction x_i , and is given as

$$394 \frac{D(\rho u_i)}{Dt} = \frac{\partial}{\partial x_j} \sigma_{ij}, \quad (6)$$

395 where $D(\cdot)/Dt$ is the material derivative and σ_{ij} are the nine components of the stress
 396 tensor. The σ_{ij} of a Newtonian fluid can be approximated by

$$397 \sigma_{ij} = -p\delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad (7)$$

398 where p and μ are the local pressure and dynamic viscosity of the fluid, respectively, and
399 the δ_{ij} is the Kronecker delta (i.e. $\delta_{ij} = 0$ when $i \neq j$ but unity otherwise). This rep-
400 resentation of σ_{ij} is approximate and assumes that the stress tensor is symmetric ($\sigma_{ij} =$
401 σ_{ji}) and the so-called second viscosity coefficient (or volume viscosity) is momentarily
402 ignored (Panton 2006).

403 In terms of *fluid properties*, ρ depends on c and strictly speaking cannot be treated
404 as constant when c varies in time or along x_i . However, this dependence is minor when
405 compared to variations in μ as demonstrated earlier (see section 1) and variations in ρ
406 will be assumed small for simplicity so that $\partial u_i / \partial x_i = 0$. In this case, the σ_{ij} repre-
407 sentation given by equation (7) is reasonable (Panton 2006). Another common assump-
408 tion in phloem transport is that μ is constant set by the loading concentration. This ap-
409 proximation is only applicable for small concentration values. However, in plants, c can
410 range from 15% to 35% (and for maple trees even up to $\approx 50\%$). In this high concen-
411 tration range, the dependence of viscosity on concentration has not been fully analyzed
412 in the context of three-dimensional water and sugar transport. Some models include this
413 dependence of μ on c in an area-averaged formulation (Thompson & Holbrook 2003, Jensen
414 et al. 2016), but area-averaged formulations that evolve concentration axially and pre-
415 sume uniform concentration along the radial direction cannot resolve radial-axial flow
416 coordination to be studied here. Therefore, the model proposed here includes the depen-
417 dence of μ on c in both axial and radial directions and tracks its consequences on the
418 shape of the $J - c$ relation as well as the magnitude of J across differing L and load-
419 ing concentrations.

420 In terms of *flow properties*, the low Reynolds number ($Re \ll 1$) and small aspect
421 ratio ($\epsilon \approx 10^{-5}$) can be used to show that under the so-called lubrication theory (where
422 the flow in one of the dimensions is significantly smaller than the others because of ge-
423 ometric constraints), the momentum balance may be approximated by (now written in
424 cylindrical coordinates)

$$425 \quad \mu \frac{\partial u}{\partial r} = \frac{r}{2} \frac{\partial p}{\partial x}, \quad \frac{\partial p}{\partial r} = 0, \quad (8)$$

426 where x remains the axial direction with $x = 0$ situated at the loading zone, r is the
427 radial direction with $r = 0$ defining the center of the tube, and $u(x, r)$ and $v(x, r)$ are
428 the axial and radial velocity components respectively at any point (x, r) . In supplementen-

429 tal materials and methods S1, the derivation of this formulation and all its assumptions
430 are presented for completeness.

431 To understand how viscosity gradients affect the flow, equation (8) can be written
432 in a compact form. First, the tube conductance is re-defined as $K_t = 1/\mu$ as in section
433 1 (the geometric factor X_f is absent since it is the result of radial averaging). Integrat-
434 ing equation (8) in the radial direction while noting that the pressure p is only a func-
435 tion of the axial position (as shown in supplemental materials and methods S1) leads to

$$436 \quad u(x, r) = \frac{1}{2} \frac{\partial p}{\partial x} \int_0^r r K_t(x, r) dr + A(x), \quad (9)$$

