

30 1. INTRODUCTION

31 In the last decades, maize (*Zea mays* L.) has reached the level of the world's largest crop, being the
32 only one to produce more than 1 billion tons per year (Contini et al., 2019), which makes it a crop of high economic
33 importance, due also to its multiple uses, such as human and animal nutrition, ethanol fuel production and in the
34 pharmaceutical industry. Although maize yield has been growing, the development of new cultivars adapted to
35 the specific edaphoclimatic conditions of different regions at different planting times is still necessary (Andrade
36 et al., 2016).

37 As an allogamous species of great agronomic interest, maize has already been extensively studied by
38 breeding programs, and the increase in productivity in this species is mainly dependent on the development of
39 single- cross cultivars, also known as hybrids, where the hybridization is used to explore the expression of
40 heterosis, first described by Shull (1908), which is quite expressive and well known in maize. To released new
41 cultivars capable of high yields and great performance of other agronomical characteristics, maize breeding
42 programs develop thousands of hybrids each year that need to be evaluated in field experiments; however,
43 resources are limited, and evaluations are expensive and labor-intensive. The time, area, labor, and budget required
44 to evaluate all those materials in all the desired locations and for all the traits of interest each year are very high
45 and, in most cases, unfeasible. Therefore, technologies, such as genomic prediction (GP), capable of predicting
46 the performance of those materials early, with no need to wait until the end of the crop cycle to discard unwanted
47 materials, are of great interest to the sector (Schrag et al., 2009; Werner et al., 2020).

48 GP emerged with the promise of increasing genetic gain per unit of time and reducing costs (Meuwissen
49 et al., 2001), and has been widely studied for different crops like maize, wheat, rice, coffee, and brachiaria (Cossa
50 et al., 2017; Carvalho et al., 2020; Matias et al., 2019), as well as for livestock and forest trees. Genomic selection
51 (GS) has been used for many purposes, for example, to predict the performance of lines and double haploids
52 during the initial stages of development (Krchov & Bernardo, 2015; Werner et al., 2020), including the quality
53 (Ibba et al., 2020; Lado et al., 2018), resistance to diseases (Rutkoski et al., 2012) and performance of single-
54 crosses (Bandeira e Sousa et al., 2017; Lyra et al., 2017; Alves et al., 2019). Several groups have shown that there
55 are advantages with the inclusion of multiple traits (MT) in GP, since they explore the correlation between traits
56 and their heritability in the prediction process, surpassing single-trait models' predictive ability (Jia & Jannink,
57 2012; Lado et al., 2018; Schulthess et al., 2018). Moreover, the use of multi-environment models (MET) seems
58 unquestionable (Guo et al., 2020; Oakey et al., 2016). Consequently, the combination of both, i.e., multi-trait
59 multi-environment models (MTMET), may improve the accuracy and save labor costs (de Oliveira et al., 2020;
60 Montesinos-López et al., 2016, 2019; Wang et al., 2018).

61 However, regardless of the GP method, the training set population (TRN) needs to be genotyped and
62 high-quality phenotyped, while the testing set population (TST) only needs to be genotyped. The establishment
63 of the TRN, which should be representative in terms of size, diversity, and the relationship of the individuals to
64 be predicted, is the key to success in GS (Jannink et al., 2010; Akdemir et al., 2015; Cossa et al., 2017; Varshney,
65 2017; Ibba et al., 2020). For that, the main objectives are to minimize costs associated with phenotyping by
66 selecting smaller training populations, and maximize the predictive ability for the individuals of the TST through
67 efficient resource allocation (Isidro et al., 2015; Lado et al., 2018; Pinho Morais et al., 2020; Riedelsheimer &
68 Melchinger, 2013; Technow et al., 2014). Additionally, there is a lack of knowledge on how to distribute

69 genotypes optimally in multi-environment trials in order to achieve the best balance between the number of
70 genotypes tested in the field and the predictive capacity of GP models, and maximize the selection gain with fixed
71 area and budget resources (Jarquin et al., 2020). Furthermore, in MTMET, we could also ask: which traits should
72 be evaluated in each genotype:trial combination?

73 Hence, the strategy is to design optimized populations for GP, which allows keeping the accuracy of
74 prediction at satisfactory levels using a training population that is smaller, but representative in terms of
75 information (Fritsche-Neto et al., 2018). In this context, many studies have been carried out aiming to establishing
76 the balance between investment and efficiency through different methods, experiment design, statistical analysis,
77 and TRN composition. For instance, the genetic algorithm to design training populations developed by Akdemir
78 (2017) was tested by Pinho Morais et al. (2020) for several population sizes. The responses were compared with
79 randomly selected populations, noting that optimizing TRN can be effective to obtain satisfactory accuracies.
80 Using MT models, Lado et al. (2018) tested other resource allocation strategies by comparing the PA with
81 different levels of availability of phenotypic information for the target trait (expensive and labor-intensive). For
82 that, they decreased the TRN sizes from 80 to 10%, then included the phenotypic data of all individuals for
83 correlated traits (less laborious and less costly), and finally, considered balanced and unbalanced scenarios. The
84 results showed no loss in PA when reducing TRN for a target trait up to 30% but using full information of
85 correlated traits; additionally, the unbalanced phenotyping approach for correlated traits performed better than the
86 balanced one for the same purpose of reducing TRN. Another strategy was proposed by Costa-Neto et al. (2021a),
87 who investigated the inclusion of dominance effects and envirotyping data into a single-trait MET scenario. The
88 authors found that, especially for traits with low heritability and highly influenced by the environment, the
89 environmental covariables (EC) can increase PA for new environments or newly developed hybrids by tracking
90 variation sources, environment resources, and reducing the error variance.

91 As described above, the use of accurate genetic algorithms for optimizing training populations can help
92 to reduce the number of genotypes that compose the TRN, as well as reduce costs and field labor, while
93 maintaining good values of predictive accuracy (Akdemir, 2017; Akdemir et al., 2015; Misztal et al., 2014;
94 Misztal, 2016). Additionally, the collection and processing of environmental data can help in the optimization
95 process. Instead of using a simple incidence matrix of environments to model the $G \times E$ interaction, processed
96 environmental data better describe specific relationships between environments and crop phenology, called
97 envirotyping. Through envirotyping, it is possible to describe the quality of an environment and estimate the
98 resources available to satisfy the crop needs. When it comes to multi-environment trials (MET), environmental
99 quality ends up as a global average of the entire experimental network. With the aid of some tools, such as the
100 EnvRtype R package of Costa-Neto et al. (2021b), it is possible to compose a covariance matrix (W) between
101 trials, which then makes it possible, among other things, to dissect the $G \times E$ interaction, and to build
102 environmental relationship matrices for genomic prediction, which better explain the sources of non-genetic
103 variation, such as the influence of environments on phenotypic variation. Finally, the envirotyping information
104 can be associated with genomic data in genetic algorithms to better select genotypes and target environments that
105 are more informative in terms of $G \times E$ (Costa-Neto et al., 2021a).

106 Compiling these ideas, to optimize TRN sets, MT models may help predict quantitative target traits
107 based on correlated characteristics. MET models also allow the inclusion of the $G \times E$ interaction term, which
108 undoubtedly helps predict non-phenotyped individuals. Finally, envirotyping is an emerging component for

109 selecting fewer but well-optimized trial locations. Therefore, our goal was to test the performance of optimized
110 training sets (OTS) for multi-trait multi-environmental trials (MTMET), and the use of environmental covariables
111 (W) in genomic prediction models, with the aim of diminishing the phenotypic labor due to lower but optimally
112 selected population sizes, while keeping the predictive ability at satisfactory levels, and then compare these results
113 with benchmarks. For that, we (i) fitted and compared the performance of five different prediction models,
114 progressively including environmental covariables and interaction terms ($G \times E$ and $G \times W$); (ii) estimated the
115 genomic prediction ability of the five prediction models for STMET and MTMET, to use as benchmarks values;
116 and (iii) estimated the genomic prediction ability using OTS with controlled unbalancing of G, E and trait
117 information, selected by a genetic algorithm.

118 2. MATERIALS AND METHODS

119 2.1. Plant material

120 The phenotypic data consisted of two datasets of tropical maize single-cross hybrids. Plant material
121 was evaluated for the following three traits of agronomic interest: grain yield (GY, in ton ha⁻¹), plant height (PH,
122 in cm), and ear height (EH, in cm). For GY assessment, ears were harvested at physiological maturity, grains were
123 adjusted to 13% moisture, and the yield was corrected by area and plant population. PH and EH were measured
124 from the soil surface to the flag leaf collar and the highest ear, respectively, on five representative plants within
125 each plot.

126 HEL dataset

127 Provided by Helix Seeds (HEL), the first dataset was composed of phenotypic and genotypic data of
128 452 maize hybrids obtained from single crosses in a partial diallel mating design among 106 tropical maize inbred
129 lines. In order to balance the data, only genotypes that were evaluated in all locations for all traits were considered,
130 so that 247 remained for analysis. Balancing the data will later allow the creation of controlled imbalances. The
131 experimental design used was randomized complete blocks with two replications per genotype per location.
132 Hybrids were evaluated in trials carried out over the 2014/15 growing season at three locations in Brazil: Ipiacú
133 (IP) and Pato de Minas (PM) in the state of Minas Gerais, and Sertanópolis (SE) in the state of Paraná.

