

1 **Title:**

2 **Volume growth in animal cells is cell cycle dependent and shows additive fluctuations**

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15

16 **Abstract**

17 The way proliferating animal cells coordinate the growth of their mass, volume, and other relevant size
18 parameters is a long-standing question in biology. Studies focusing on cell mass have identified patterns
19 of mass growth as a function of time and cell cycle phase, but little is known about volume growth. To
20 address this question, we improved our fluorescence exclusion method of volume measurement (FXm)
21 and obtained 1700 single-cell volume growth trajectories of HeLa cells. We find that, during most of the
22 cell cycle, volume growth is close to exponential and proceeds at a higher rate in S-G2 than in G1.
23 Comparing the data with a mathematical model, we establish that the cell-to-cell variability in volume
24 growth arises from constant-amplitude fluctuations in volume steps, rather than fluctuations of the
25 underlying specific growth rate. We hypothesize that such “additive noise” could emerge from the
26 processes that regulate volume adaptation to biophysical cues, such as tension or osmotic pressure.

27

28 **Introduction**

29 The regulation of animal cell growth is a central question in cell biology¹⁻³, but our knowledge is limited
30 by the lack of methods to reliably measure cellular growth at the single-cell level. In the last decade,
31 several sophisticated approaches measuring buoyant mass^{4,5}, dry mass⁶⁻⁹ and volume^{10,11} have produced
32 new data revealing unexpected features at several levels. In particular, in contrast to what has been
33 observed in unicellular organisms such as *S. pombe*^{12,13}, *S. cerevisiae*¹⁴ or *E. coli*¹⁵, growth patterns of single
34 animal cells *in vitro* cannot easily be associated to a simple growth mode, such as mono-exponential,
35 linear or bi-linear. Instead, single animal cells show complex growth patterns that remain poorly
36 understood to date. Note that to avoid ambiguity we hereon call “growth speed” the time-derivative of
37 mass or volume (e.g., $\frac{dV}{dt}$) and “mass- or volume-specific growth rate”, the growth speed divided by mass
38 or volume (e.g., $\frac{1}{V} \frac{dV}{dt}$).

39

40 So far, studies on single animal cell growth have focused on patterns observed at timescales ranging from
41 several hours to a cell cycle. For HeLa cells, growth was reported to couple with cell size, thus contributing

42 to cell size homeostasis^{10,16}. These cells were reported to grow, while in G1, at a faster-than-average
43 volume-specific growth rate if they were born smaller than average¹⁰. This finding was paralleled by the
44 observation that inhibition of cell cycle progression or growth pathways has antagonistic effects on mass-
45 specific growth rate or cell cycle progression respectively¹⁷. A second type of growth pattern was
46 identified in studies measuring cell mass, and showed an association between mass-specific growth rate
47 and cell cycle progression or cell age. One study showed that in L1210 cells that undergo polyploidization,
48 mass-specific growth rate follows a bell-shaped dependency on mass over the course of each cell cycle,
49 independently of the increasing mass of the cell as ploidy increases¹⁸. This suggests that the bell-shape
50 pattern of mass-specific growth rate is a function of cell cycle progression, not mass itself. Two other
51 studies reported that the mass growth speed of several cell types displayed periodic oscillations. Although
52 the characteristics of the oscillations identified differ in the two studies, these results show that mass
53 growth follows an oscillatory pattern that depends on time since birth¹⁹ or time until division⁹.