437 where $A(x)$ is an integration function that varies in x . Using the no-slip boundary con-
438 dition on the longitudinal velocity component $u(x, a) = 0$ at the membrane surface set
439 as $r = a$ leads to $A(x) = -a^2 \langle K_t \rangle / 4 (\partial p / \partial x)$ where $\langle K_t \rangle$ is the radially averaged
440 tube conductance. Similarly, the term $\int_0^r K_t dr$ can be written as $r^2 \langle K_t \rangle_r / 2$ where the
441 subscript r denotes radial averaging until the current radial position ($r \leq a$). Using this
442 form, equation (9) describes the axial velocity

$$443 \quad u(x, r) = \frac{1}{4} \frac{\partial p}{\partial x} r^2 \langle K_t \rangle_r - a^2 \langle K_t \rangle, \quad (10)$$

444 where both terms $\langle K_t \rangle_r$ and $\langle K_t \rangle$ are functions of x but only $\langle K_t \rangle_r$ is a function of r .

445 Equation (10) is different from the HP expression because the variable viscosity de-
446 pends on $c(x, r)$ that itself varies radially and axially. To be clear, this dependence is non-
447 local because of the integral operator in the r direction. However, if a constant viscos-
448 ity is used at a given x , $\langle K_t \rangle_r$ and $\langle K_t \rangle$ are equal to $1/\mu$, and the aforementioned con-
449 servation of momentum equation becomes equivalent to the HP expression with an ad-
450 justment. This adjustment is due to osmosis that generates a radial inflow of water lead-
451 ing to $\partial^2 p / \partial x^2 \neq 0$, which then leads to a variable pressure gradient instead of a con-
452 stant one as is common in HP applications in pipes (Phillips & Dungan 1993). However,
453 the partial $\partial p / \partial x$ not being constant does not violate or invalidate the HP equation as
454 discussed elsewhere (Thompson & Holbrook 2003, Nakad et al. 2021).

455 The effect of viscosity gradients is not directly apparent in equation (10). To make
456 it explicit, the continuity equation (5) in cylindrical coordinates is now considered. It
457 is given as

$$458 \quad \frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial rv}{\partial r} = 0. \quad (11)$$

459 Here ρ variation with c is once again assumed to be small compared to the viscosity vari-
460 ations with c as stated before. Using the expression for the axial velocity from equation
461 (10) in the continuity equation (11), one can see how axial viscosity gradients impact the
462 radial velocity v , which is not identically zero due to osmosis. Moreover, the viscosity
463 gradient is not only the result of the area-averaged tube conductance $\langle K_t \rangle$ but also stems
464 from the radially-averaged (or non-local) tube conductance $\langle K_t \rangle_r$ that depends on ra-
465 dial position r . Equation (8) with equation (11) can be used to describe the flow of wa-
466 ter characterized by $u(x, r)$ (axial velocity) and $v(x, r)$ (radial velocity) inside the tube
467 as a function of position x, r .

468 Equations (8) and (11), however, remain incomplete since there are two equations
469 with three unknowns u, v , and p . This mathematical setup is in sharp contrast to flow
470 in closed pipes where $v = 0$ everywhere due to the solid wall boundary condition at $r =$
471 a and symmetry considerations at the pipe center. In phloem, osmosis necessitates a fi-
472 nite v at the pipe walls while symmetry considerations alone result in $v = 0$ at the cen-
473 ter of the pipe. Thus, the third equation that relates v to total pressure inside the tube
474 must be provided by osmoregulation. This equation is best formulated as a boundary
475 condition describing a flow through a porous media (a thin membrane here) at $r = a$.
476 Such a boundary condition may be given by a Darcy-type formulation assuming a very
477 low Reynolds number for the radial flow into or out of the pipe walls. This boundary
478 condition yields an expression for v at $r = a$ given by

$$479 \quad v|_{r=a} = k(p - \Pi|_{r=a}), \quad (12)$$

480 where $\Pi|_{r=a}$ is the osmotic potential at the membrane and k is the membrane perme-
481 ability assumed constant and independent of v (i.e. no Forscheimer or quadratic correc-
482 tions to Darcy's law). This osmotic potential can be related to c using the Van't Hoff
483 relation, $\Pi = R_g T c(x, a)$ as before. This approximation is reasonable for low c and com-
484 patible with the assumption that the density and molecular diffusion (discussed below)
485 do not vary appreciably with c when compared to viscosity.