134 USP dataset

135 The second dataset belongs to the University of Sao Paulo (USP). The data consist of 903 maize single
136 crosses obtained from a diallel mating design between 49 inbred lines. After balancing the data, 623 genotypes
137 remained for analysis. Hybrids were evaluated at two locations in Brazil: Piracicaba (PI) and Anhumas (AN), in
138 São Paulo. They were evaluated for two years during the second growing season of years 2016 and 2017. The
139 experimental design was an augmented block, with two commercial hybrids as checks per block. Although the
140 areas are relatively close on the map, the soil and climate conditions are quite contrasting, and thus characterize
141 different environments, allowing us to consider each location × year combination as an environment: AN.16,
142 PI.16, AN.17, and PI.17.

143 Further details about both datasets can be found in Alves et al. (2019), Bandeira e Sousa et al. (2017)
144 and Lyra et al. (2017).

145 2.2. Genotypic data

146 Parental inbred lines from HEL and USP datasets were genotyped with an Affymetrix® Axiom® Maize
147 Genotyping SNP array of 616 K (Unterseer et al., 2014). The genomic quality control (QC) was performed using
148 the SNPRelate package (Zheng et al., 2012) from R software. Markers with a call rate ≤ 0.95 for HEL and a call
149 rate ≤ 0.90 for USP, heterozygous loci in at least one of the parental lines, and monomorphic loci were removed.

150 The genotypic data of the hybrids were obtained by combining the homozygous markers of their
151 parental lines. The imputation of the lines and genotypes was performed by Synbreed (Wimmer et al., 2012) using
152 the Beagle 4.0 algorithm (Browning & Browning, 2008). Allele frequencies and linkage disequilibrium were
153 computed using the genotypes of the hybrids. Then, markers with minor allele frequency (MAF) ≤ 0.05 were

154 removed. After QC, 30,467 and 62,409 high-quality SNPs were available to analyze the HEL and USP datasets,
155 respectively. All the analyses were performed in the R software (R Core team, 2020).

156 **2.3. Enviromic data**

157 Enviromental covariables (EC) were obtained from the EnvRtype R package (Costa-Neto et al., 2021b),
158 to be used as descriptors of the environment for prediction purposes, aiming to increase predictive accuracy (PA)
159 in multi-environment GP scenarios. EnvRtype is a very practical package to acquire and process weather data.
160 Based on trial network information like geographical coordinates (WGS84), plant date, and harvest date, the
161 package collects and processes remote weather data from NASAPower. The environmental factors can be
162 summarized according to the plant phenology intervals of growth or preestablished fixed time intervals. For this
163 research, we used five time intervals according to the maize cycle phenology, defined as 0-14, 15-35, 36-60, 61-
164 90, and 91-120 days after emergency. The environmental factors used were: radiation-related (sunshine hours, in
165 hours, and total daylength, in hours), radiation balance (insolation incident on a horizontal surface, shortwave,
166 and downward thermal infrared radiative flux, longwave), and atmospheric demands (rainfall precipitation, in
167 mm, and relative air humidity, in %) as described in Costa-Neto et al. (2021a). The ECs can be estimated from
168 mean air temperature and accumulated precipitation over the period, for example, and then used to establish G ×
169 E interaction. This process creates a covariate matrix of ECs called W, which produces environmental relationship
170 matrices for genomic prediction. Then we can calculate an enviromic kernel equivalent to a genomic relationship
171 matrix, as follows (Costa-Neto et al., 2021b):

$$172 \quad K_E = \frac{WW'}{\text{trace}(WW')/\text{nrow}(W)}$$

173 where K_E is the enviromic-based kernel for the similarity between environments and W is the matrix
174 of ECs.

175 For the HEL dataset, each environment was characterized by 217 ECs, and for the USP dataset, each
176 environment was characterized by 238 ECs, resulting in matrices of dimensions 3×217 and 4×238 , then used
177 to estimate the W matrix.

178

179 **2.4. Variable transformation**

180 We established an index for EH that represents the distance from the actual EH to an ideal ideotype,
181 defined here as 80 centimeters, according to the following formula:

$$182 \quad EH_{tr} = |EH_{ij} - 80| * (-1)$$

183 where EH_{tr} is the transformed EH and EH_{ij} is the EH for genotype i at environment j . According to
184 this index, the closer to zero, the closer to our ideal height. For PH, values were normalized in order to obtain a
185 normal distribution interval. To fit the models, all phenotypic data were centered and standardized.

186 2.5. Statistical analysis

187 2.5.1. Phenotypic analysis

188 We used a linear mixed model for the two-step analysis to calculate the best linear unbiased estimates
189 (BLUEs) of each trait's hybrids. BLUEs were obtained within environments for the USP and HEL datasets by the
190 following respective models:

$$191$$
$$192 \mathbf{y}_{USPij} = \boldsymbol{\mu} + \mathbf{g}_i + \mathbf{g}^* + \mathbf{bl} + \boldsymbol{\varepsilon}_{ij}$$
$$193 \mathbf{y}_{HELij} = \boldsymbol{\mu} + \mathbf{g}_i + \mathbf{bl} + \boldsymbol{\varepsilon}_{ij}$$
$$194$$

195 where \mathbf{y}_{ij} is the estimated phenotypic value of genotype i at environment j , $\boldsymbol{\mu}$ is the general mean or
196 intercept, \mathbf{g}_i is the fixed effect of hybrid genotype i , \mathbf{g}^* is the fixed effect of check genotypes, \mathbf{bl} is the random
197 effect of blocks for the USP dataset ($\mathbf{bl} \sim \text{NM}(0, \sigma_{bl}^2)$) and the fixed effect of blocks for the HEL dataset, and
198 finally, $\boldsymbol{\varepsilon}_{ij}$ is the residual error for genotype i at environment j , where $\boldsymbol{\varepsilon} \sim \text{NM}(0, \sigma^2)$.

199 Phenotypic analyses were performed using the ASReml-R package (Butler, 2018) of R software (R
200 Core Team, 2020) and subsequently used in our genomic prediction models.

201 The variance components estimated for each model's effect will be used to estimate the average broad-
202 sense heritability H^2 .

203 2.5.2. Genomic prediction scenarios

204 In order to obtain a benchmark value of PA for the models, we first tested these models in full single-
205 trait multi-environment trials (STMET) and multi-trait multi-environment trials (MTMET) genomic prediction
206 analyses. From those, we were able to obtain the highest possible PA for the specific datasets under study because
207 we used all the information we had available (Fritsche-Neto et al., 2018), through cross-validation schemes with
208 replication.

209 The algorithms APY (Misztal et al., 2014; Misztal, 2016) and LA-GA-T, from the STPGA R package
210 (Akdemir, 2017), were used in optimization scenarios. The APY is used for determining the size of samples by
211 singular value decomposition. LA-GA-T is a genetic-based algorithm used to select representative individuals
212 from the population and compose the samples. For this purpose, two different kernels were built from the
213 Kronecker product between the variance-covariance matrices of genotypes (G), environments (E), environmental
214 covariables (W), and traits (T) as follows: $\Sigma_G \otimes \Sigma_E \otimes \Sigma_T$ and $\Sigma_G \otimes \Sigma_W \otimes \Sigma_T$, hereafter called GET and GWT,
215 respectively.

216 These kernels, used as inputs for the algorithms, assemble combinations between our variables. Thus
217 APY gives us the number of components that explain 98% of the variation within the population, and LA-GA-T
218 selects that number of representative information inside the kernels. Moreover, a genotype was added as a check
219 and therefore evaluated in all environments to create a connection between environments.

220 The optimized samples from LA-GA-T were obtained three times for each dataset and considered the
221 training set (TRN), while the remaining individuals were used as a testing set (TST). From these three samples
222 (OTS 1), two other scenarios were created, always within kernels. The former one was created by combining the

223 samples two by two (OTS 2), which resulted in three replicates. In the latter, the three independent samples were
224 added together (OTS 3), resulting in just one and bigger optimized training set (OTS).

225 **2.5.3. Genomic prediction via single and multi-trait multi-environment models with additivity and** 226 **dominance effects**

227 The genomic prediction was first performed by five GBLUP additive + dominance models for STMET
228 and MTMET scenarios. The following models were already tested (for further details, see Costa-Neto et al.,
229 2021a).