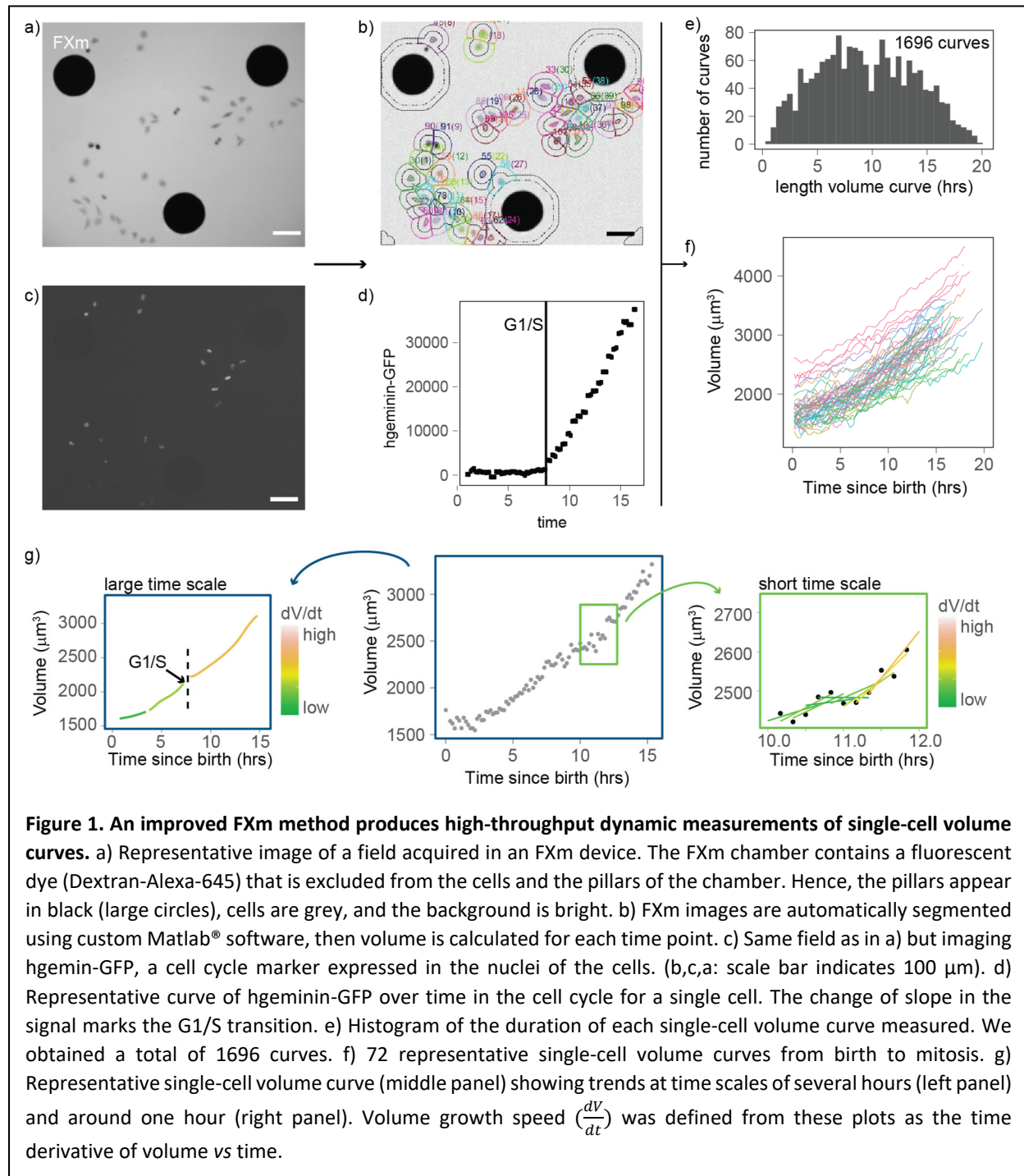
54
55 At shorter timescales (one hour or less) single adherent cells display cell volume¹⁰ and cell mass^{5,9} growth
56 trajectories that vary in time and across cells. These fluctuations have not yet been analyzed in detail, and
57 their origin remains poorly understood. Mass or volume changes in a given time interval are the combined
58 consequence of biosynthesis (via transcription and translation) and mechanisms that import or export
59 mass (import of molecules via endocytosis/exocytosis)²⁰ or volume (import of water via osmotic balance,
60 hydrostatic pressure, and membrane turnover)¹. Variability in growth can thus result from variability in
61 either or both categories of mechanisms. While seminal studies have revealed the origins of “noise” in
62 transcription in animal cells^{21,22}, the processes leading to noisy mass and volume growth in animal cells
63 still need exploration.