486 The last equation needed to describe the physics of sugar transport is the conser-
487 vation of solute mass, which is also needed to solve for u, v , and p . This equation is de-
488 rived using Reynolds transport theorem that describes the movement of solutes (mainly
489 sugar here) due to advection and molecular diffusion. In cylindrical coordinates, it is given

490 by

$$491 \quad \frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(uc) + \frac{1}{r} \frac{\partial}{\partial r}(rvc) = D \frac{\partial^2 c}{\partial x^2} + D \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c}{\partial r} \right), \quad (13)$$

492 where $D = \nu Sc^{-1}$ is the molecular diffusion coefficient of sugar in water assumed to
493 be again insensitive to c variations when compared to ν , and $Sc \gg 1$ is the molecular
494 Schmidt number for sugars in water (usually of order 10^4).

495 The final step for describing the physics of sugar transport is to specify the bound-
496 ary and initial conditions. These are problem-specific and are selected here to illustrate
497 one restrictive 'end-member' case of flow in a closed tube with no sugar sinks. This case
498 is dynamically interesting because sugar concentration keeps building up as no sugars
499 are removed from the tube. The other 'end-member' case is where sugars are instantly
500 consumed at the end of the pipe (i.e. $c(L, r) = 0$ and sugar sinks are treated as 'infi-
501 nite'). This latter case is expected to lead to a much larger J in the tube, which is why
502 the focus is on the more restrictive former case. In plants, $c(L)/c(0) \ll 1$ and thus
503 osmotic gradients are much higher in the presence of sinks than those set by the closed
504 tube assumption. Thus, the physics of closed tubes must require higher loading concen-
505 trations to drive the water velocity, which is why they are more restrictive and thus dy-
506 namically interesting from the perspective of exploring limitations on the Münch hypoth-
507 esis. In a pipe closed at both ends $u(x = 0) = u(x = L) = 0$, water flow must accel-
508 erate to a well-defined maximum and then decelerate to zero velocity along L . For ini-
509 tial conditions selected here, sugar is released as an axially smooth function $c(x, t = 0) =$
510 $f(x)$ with no radial variation, meaning that radial diffusion is initially fast enough to en-
511 sure a uniform distribution of sucrose along r . The closed tube assumption with no sinks
512 requires sugar mass to be conserved inside the tube during the entire period resulting
513 in

$$514 \quad \frac{\partial c}{\partial x} \Big|_{x=0} = \frac{\partial c}{\partial x} \Big|_{x=L} = 0, \quad (14a)$$

$$516 \quad (vc) \Big|_{r=a} - D \frac{\partial c}{\partial r} \Big|_{r=a} = 0. \quad (14b)$$

517 Finally, a no-slip boundary condition at the membrane in the axial direction only, i.e.
518 $u \Big|_{r=a} = 0$, and symmetry considerations at the center of the tube, where $v \Big|_{r=0} = 0$
519 and $\partial c / \partial r \Big|_{r=0} = 0$, are all enforced.

4.1.2 Non-dimensional form and key dimensionless numbers

To elucidate the key dimensionless numbers governing water and sugar movement, and make interpretation of the equations easier, this section describes the scaling analyses and the non-dimensional form of the equations used. To write the equations in dimensionless form, the following scales were adopted: The x and r were scaled by the tube length L and radius a , respectively, leading to $x = LX$, $r = aR$. Time, and axial and radial velocity as well as pressure and concentration were scaled by their respective initial values at $x = 0$ (subscript 0) leading to $t = t_0\tau$, $u = u_0U$, $v = v_0V$, $p = p_0P$, $c = c_0C$ and $\mu = \mu_0\tilde{\mu}$. The dynamic viscosity μ was scaled by μ_0 , determined from c_0 and T . This μ_0 will also be used in a constant viscosity model (called the Poiseuille model hereafter) as a reference to assess the impact of accommodating variable viscosity in σ_{ij} and its spatial gradients. Using these scales, the non-dimensional equations for the two velocity components and pressure are

$$\frac{\partial U}{\partial X} + \frac{1}{R} \frac{\partial RV}{\partial R} = 0. \quad (15a)$$