230

231 **Model 1 (M1): Environment and main additive plus dominance genomic effects (EAD)**

232 M1 is the most basic model tested, described as follows:

$$233 \quad \mathbf{y} = \mathbf{Z}_E \boldsymbol{\beta} + \mathbf{Z}_G \mathbf{u}_A + \mathbf{Z}_G \mathbf{u}_D + \boldsymbol{\varepsilon}$$

234 where \mathbf{y} is the adjusted observed values (BLUEs) obtained from the first step for the hybrids. The fixed
235 effects of environment were modeled by $\mathbf{Z}_E \boldsymbol{\beta}$ with the incidence matrix \mathbf{Z}_E , and \mathbf{Z}_G is the incidence matrix for
236 the genotypic effects. \mathbf{u}_A is the vector of additive genetic effects, where $\mathbf{u}_A \sim MN(\mathbf{0}, \mathbf{G}_a \sigma_A^2)$, \mathbf{u}_D is the vector of
237 dominance effects, where $\mathbf{u}_D \sim MN(\mathbf{0}, \mathbf{G}_d \sigma_D^2)$, and $\boldsymbol{\varepsilon}$ is the random residual effect, where $\boldsymbol{\varepsilon} \sim MN(\mathbf{0}, \sigma_e^2 \mathbf{I})$. \mathbf{G}_a and
238 \mathbf{G}_d are the genomic relationship matrices (GRM) for additive and dominant effects, respectively, given according
239 to VanRaden (2008) as follows:

$$240 \quad \mathbf{G}_a = \frac{W_A W_A'}{2 \sum_i^n p_i (1 - p_i)}$$

241 where the values from the incidence matrix W_A are equal to 0, 1 and 2, for genotypes markers of A_1A_1 ,
242 A_1A_2 and A_2A_2 , respectively, and p_i is the frequency of one allele from i locus.

$$243 \quad \mathbf{G}_d = \frac{W_D W_D'}{4 \sum_i^n \{(p_i(1 - p_i))\}^2}$$

244 where W_D contains the values equal to 0 (zero) for both homozygotes A_1A_1 and A_2A_2 , and equal to 1 for
245 heterozygotes A_1A_2 .

246

247 **Model 2 (M2): Environment, main effects plus block diagonal GE (EAD+GE)**

248 This model is an update of M1 that accounts for the main effects (A and D), adding the additive \times
249 environment and dominance \times environment interactions effects (AE and DE).

$$250 \quad \mathbf{y} = \mathbf{Z}_E \boldsymbol{\beta} + \mathbf{Z}_G \mathbf{u}_A + \mathbf{Z}_G \mathbf{u}_D + \mathbf{u}_{AE} + \mathbf{u}_{DE} + \boldsymbol{\varepsilon}$$

251 where \mathbf{u}_{AE} and \mathbf{u}_{DE} are the vectors of random effects of the interactions. \mathbf{u}_{AE} and \mathbf{u}_{DE} have a
252 multivariate normal distribution, $\mathbf{u}_{AE} \sim MN(\mathbf{0}, [\mathbf{Z}_G \mathbf{A} \mathbf{Z}_G'] \odot [\mathbf{Z}_E \mathbf{Z}_E'] \sigma_{ae}^2)$ and $\mathbf{u}_{DE} \sim MN(\mathbf{0}, [\mathbf{Z}_G \mathbf{D} \mathbf{Z}_G'] \odot$
253 $[\mathbf{Z}_E \mathbf{Z}_E'] \sigma_{de}^2)$, where σ_{ae}^2 and σ_{de}^2 are the variance components for \mathbf{u}_{AE} and \mathbf{u}_{DE} interaction effects, respectively
254 (Bandeira e Sousa et al., 2017; Jarquín et al., 2014; Lopez-Cruz et al., 2015).

255

256 **Model 3 (M3): Main effects plus main environmental covariable information (EAD+W)**

257 This third model includes environmental covariables information (W) from envirotyping data.

258
$$\mathbf{y} = \mathbf{Z}_E \boldsymbol{\beta} + \mathbf{Z}_G \mathbf{u}_A + \mathbf{Z}_G \mathbf{u}_D + \mathbf{u}_W + \boldsymbol{\varepsilon}$$

259 where \mathbf{u}_W is the matrix of environmental covariables, as according to Costa-Neto et al. (2021a), it is
260 non-genetic information that fills the gap between the genomic phenotypic information that remains across
261 environments.

262

263 **Model 4 (M4): Main effects EADW plus reaction norm for GE (EAD+W+GE)**

264 This model is an extension of the previous model (M3), adding the environment's additive and
265 dominance interactions.

266
$$\mathbf{y} = \mathbf{Z}_E \boldsymbol{\beta} + \mathbf{Z}_G \mathbf{u}_A + \mathbf{Z}_G \mathbf{u}_D + \mathbf{u}_W + \mathbf{u}_{AE} + \mathbf{u}_{DE} + \boldsymbol{\varepsilon}$$

267

268 **Model 5 (M5): Main effects EAD plus W plus reaction norm for GW (EAD+W+GW)**

269 This model is a modification of the latter (M4) reaction-norm variation; it replaces the genomic \times
270 environment interactions with the genomic \times enviromic effects interactions.

271
$$\mathbf{y} = \mathbf{Z}_E \boldsymbol{\beta} + \mathbf{Z}_G \mathbf{u}_A + \mathbf{Z}_G \mathbf{u}_D + \mathbf{u}_W + \mathbf{u}_{AW} + \mathbf{u}_{DW} + \boldsymbol{\varepsilon}$$

272

273 where \mathbf{u}_{AW} and \mathbf{u}_{DW} are the vectors of random effects of interactions. \mathbf{u}_{AW} and \mathbf{u}_{DW} have a multivariate
274 normal distribution, $\mathbf{u}_{AW} \sim MN(\mathbf{0}, [\mathbf{Z}_G \mathbf{A} \mathbf{Z}_G'] \odot [\mathbf{W}\mathbf{W}'] \sigma_{aw}^2)$ and $\mathbf{u}_{DW} \sim MN(\mathbf{0}, [\mathbf{Z}_G \mathbf{D} \mathbf{Z}_G'] \odot [\mathbf{W}\mathbf{W}'] \sigma_{dw}^2)$.
275 Here we can assume that there are different levels of relationship between genotypes and environments.

276 All models were fitted with the Bayesian Generalized Linear Regression BGLR R package (Pérez &
277 de los Campos, 2014a; Pérez & De Los Campos, 2014b), using a Gibbs sampler with 10,000 iterations, assuming
278 a burn-in of 1,000, and a thinning of 2.

279 It is important to point out that the "Multitrait" function of the BGLR package has some basic premises
280 that must be met for the model to work, one of which is the availability of complete information from at least one
281 genotype. Here, as we used multiple traits with the multiple environments approach, we established a genotype
282 as a check, with complete phenotypic data available, common to all environments. This way, it was possible to
connect the environments, especially when we explored the $G \times E$ interaction.

283 **2.6. Assessing the predictive ability of the models**

284 Two cross-validation schemes were used to access GP models' predictive ability, proposed by
285 Burgueño et al. (2012).

286 The first validation scheme, known as CV1, was applied considering 50 random partitions with 70%
287 of phenotypic and genotypic (genotypes phenotyped for all traits in all environments) information as TRN, while
288 the remaining 30% (genotypes not phenotyped in any of the environments) were predicted, using only their
289 genotypic information. This scheme aims to quantify GP models' ability and reproduce a scenario frequently faced
290 by breeders when predicting new genotypes in a network of already known environments, i.e., newly developed
291 maize hybrids never evaluated in any environment. The second scheme, CV2, mimics another common situation
292 when genotypes are tested in unbalanced field trials (or incomplete field trials), i.e., some genotypes are evaluated
293 in some environments but not in the entire experimental network. For this scheme, we also used 50 random
294 partitions with 70% of the information (genotype–environment combinations) as TRN, and the remaining 30% as
295 TST.

296 For each TRN-TST partition, models were fitted using the TRN, and we performed Pearson's
297 correlation coefficient between the predicted value and the observed value or BLUE of the TST individuals within
298 each environment, for each one of the 50 partitions. Then these correlations were used to assess the accuracy and
299 compare the performance of each model. Since the BLUEs were calculated by environment, the PAs were also
300 calculated by environment. The same 50 TRN-TST partitions were used to fit each model, allowing access to the
301 best performance model.

302 For OTS, the predictive ability was also calculated as the Pearson's correlation coefficient between the
303 predicted value and the adjusted observed value or BLUE of the TST individuals within each environment for
304 each trait; then the average of environments was taken.

305 **2.7. Response to selection per unit invested**

306 The genetic gain per dollar invested was estimated to compare the efficiency of the scenarios tested in
307 this work with pure phenotypic selection (PS). The methodology was based on information from Krchov &
308 Bernardo (2015) and Muleta et al. (2019). The phenotyping costs assumed were: 2 US dollars (USD) per plot per
309 trait for PH and EH; 4 USD per plot for GY. For genotyping, we considered 20 USD per sample. As we are
310 dealing with F1 maize hybrids, the parental inbred lines were genotyped, and the hybrid genotype was assembled
311 in silico. This way, the total cost was the sum of the expenses with genotyping (20 USD × number of lines) plus
312 phenotyping the TRN. This calculation was made for each dataset × scenario, considering the three OTS scenarios
313 (OTS 1, OTS 2 and OTS 3) for each kernel and the MTMET CV2 standard scenario. For the phenotypic selection
314 scenario, the average accuracy ($\sqrt{H^2}$ of each trait) was divided by phenotyping cost, 8 USD per plot for all traits,
315 for the complete dataset. The genetic gain was estimated by dividing the PA by the corresponding cost and
316 subsequently transformed to the base of 10,000 USD, given the fact that the other components of the breeder's
317 equation (Lush, 1937) were considered as fixed.