64
65 Crucially, with most of the above-cited studies focusing on cell mass, volume growth remains poorly
66 characterized, although it is clear that it follows independent patterns from cell mass throughout the cell
67 cycle^{11,23,24}. Regarding cell volume, open questions remain regarding both the identification of a mean
68 growth mode (e.g. linear or mono-exponential), and on the determination of the fluctuations around this
69 trend. While the study of mean trends is possible with medium size datasets (hundreds of cells), the study
70 of fluctuations in growth requires much larger datasets (thousands of cells), which remain difficult to
71 generate for animal cells. In a previous study¹⁰, we dynamically measured the volume of single adherent
72 animal cells using a fluorescence exclusion technique (FXm)²⁵. Our results indicated that, on average,
73 volume growth speed increases with cell volume but the data obtained were not sufficient for deeper
74 analysis. Here, we report an improvement of the FXm method that produces higher throughput in volume
75 readouts. We obtained a data set of around 1700 single-cell volume curves of HeLa cells, combined with
76 the tracking of key cell cycle transitions (birth, G1/S, and mitosis). The data show that volume-specific
77 growth rate depends on both cell cycle phase and cell volume. Our unprecedented statistical resolution
78 also allows us to investigate the variability in volume growth and to show that it arises from constant-
79 amplitude additive fluctuations of growth speed rather than from fluctuations of the specific growth rate.

80 81 **Results**

82 83 **An improved FXm method produces high-throughput dynamic measurements of single-cell volume**

84 To obtain high-throughput measurements of volume growth of single cells, we improved the
85 fluorescence-exclusion based measurement (FXm) of cell volume we previously developed^{11,25}. Briefly,
86 this method relies on seeding cells in chambers of known height in the presence of a fluorescent probe
87 (10 kDa dextran) that does not enter or harm the cell (Figure 1 a-b). Since the cell excludes the dye, the
88 measured fluorescence in the area containing a cell is negatively proportional to the volume of that cell.