$$\tilde{\mu} \frac{\partial U}{\partial R} = \frac{R}{2} \frac{\partial P}{\partial X} \quad (15b)$$

$$V|_{R=1} = MP - C|_{R=1} \quad (15c)$$

where $u_0 = kR_g T c_0 \epsilon^{-1}$, $v_0 = \epsilon u_0$, $p_0 = \mu_0 L u_0 a^{-2}$, and $M = k \mu_0 L^2 a^{-3}$ is the Münch number defined as the ratio of axial resistance over radial resistance as discussed in (Jensen et al. 2009, Nakad et al. 2021). The complete scaling analysis for the Navier-Stokes equations is shown in supplemental materials and methods S1.

The non-dimensional form of the conservation of sugar mass, i.e. equation (6), is

$$\frac{\partial C}{\partial \tau} + Pe \frac{\partial}{\partial X} (UC) + Pe \frac{1}{R} \frac{\partial}{\partial R} (RV C) = \epsilon \frac{\partial^2 C}{\partial X^2} + \frac{1}{R} \frac{\partial}{\partial R} R \frac{\partial C}{\partial R}, \quad (16)$$

where $t_0 = a^2 D^{-1}$ is the radial diffusion timescale and $Pe = v_0 a D^{-1}$ is the radial Peclet number defined by the ratio of radial advection to radial diffusion. This expression explicitly shows the relative contributions of radial flow dynamics (through Pe) and simplified geometry (through the slender ratio ϵ) to mass transport.

4.2 Model calculations

The proposed model calculations provide the variations of $J(x, r)$, $c(x, r)$, $u(x, r)$, $v(x, r)$, and $p(x, r)$ at every t . A description highlighting the numerical method used to

551 obtain the results is presented in supplemental materials and methods S2. To link a rep-
552 resentative J with a c_0 in a manner that allows comparison with the prior J_{max} and op-
553 timal c analysis, the following steps and approximations were taken in post-processing
554 the model results. With a radial Peclet number $Pe \gg 1$, it is reasonable to assume that
555 the mass flow primarily occurs in the axial direction (Nakad et al. 2021). The molecu-
556 lar diffusion can also be neglected since the axial Peclet number, defined by the ratio of
557 axial advection to axial diffusion and derived from Pe by $Pe_l = \epsilon^{-2}Pe$, is large (i.e.
558 $Pe_l \gg 1$). Using these assumptions, the area-averaged sugar flux can be reasonably de-
559 termined from post-processing time variations of the sugar front position x_f . This front
560 is also delineated from maximal $|\partial c(x, r)/\partial x|$. To determine x_f , we fitted an exponen-
561 tial relation between x_f and t using (Jensen et al. 2009, Nakad et al. 2021)

$$562 \quad x_f = L - (L - l) \exp \left(-\frac{t}{\hat{t}} \right), \quad (17)$$

563 where $l \approx 0.2$ is the initial sucrose front position, $L = 1$ is the length of the tube and
564 x_f is the front position all in non-dimensional form. Subtracting l from both sides of equa-
565 tion 17, a linear relation between t and $\ln[(X_{final} - X_f)/X_{final}]$ (where $X_{final} = L -$
566 $l \approx 0.8$ and $X_f = x_f - l$) can be obtained. Hence, linear regression applied to the 2D
567 numerical solution was then used to obtain the constant \hat{t} away from the entrance bound-
568 ary condition. The front speed was determined as $U_s = dx_f/dt$. Finally, the mass flux
569 was approximated by $J_{num} = Q_{num}C$ where Q_{num} is the numerical volumetric flow
570 rate. The Q_{num} was determined using approximated speed $Q_{num} = A_t U_s$ where A_t is
571 the cross-sectional area of the tube. We present results from the two-dimensional (2-D)
572 model simulation for the axial velocity U , radial velocity V , concentration C , and pres-
573 sure P in the dimensionless form to illustrate the effect of variable viscosity on radial
574 and axial variations of these variables. The 2-D model with variable viscosity is here-
575 after referred to as the generalized model. The model simulations were conducted us-
576 ing MATLAB programming language (Mathworks, Natick, MA).

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