318 3. RESULTS

319 3.1. Descriptive statistics

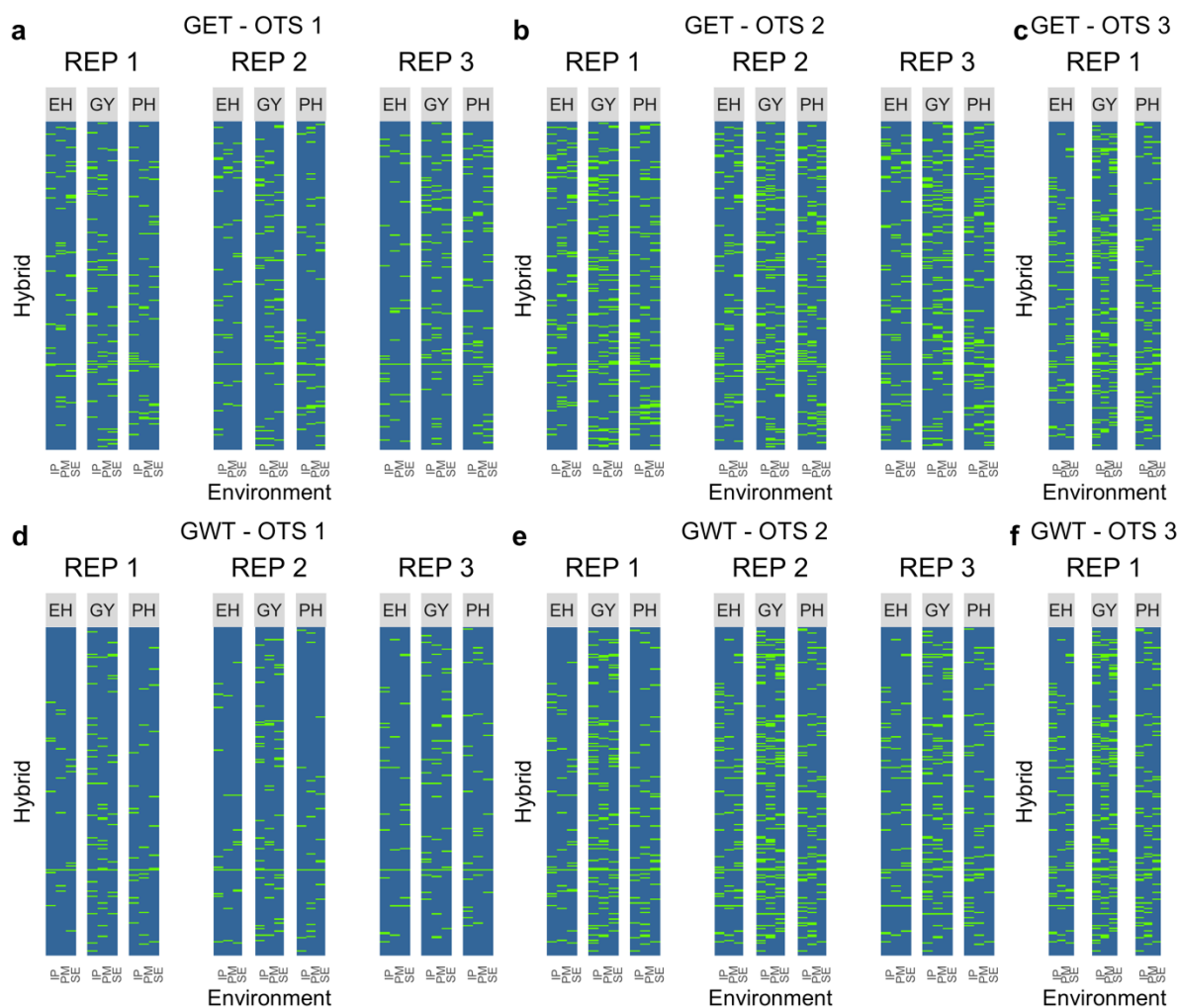
320 Pearson's correlation between traits was calculated for each dataset using the BLUEs obtained in the
321 first step. As a consequence of the EH transformation, GY assumed a negative correlation with the other traits.
322 For the HEL dataset, GY had a moderately negative correlations with PH and EH, of -0.55 and -0.58 ,
323 respectively, while PH and EH had a high positive correlation of 0.82 . For the USP dataset, GY had weak negative
324 correlations with PH and EH, of -0.44 and -0.33 , respectively, while PH and EH had a high positive correlation
325 of 0.70 .

326 Estimated heritability was intermediate to high: for the HEL dataset, trait heritability was 0.62 , 0.78
327 and 0.80 for GY, PH and EH respectively; for the USP dataset, heritability was 0.56 for GY, 0.84 for PH and 0.89
328 for EH.

329 As expected, the correlation for the complex trait GY was lower than for PH and EH; additionally, the
330 complex trait had a lower heritability than the auxiliary ones.

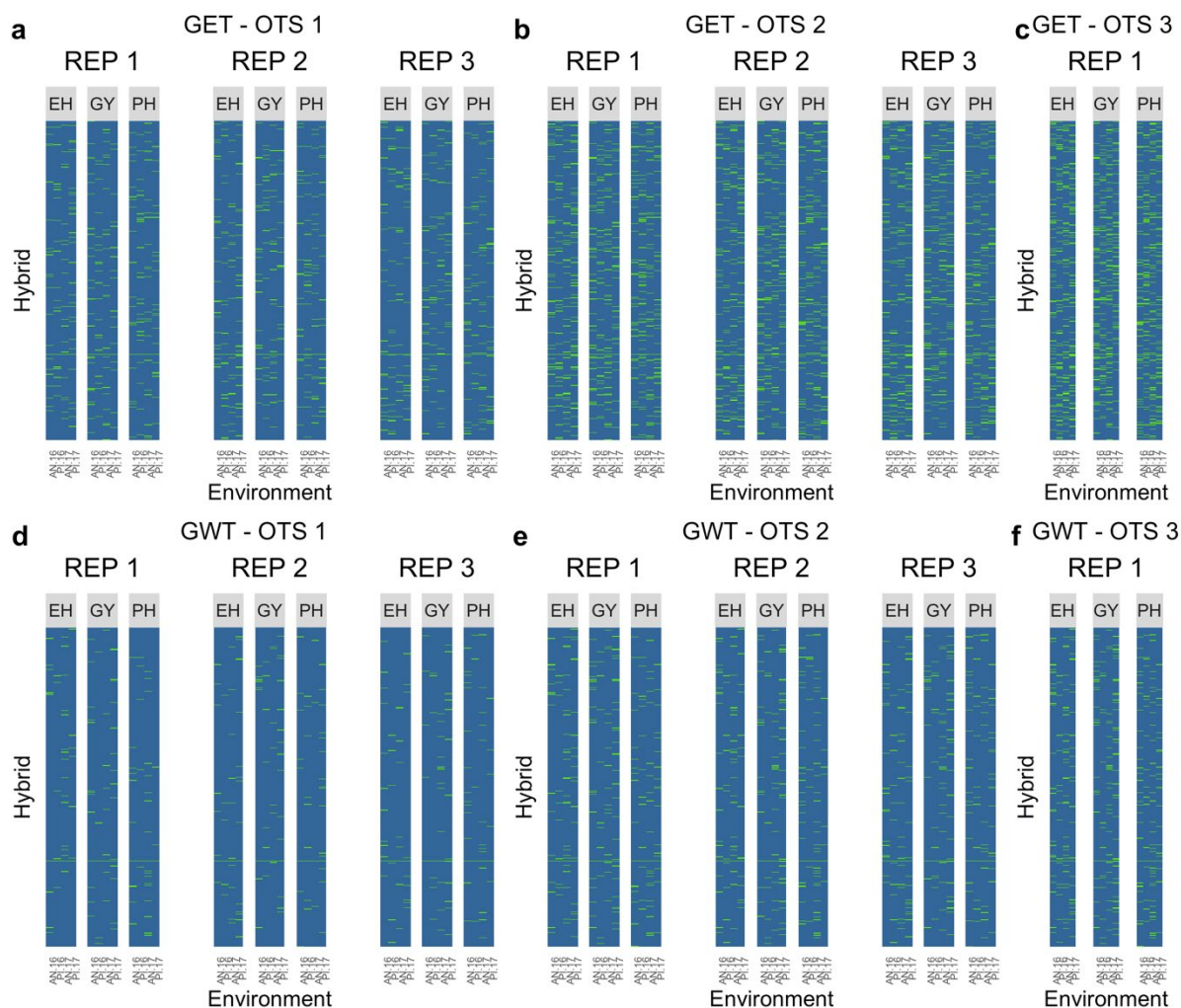
331 3.2. Optimized training sets (OTS)

332 The first result of selecting information to form the training populations, using the APY algorithm,
333 returned the effective population sizes (N_e) for each kernel, as described below. *HEL dataset*: GET – OTS 1: 155
334 combined information of genotype \times environment \times trait selected to form the TRN, which represents 7.4% of
335 observations; GWT– OTS 1: 102 combined information of genotype \times environment \times trait selected to form the
336 TRN, based on environmental covariables (W), representing 5% of observations. *USP dataset*: GET– OTS 1: 267
337 combined information of genotype \times environment \times trait selected to form the TRN set, representing 3.7% of
338 observations; GWT– OTS 1: 107 combined information of genotype \times environment \times trait selected to form the
339 TRN set, representing 1.6% of observations. Sample size differs depending on the kernel and germplasm because
340 the amount of available information varies as well as the genomic source. From this number, in order to minimize
341 the stochastic error, the LA-GA-T algorithm was performed three times to select the individuals. Thus, the first
342 validation scheme was done for OTS 1; the second combining the three basic populations, two by two, also
343 resulting in three different repetitions, where for Helix the N_e were: GET – OTS 2 = 306, representing 13.8% and
344 GWT – OTS 2 = 206, representing 9.3% of total observations, respectively, and for USP: GET – OTS 2 = 533,
345 representing 7.1% and GWT – OTS 2 = 224, representing 3% of total observations, respectively. Finally we added
346 the three repetitions of the base population to form a larger, but optimized, training population, which corresponds
347 to Helix: GET – OTS 3 = 436, representing 19.6% and GWT – OTS 3 = 300 representing 13.5% of total
348 observations, and for USP: GET – OTS 3 = 775, representing 10.4% and GWT – OTS 3 = 326, representing 4.4%
349 of total observations, respectively. The difference in selecting information between the three repetitions from the
350 different OTS tested scenarios can be seen in the heatmaps for Helix (**Fig. 1a-f**) and USP (**Fig. 2a-f**).