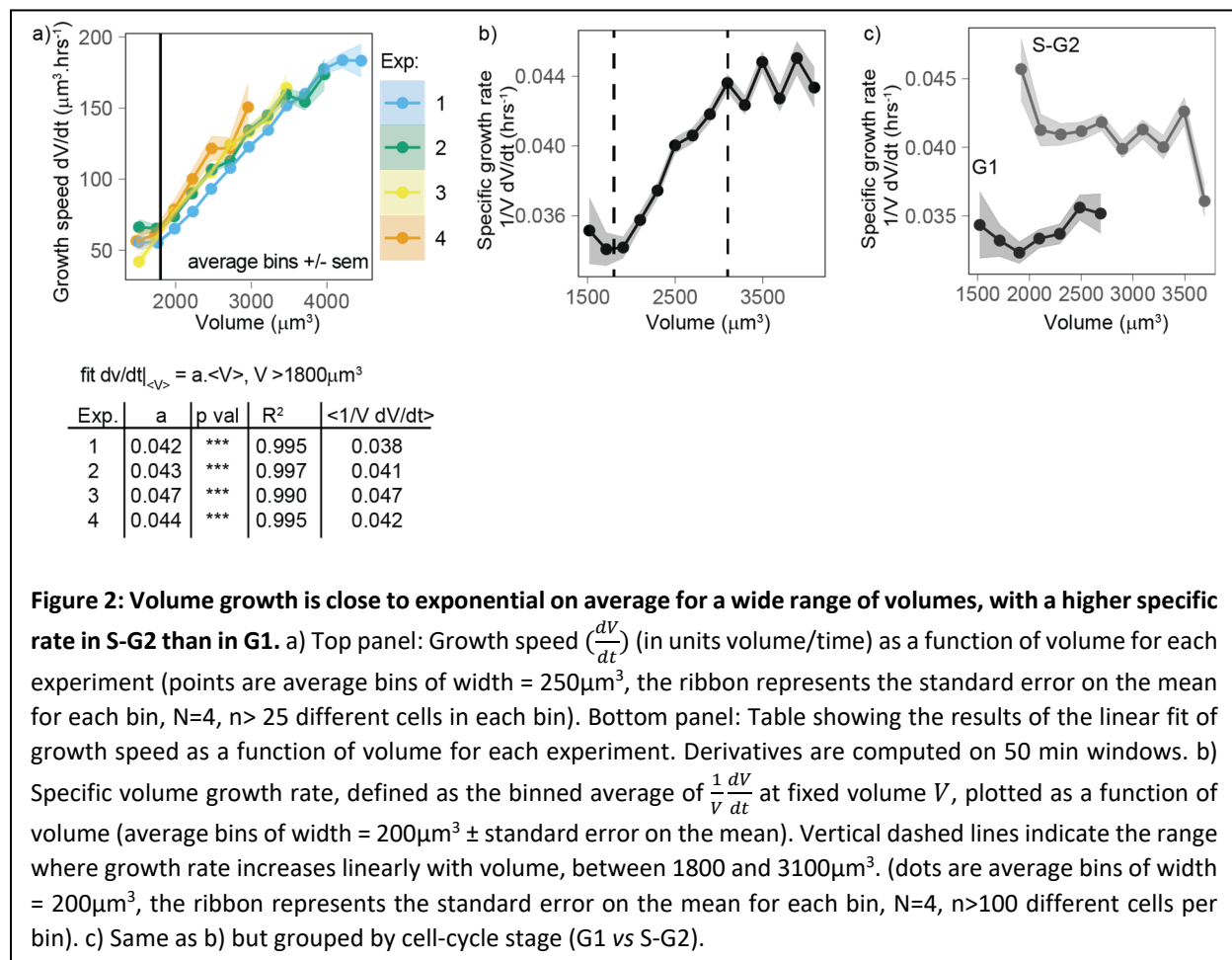


89 We combined these volume measurements with cell cycle phase analysis using the hgeminin-GFP part of
 90 the FUCCI system^{26,27} the expression of which marks S phase entry (Figure 1c). We performed four
 91 independent 24-hour-long experiments in which we imaged thousands of cells growing asynchronously in
 92 the FXm chambers (Figure 1 – supplement 1). To extract cell volume through time for each individual cell,
 93 we developed an automated cell tracking algorithm (Figure 1b) and we verified that the segmentation and
 94 lineage tracing (recording of mitotic events) were accurate by manual inspection of each single cell trace.
 95 Birth was defined as the onset of cytokinesis, the G1/S transition was defined as the onset of increase in

96 hgeminin-GFP intensity (Figure 1c) and mitosis was defined as the onset of the mitotic volume
 97 overshoot^{10,11,24}. Using this approach, we obtained 1696 verified single-cell volume trajectories that
 98 contained all available cell cycle information (time of birth, time of G1/S and/or time of entry into mitosis,
 99 Figure 1e-f). This high-quality, high-throughput measurement of animal cell volume was used to analyze
 100 the patterns and regulation of cell volume growth with unprecedented statistical resolution.

101 Volume growth is close to exponential for a wide range of volumes

102 First, we considered the growth mode of cells - a central question to understanding both cell growth and
 103 size homeostasis^{28,29}. Typical limit cases are linear or exponential growth. As previously reported for
 104 adherent cell types^{9,10}, single-cell trajectories show highly variable behavior (Figure 1g), making it difficult
 105 to associate them with any simple behavior. We turned to an alternative method based on population
 106 averages. We reasoned that, in the case of an average simple exponential growth model, growth speed
 107 should increase, on average, linearly with volume and the slope α followed by $\frac{dV}{dt}$ vs V (formally the trend
 108 of a conditional average) can be used to define an average volume-specific growth rate³⁰. All four
 109 experiments consistently showed that, for volumes higher than $1800 \mu\text{m}^3$ (and lower than $4000 \mu\text{m}^3$),
 110 growth speed increases on average linearly with volume, with a slope α that was very close to the
 111 (unconditional) average of $\langle 1/V dV/dt \rangle$ (Figure 2a). The agreement of these two different estimates
 112 supports the idea that average exponential growth describes these data well. All four experiments were