351

352 **Fig. 1** Heatmap of OTS (optimized training sets) graph for the Helix dataset. (a) OTS 1 for kernel GET. (b) OTS
 353 2 for kernel GET. (c) OTS 3 for kernel GET. (d) OTS 1 for kernel GWT. (e) OTS 2 for kernel GWT. (f) OTS 3
 354 for kernel GWT. In green are the hybrids selected to form the training population, for each trait × environment
 355 and repetition inside kernels. The solid line that crosses all the graphs represents the genotype used as a check.
 356 The environments on the x-axis: IP (Ipiacú), PM (Patos de Minas), and SE (Sertanópolis); the traits under study:
 357 EH (ear height), GY (grain yield), and PH (plant height). The kernels: GET (genotype × environment × trait) and
 358 GWT (genotype × environmental covariables × trait) used as the base to select information



359

360 **Fig. 2** Heatmap of OTS (optimized training sets) graph for the USP dataset. (a) OTS 1 for kernel GET. (b) OTS
 361 2 for kernel GET. (c) OTS 3 for kernel GET. (d) OTS 1 for kernel GWT. (e) OTS 2 for kernel GWT. (f) OTS 3
 362 for kernel GWT. In green we see the distribution of the hybrids selected to form the training population, for each
 363 trait x environment and repetition inside kernels. The solid line that crosses all the graphs represents the genotype
 364 used as a check. The environments on the x-axis: AN.16 (Anhembi 2016), PI.16 (Piracicaba 2016), AN.17
 365 (Anhembi 2017) and PI.17 (Piracicaba, 2017); the traits under study: EH (ear height), GY (grain yield) and PH
 366 (plant height). The kernels: GET (genotype × environment × trait) and GWT (genotype × environmental
 367 covariables × trait) are used as the base to select information

368 3.3. Predictive abilities of the five models over STMET and MTMET scenarios

369 HEL dataset

370 *Single-trait multi-environment trial analysis:* Results for CV1 showed, on average, PAs varying from
 371 0.53 to 0.73 (**Supplementary Table 1**). However, individually, environment SE was inferior in predicting GY,
 372 mainly when models without $G \times E$ interactions (**M1** and **M3**) were used. In contrast, models including $G \times E$
 373 (**M2** and **M4**) increased PA from 100 to 104% and $G \times W$ (M5) increased PA by 92% for this specific trait ×
 374 environment; considering the average within environments, PA for GY between models varies from 3 to 19%.
 375 For EH and PH, PAs were higher, from 0.70 to 0.73, and differences between models were minimal, from 0 to
 376 2%. Results for CV2 (Supplementary Table 1) produced the same patterns as those for CV1, ranging from 0.52
 377 to 0.79 when the overall average of environments was considered. For GY in the SE environment, PA could

378 increase between 61 to 68% when models with $G \times W$ (**M5**) and $G \times E$ interactions (**M2** and **M4**) were used,
379 respectively. For EH and PH, the accuracies were similar within models, with a maximum difference of 2%.
380 Comparing CV1 with CV2, PA means increased from 3 to 8%, and differences were higher for PH and EH. In
381 general, **M4** showed the best performance. Overall, the greatest difference was observed for GY, between models
382 **M1** and **M3** (without $G \times E$ interaction) and **M2** and **M4** (with $G \times E$ interaction), where **M4** (**M2**) outperformed
383 **M3** (**M1**) by between 25 and 30%, however, M5 also outperformed **M3** (**M1**) by 29%.

384 *Multi-trait multi-environment trial analysis:* As for the STMET analysis, the results of the multi-trait
385 analysis showed a similar pattern of responses, both for CV1 and CV2, varying from 0.54 to 0.73 and 0.53 to
386 0.79, respectively (**Supplementary Table 2**). Nevertheless, we noticed that PAs increased between 61 and 68%
387 for CV1 in the SE environment, which showed the lowest PA for GY in MTMET, like for STMET, when not
388 exploring $G \times W$ and $G \times E$ interaction effects. Considering the average within environments, the PA variation
389 for GY between models was practically identical to that of STME (0-24%). For EH and PH, accuracy varied from
390 0.70 to 0.73 and 0.75 to 0.79 for CV1 and CV2, respectively, but in general, the means ranged from 0 to 1%,
391 where **M4** performed better. Comparing CV1 with CV2, the PAs increased from 2.4 to 7.7%, with higher PA
392 differences for GY. Comparing STMET with MTMET, PAs rose from 0 to 1.4%, where the differences were
393 higher for GY and nonexistent for PH.

394 USP dataset

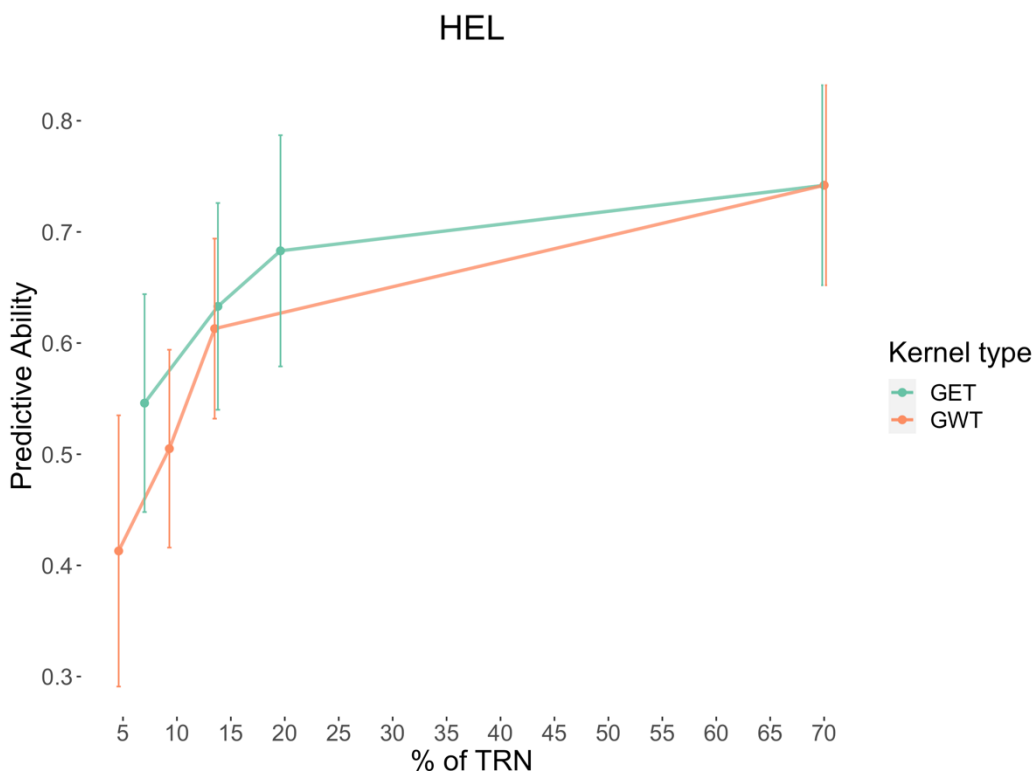
395 *Single-trait multi-environment trial analysis:* Results for CV1 showed PAs varying from 0.46 to 0.65.
396 The PI.17 environment isolated was inferior for predicting EH when models without $G \times E$ interactions (**M1** and
397 **M3**) were used (**Supplementary Table 3**). In contrast, models with $G \times E$ (**M2** and **M4**) could increase the
398 accuracy up to 440% in the PI.17 environment for EH (considering the overall average within environments, PA
399 for EH varied from 0 to 7%) and model with $G \times W$ increased PA by 420% for the same environment \times trait;
400 other environments performed very well for EH, with PA from 0.66 to 0.80. For GY, PA ranged from 0.42 to
401 0.51. For PH, PA were high, from 0.63 to 0.68 between environments, with no difference between models. Results
402 for CV2 produced almost the same patterns as those for CV1. For EH in the PI.17 environment, PA could increase
403 by 340% when models with $G \times E$ interactions were used and 320% when using $G \times W$. Despite the environment,
404 PI.17 for EH, the accuracy of all other traits and environments was similar within models, with differences of
405 around 0 to 5%. In general, **M4** showed the best performance. Comparing CV1 with CV2, the PA increased from
406 4 to 8% for GY and PH, respectively.

407 *Multi-trait multi-environment trial analysis:* As for the STMET analysis, the results of the multi-trait
408 analysis showed similar response patterns, both for CV1 and CV2 (**Supplementary Table 4**). Nevertheless, PAs
409 increased between 4 and 8%. Also for STMET, the PI.17 environment showed low PA for EH in MTMET, but
410 by exploring $G \times E$, accuracy increased up to 333%. For GY, accuracies varied from 0.42 to 0.52, and for PH,
411 from 0.63 to 0.72, but for both, in general, the means did not vary. Comparing STMET with MTMET, PAs
412 changed from 0 to 0.4%. Including interaction effects (no matter if $G \times E$ or $G \times W$) always increased PA.

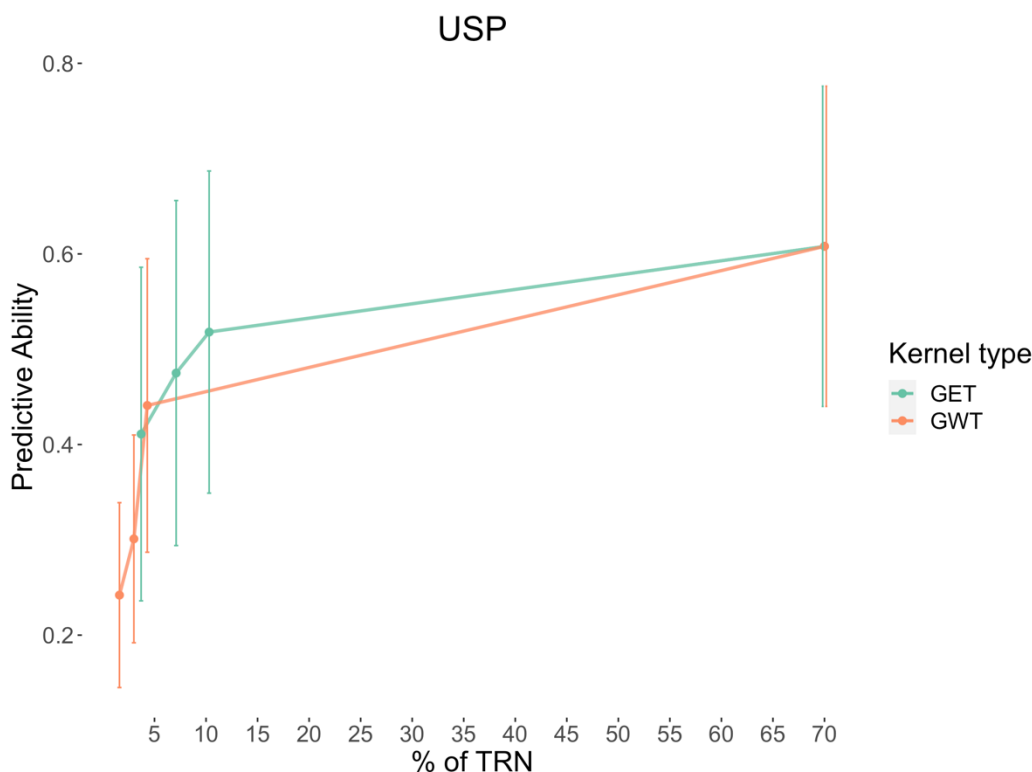
413

414 **3.4. Predictive ability for OTS scenarios**

415 Similar to what was done previously, for the optimized training sets, the five models were also tested.
416 The model that achieved the best performance was the **M4**, so only this result will be presented. *Helix dataset*: In
417 the overall average of traits, for OTS 1, PAs were 0.55 and 0.41 for kernels GET and GWT, respectively. Those
418 values increased to 0.63 (+15.9%) and 0.50 (+22.6%), then 0.68 (+7.7%) and 0.61 (+21.4%), for OTS 2 then OTS
419 3, while the maximum PA obtained by the benchmark CV2 was 0.74 (**Table 1** and **Fig. 3**). *USP dataset*: in the
420 overall average of traits, for OTS 1, PAs were 0.41 and 0.24 for kernels GET and GWT, respectively. Following
421 the same pattern, when we increased the size of TRN, those values increased to 0.47 (+15.6%) and 0.30 (+24.4%)
422 for OTS 2, then 0.52 (+9%) and 0.44 (+46.5%) for OTS 3, while the maximum PA obtained by the benchmark
423 CV2 was 0.61 (**Table 1** and **Fig. 4**).



424 **Fig. 3** The trend graph shows the increase in the average PA (predictive ability, mean of environments and traits)
425 according to increases in the Helix dataset's training set size. GET and GWT are the kernels used as the basis for
426 selection of information by the LA-GA-T algorithm. The 4 points on each line correspond to OTS1, OTS2, OTS3
427 and finally CV2, which is a benchmark or the highest value achieved with this dataset under this particular study's
428 conditions
429



430

431 **Fig. 4** The trend graph shows the increase in the average (predictive ability, mean of environments and traits)
 432 according to increases in the USP dataset's training set size. GET and GWT are the kernels used as the basis for
 433 selection of information by the LA-GA-T algorithm. The 4 points on each line correspond to OTS1, OTS2, OTS3
 434 and finally CV2, which is our benchmark or the highest value achieved with this dataset under this particular
 435 study's conditions

436 **Table 1** Average increase, per trait, in predictive ability, according to an increase in the training set (denoted here
 437 as OTS 1, 2 and 3), for both GET and GWT kernels, in the Helix and USP datasets. Ne: number of information
 438 used as the training set; prediction accuracies for EH: ear height, GY: grain yield and PH: plant height. Values in
 439 parentheses show the percentage increase between that value and the value immediately preceding it

		GET				GWT			
	OTS	Ne	EH	GY	PH	Ne	EH	GY	PH
Helix	1	155	0.61	0.47	0.56	102	0.43	0.42	0.38
	2	306 (+97%)	0.67 (+11,2%)	0.56 (+18%)	0.66 (+19,2%)	206 (+100%)	0.54 (+23,5%)	0.50 (+19,8%)	0.48 (+24,3%)
	3	436 (+42%)	0.72 (+7%)	0.60 (+7,7%)	0.72 (+8,5%)	300 (+45%)	0.64 (+18,4%)	0.60 (+19,3%)	0.63 (+26,7%)
		GET				GWT			
	OTS	Ne	EH	GY	PH	Ne	EH	GY	PH
USP	1	267	0.47	0.28	0.48	107	0.22	0.19	0.32
	2	533 (+91.9%)	0.52 (+12.6%)	0.34 (+19.8%)	0.56 (+16%)	224 (+87.5%)	0.31 (+42.2%)	0.23 (+23%)	0.36 (+12.8%)
	3	775 (+45%)	0.56 (+5.9%)	0.40 (+17%)	0.61 (+8.7%)	326 (+43%)	0.43 (+39%)	0.35 (+51.5%)	0.54 (+49.2%)

440

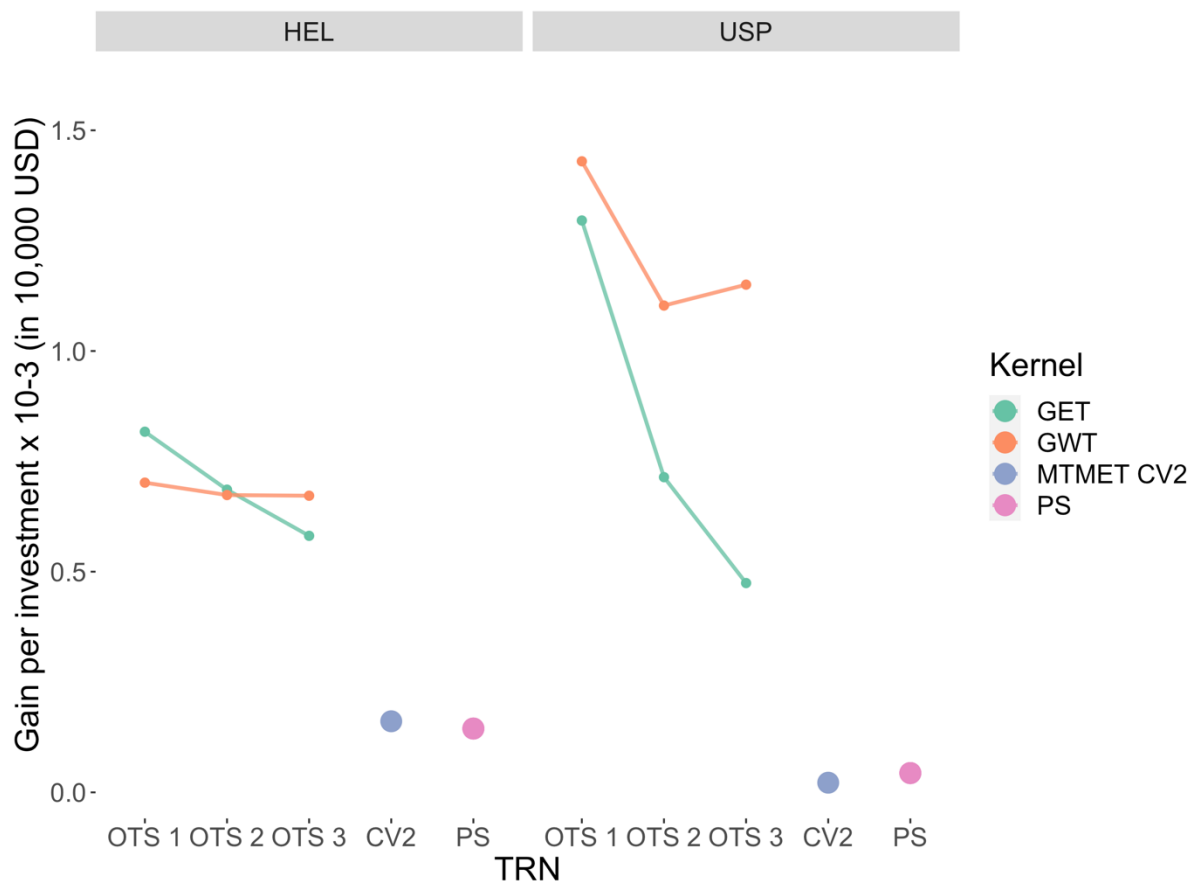
441 3.5. Response to selection per amount invested