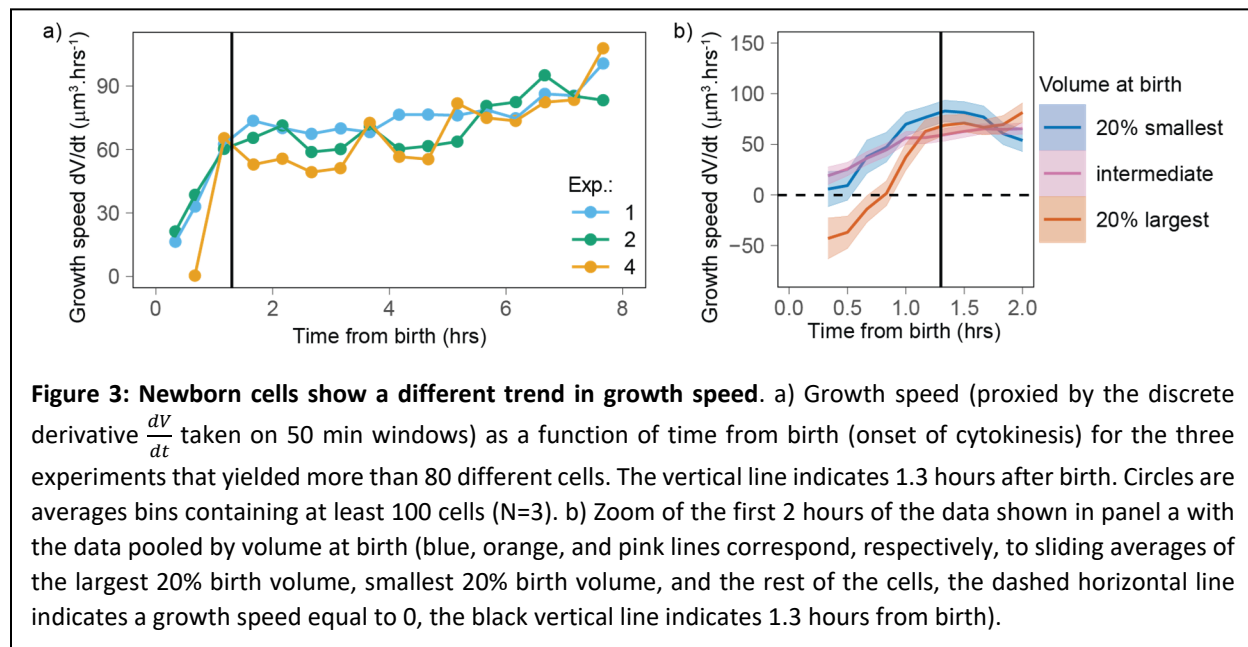
113 also very similar with values of α ranging from 0.038 to 0.047 h⁻¹. Thus, we conclude that volume growth
114 is faster than linear, and on average close to exponential in this range of cell sizes. Cells with volumes
115 below 1800 μm^3 did not follow the same trend, likely due to a different pattern of growth early in the cell
116 cycle (see below).

117 Specific volume growth rate depends on cell-cycle progression

118 When we considered in more detail an estimated volume-specific growth rate, defined by the conditional
119 average of $\frac{1}{V} \frac{dV}{dt}$ vs volume V (which has units 1/time), as a function of volume, we observed a slight but
120 significant increase with volume for cells between 1800 and 3100 μm^3 (Figure 2b). Outside this range, the
121 trend is more complex but the robustness of the observed behavior may be limited due to the lower
122 number of observations at these extreme sizes. We hypothesized that one potential cause of increase in
123 volume-specific growth rate during the cell cycle could be a cell-cycle stage dependency, which we could
124 test using data on the transition from G1 to S-G2 phase. To address this question, we repeated the plot of
125 volume-specific growth rate as a function of volume, also grouping the data by cell cycle phase (G1 vs S-
126 G2). This analysis shows that estimated volume-specific growth rate is nearly constant with volume for a
127 given phase, and growth rate in S-G2 is about 15% higher than in G1 (Figure 2c). These results show that
128 progression from G1 into S/G2 is accompanied by an increase in the volume-specific growth rate.