442 The genetic gain per dollar per 10,000 USD invested was estimated for each dataset × OTS' scenario,
 443 primarily to compare the efficiency at the kernel level. Then, between scenarios: OTS *versus* MTMET CV2 and
 444 OTS *versus* phenotypic selection, within datasets.

445 For the *HEL dataset*, the results showed an inversion of gain between the kernels over scenarios. While
 446 the GET kernel started at 0.80×10^{-3} and went to 0.58×10^{-3} , the GWT kernel started at 0.70×10^{-3} and went to
 447 0.67×10^{-3} gain per 10,000 USD invested. For PS, the gain was 0.14×10^{-3} (**Fig. 5**).

448 For the *USP dataset*, the GET kernel started with a gain of 1.29×10^{-3} , and went to 0.47×10^{-3} , while
 449 the GWT kernel started at 1.42×10^{-3} and went to 1.15×10^{-3} . For PS, the gain was 0.04×10^{-3} .

450 As a matter of comparison, we did the same procedure for the standard scenario MTMET CV2 (for
 451 **M4**) and the gains per investment were 0.16×10^{-3} for HEL and 0.02×10^{-3} for the USP dataset.



452 **Fig. 5** Gain per cost $\times 10^{-3}$ (per 10,000 dollars invested) for the HEL and USP datasets, comparing the two
 453 optimization kernels (GET and GWT) and the standard scenario (MTMET CV2, TRN = 70%). The cost includes
 454 the phenotyping of TRN (3 USD per trait per plot) and the whole dataset's genotyping (20 USD per sample)
 455

456

457 4. DISCUSSION

458 The major goal of GS can be defined as increasing the genetic gain with no increase in costs compared
459 to phenotypic selection only (Crossa et al., 2017; Werner et al., 2020), thus compensating the loss in predictive
460 ability by the gains in response to selection.

461 The traits evaluated here are moderately to strongly positively correlated by Pearson's correlation
462 coefficient. However, the selection targets for them in a real breeding program are in opposite directions. While
463 we want to increase grain yield, we want to decrease plant height and stabilize ear height. Dwarf plants with high
464 yield are already a reality in other crops like wheat and rice. However, in maize, dwarfing mutant genes have been
465 studied. Unlike wheat and rice, PH in maize is a quantitative trait that affects other plant characteristics like yield
466 losses, making it difficult to apply (Chen et al., 2018). Since we want all those attributes simultaneously, we
467 created a selection index for EH, which measures the distance from the ideal ideotype defined here as 80
468 centimeters. Then, traits assume a negative correlation between them due to the index, allowing us to select for
469 all the traits concurrently (Wang et al., 2018).

470 Similar to what Ibba et al. (2020) and Werner et al. (2020) reported and to what was expected, accuracy
471 is specific to the population, which depends on many other factors like the model chosen, the traits to be predicted,
472 trait heritability, the correlation between traits, the environments, and the correlation between environments. So,
473 the results here were no exception. Differences in predictive ability between the two datasets, which occurred for
474 the benchmarks (CV1 and CV2) and OTS's kernels (GET and GWT), can be partially attributed to differences
475 in Pearson's correlation coefficient between traits, since PA is directly related to the correlation between traits
476 (Lado et al., 2018).

477 We could see that some models overperformed others, which was also expected since they contain
478 additional terms or variance components such as environmental covariables (**W**) and interaction terms (**G × E**
479 and **G × W**) that better capture the portion of variance explained by the model (Alves et al., 2019). Results of
480 Dias et al. (2018) suggested that GBLUP models that contain additivity, dominance and $G \times E$ interaction should
481 be preferred for predicting the performance of newly developed hybrids in any MET analyses, as is the case with
482 STMET and MTMET under CV1, and OTS.

483 In this study, two cross-validation schemes, CV1 and CV2 (Burgueño et al., 2012), were used to
484 evaluate PA for both STMET and MTMET models. For all combination scenarios of dataset, model, single or
485 multi-trait, CV2 outperformed CV1, because, in this scheme, we have phenotypic information of genotypes in
486 some environments, but not in others, which helped increase the PA of the models. With CV1 and CV2, single-
487 trait and multi-trait genomic predictions with multi-environment trials are well described and established in the
488 general literature and for the data used in this study (Bandeira e Sousa et al., 2017; Alves et al., 2019). Random
489 cross-validation schemes CV2 and CV1 combined with STMET and MTMET were then used here as benchmarks
490 and as a matter of comparison for the five different prediction models tested (Werner et al., 2020). Thus, based
491 on this prior validation, the model with the highest accuracies (**M4**) was chosen for the prediction using the
492 optimized training set populations, and those validations were also used in a scale as a comparison parameter for
493 prediction accuracies while increasing the effective population sizes of TRN (see **Fig. 3** and **Fig. 4**). Note that the
494 model whose performance was superior includes the $G \times E$ interaction component, in agreement with what has
495 already been reported by several authors (Acosta-Pech et al., 2017; Burgueño et al., 2012; Dias et al., 2018;

496 Jarquin et al., 2020; Montesinos-López et al., 2019; Robert et al., 2020) and more recently, by Jarquin et al. (2021).
497 As well, adding the interaction effect $G \times W$ (M5) increases PA when compared to the main effects models (M1
498 and M3), but not as much as models containing the $G \times E$, similar to what was already presented by Robert et al.
499 (2020) and Jarquin et al. (2021); however, $G \times W$ has the advantage of predicting new environments (Costa-Neto
500 et al., 2021a; Jarquin et al., 2021; Robert et al., 2020), which has not been tested here. Also, MTMET models
501 gave a small increase in PA compared with STMET, especially for models containing the $G \times E$ term (Lyra et
502 al., 2017; Mendonça & Fritsche-Neto, 2020). Furthermore, as Costa-Neto et al. (2021a) reported, including the
503 W matrix helps increase PA, especially in the case of untested hybrids and/or untested environments, by better
504 explaining sources of variation, capturing the environment potential per se and its interaction with the genotypes.
505 Nevertheless, in the present study, and according to what was presented by Jarquin et al. (2021), the inclusion of
506 W matrix alone (M3) did not improve PA, but was similar to the M1; and the inclusion of W with $G \times E$ in M4,
507 gave similar results as the $G \times E$ alone (M2). Despite that, the W matrix allows optimizing complex information,
508 as we saw in our OTS's scenarios, then optimizing trials (Jarquin et al., 2021).

509 Studies involving SNP data, such as GP, require the inverse of the genomic relationship matrix (GRM).
510 However, as the number of individuals to be evaluated increases, the computational cost of this matrix's inversion
511 is relatively high, with limitations in memory and time. In order to minimize this problem, Misztal (2016)
512 proposed the algorithm for proven and young animals (APY). Using the APY it is not necessary to make a
513 complete inversion of the GRM, since its result returns the number of individuals (n) needed to sample 98% of
514 the population variation; then the inverse of GRM can be obtained by recursion, based on the information of the
515 core individuals. Despite reducing the number of individuals, this algorithm does not specifically indicate which
516 genotypes we should select as core individuals. Hence, it is necessary to take a random sample of population.
517 Here we extended the APY to plants. Since the APY does not provide information on which genotypes should be
518 included in the core population, another algorithm was used to efficiently select these individuals. We used the
519 genetic optimization algorithm in the selection of sub-populations, the LA-GA-T (look ahead genetic algorithm
520 with taboo) proposed by Akdemir (2017) in his STPGA (selection of training populations by genetic algorithms)
521 R package. Genetic algorithms work based on the principles of biological evolution, so that they solve their
522 problems using evolutionary strategies, where at each iteration, the best individuals are selected and elite
523 individuals and so on form the next population. Still, the term taboo indicates that the solutions recently tested
524 will be avoided in the next attempts, avoiding unnecessary evaluations, and limiting the number of iterations
525 necessary to reach convergence. Thus, LA-GA-T optimizes the selection, on a genetic basis, of the n genotypes
526 informed by APY to compose our optimized training set (OTS) (Fritsche-Neto et al., 2018). In this context,
527 Mendonça & Fritsche-Neto (2020) used the algorithm designed by Akdemir (2017) to select the most
528 representative genotypes to build the training population. Similar to our findings, they did not notice an increase
529 in PA while using OTS, but reduced the budget.

530 Nevertheless, in the present work, we extended these algorithms to more complex relationship matrices,
531 as the Kronecker product of the genomic relationship matrix (GRM), with the environmental variances and
532 covariances matrix (W) and the traits (T) of interest ($G \otimes E \otimes T$ or $G \otimes W \otimes T$). Although these scenarios cause
533 a high level of imbalance in the data, they will give an idea about which genotypes need to be phenotyped, in
534 which locations and for which traits in order to form a smaller but optimized training set, reducing fieldwork and
535 the financial resources spent on evaluations, and how much data imbalance the models support for the prediction

536 of hybrids, without important losses in PA. Thus, it allowed us to identify which genotypes should be evaluated
537 in which environments and for which traits, to form a super-optimized training population, capable of predicting
538 the performance of the entire population for all traits and environments, filling gaps in GS and answering questions
539 about the optimal partition of genotypes across environments (Jarquin et al., 2020).

540 Voss-Fels et al. (2019) said that the TRN have to be exceptionally large. Similarly, Wang et al. (2018)
541 stated that the larger the TRN, the better the estimation of genetic effects and therefore, the greater the accuracy,
542 mainly for low heritability traits. Here, the amount of information for each OTS, regardless of the kernel, is small,
543 representing between 1.6 and 19.6% of the total available information. Moreover, the samples have a good
544 distribution of genotypes, including those genotypes that perform well, and those that are not so good, bringing
545 positive impacts on PA (Michel et al., 2020). As expected and similar to what Pinho Morais et al. (2020) found,
546 with a small effective population size, PA is diminished, since the sample contains small genetic variability.
547 However, we still obtained satisfactory prediction values, with an overall mean up to 0.41 and 0.54 for USP and
548 Helix, respectively, with the advantage that costs were reduced by more than 1000% and the labor with the TRN
549 was also reduced. According to Krchov and Bernardo (2015), accuracies should be greater than 0.50 so that the
550 GS is superior and chosen instead of the phenotypic selection.

551 It is interesting to notice that, with a similar increase in effective population size of approximately 90-
552 100% from OTS 1 to OTS 2, and from OTS 2 to OTS 3, respecting the particularities of each training set
553 population, the increases in PA are different for each kernel-dataset combination, as given in **Fig. 1**, **Fig. 2** and
554 **Table 1**, which show that PA practically doubled for GWT when compared to GET, mainly for the USP dataset,
555 whose amount of information used as TRN represents a very small portion of the total dataset. Therefore, it means
556 that a small increase in the training population results in satisfactory PA increases, especially when considering
557 the response to selection. However, GET proved less efficient for optimizing TRN, because it assumes that the
558 environments are not related, and thus needs more information to explain variations in the whole dataset, while
559 GWT proved more efficient for optimizing TRN, than using the W matrix for optimization. With all the
560 aggregated information it carries, GWT is able to select individuals more assertively, and then needs less
561 information to form the OTS, so the TRN size is smaller, providing the added advantages of lower cost and labor.
562 Shown here for a complex case, the global idea is similar to what was observed in Costa-Neto et al. (2021a),
563 adding information to help in prediction.

564 Since the costs of genotyping are decreasing, whereas the costs of field testing in maize are either
565 stagnant or increasing, and adding that genotypic information is stable, not liable to seasonal variation, less effort
566 is expended saving money resources, genotypic selection is, from this perspective, more efficient (Krchov &
567 Bernardo, 2015). It is worth noting that with a fixed budget, as we decrease the training population size, using
568 OTS for example, we can have a larger test population, since the resources are reallocated from phenotyping to
569 genotyping, or even, with a fixed training population, small budget increments mean a significant increase in the
570 test population, which can be considerably expanded for greater selection gains (Riedelsheimer & Melchinger,
571 2013; Krchov & Bernardo, 2015).

572 Looking to reduce costs with phenotyping, Lado et al. (2018), working with wheat, found that there
573 were no losses in the predictive power when reducing the training population up to 30% when traits highly
574 correlated to the trait of interest are used in the multi-trait model because the correlation between the traits and
575 the heritability of each assists in the prediction of the other. Here, we could reduce the training population up to

576 approximately 4% obtaining satisfactory results when considering the selection gains per 10,000USD invested,
577 which were about 0.70×10^{-3} against 0.16×10^{-3} , with optimized (GWT) against standard scenario (MTMET
578 CV2) TRN, respectively, for the HEL dataset and 1.42×10^{-3} against 0.16×10^{-3} for the USP dataset.

579 The genetic gain per dollar invested was estimated as a basis of comparison for responses in different
580 TRN sizes, datasets and mainly to justify the use of optimization for training populations in GS. From the results,
581 we can infer that the optimized populations have advantages over the standard scenario (70% TRN-30% TST).
582 The difference in gains between the datasets is due to their particular characteristics, such as the number of inbred
583 lines and the PA reached. For the HEL dataset, however, PA was higher, and the costs of genotyping were also
584 higher, since there were a great number of inbred lines, so gains were lower; there was also an inversion of gains
585 between GET and GWT, where the efficiency of the GWT kernel remains practically constant while that of GET
586 falls. For the USP dataset it was the opposite; nevertheless, the PAs were lower, there were fewer individuals to
587 both genotype and phenotype. In this way, the costs per individual were lower and the gains were higher, and
588 GWT outperformed GET in all the scenarios. Altogether, GWT allowed reducing the TRN up to 58% compared
589 with GET. From this point of view, although the PA using GWT is the lowest, independent of the scenario, its
590 advantage can be offset by the gain in the response to selection per USD invested (see **Fig. 5**); giving special
591 attention to GWT in the USP dataset. In summary, compared with MTMET – CV2, with GWT there was a
592 reduction of up to 60% in terms of PA; however, it brings the possibility of substantially reducing the number of
593 plot:traits to be phenotyped up to 98%. Furthermore, using OTS plus W allows increasing the response to selection
594 per amount invested up to 142% compared with GET; thus, there is no gain in PA with OTS, but the reduction in
595 the training population greatly reduces costs and fieldwork, and thus, the relative genetic gains are greater.

596 In light of this, our results add to the subject of training populations, answering questions about which
597 design to use to distribute the population for evaluation, which individuals to choose to form the training
598 population, because as already seen, TRN is the key to the success of GS.

599 Multi-trait and multi-environment analyses have been applied as a way to optimize the distribution of
600 resources through GP, reaching satisfactory accuracy and gains; however, this scenario can still be worked on and
601 improved, taking advantage of new tools, like the environmental relationship matrix (W matrix) and genetic
602 algorithms (APY and LA-GA-T) to optimize the allocation of resources. To our knowledge, this is the first work
603 that tests the optimization of training set populations with genetic algorithms, which determine the size of the
604 population and select the individuals based on complex kernels, causing a high level of imbalance, and we
605 observed that, using a smaller optimized training set, we diminished the phenotypic evaluations in the field, and
606 consequently saved costs that can be reallocated for genotyping. Additionally, we calculated the gains per
607 10,000USD invested, which allowed us to infer that, in a practical way, by applying optimization and maintaining
608 a constant selection intensity, under a fixed budget, the input lines/hybrids of a breeding program can be larger, a
609 greater number of crosses can be tested per cycle and in the early stages, this will improve the gains. The initial
610 investments in GS are considerably high; however, they are offset by gains per unit of time. Nevertheless, it is
611 known that that the genetic variance of a given population decreases over the selection cycles, especially in small
612 populations sizes, which limits the selection gains (Muleta et al., 2019). Hence, we can consider the optimization
613 aiming to renovate training sets each year, to keep GS accuracy acceptable and raise the gains. Therefore, periodic
614 recalibration of the training population is important to endorse genetic variability and, in addition, when using an
615 optimized population for recalibration, the cost of evaluation (genotyping plus phenotyping) is offset by the

616 genetic gains obtained (Muleta et al., 2019). In summary, optimization gave a good balance between gain *versus*
617 costs, and between gain *versus* labor, and added new insight for using the algorithms tested here.

618 **5. CONCLUSIONS**

619 Genomic prediction models that include $G \times E$ and $G \times W$ interaction effects always increase PA,
620 performing better than main effects models; $G \times E$ interaction is the best scenario, with a small increase in multi-
621 trait multi-environment analysis when compared with single-trait multi-environment analysis. Furthermore,
622 genetic algorithms of optimization associated with genomic and enviromic data are efficient in designing
623 optimized training sets for genomic prediction and improve genetic gains per dollar invested. However, it is worth
624 remembering that there is a specific interaction between combinations of germplasm, environments, experimental
625 network and evaluated traits that must be taken into account when using the proposed approach.

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