129 Newborn cells show a distinct pattern of volume growth

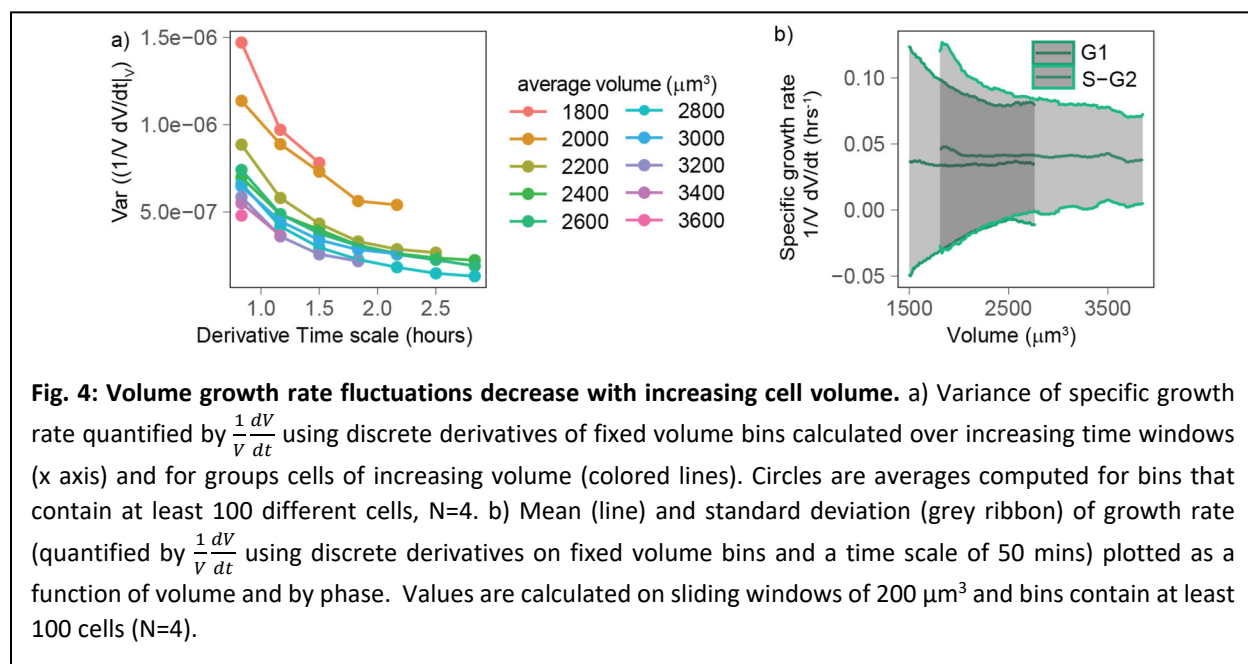
130 The observation that small cells show a different growth behavior than the rest of the cells (Figure 2a-b)
131 prompted us to test whether growing cells could follow different patterns early in the cell cycle. We
132 examined volume growth speed $\frac{dV}{dt}$ as a function of time from birth for the three experiments that had
133 more than 80 cells (to ensure that we had enough statistical power, even when analyzing each experiment
134 separately). In all three experiments, volume growth speed showed a fast increase during the first 1.3
135 hours after birth and increased more slowly and steadily after that point in time (Figure 3a). This suggests
136 that during the initial 1.3 hours after birth, cells follow patterns different from those followed during the



137 rest of the cell cycle. To gain more insight into the details of volume growth during this initial period, we
 138 pooled the three experiments together and grouped cells by their volume at birth. The largest cells at
 139 birth started their cell cycle with a negative growth speed (meaning that they were losing volume) during
 140 the first hour after birth. Small, intermediate, and large cells ultimately converged on the same growth
 141 speed at 1.3 hours after birth (Figure 3b). We note that in our data, birth is defined as the first time point
 142 of cytokinesis onset (a process which is then typically completed within 20-30 minutes¹⁰), thus the 1.3
 143 hour period comprises the end of cytokinesis as well as early G1 phase. The fact that volume follows a
 144 distinct pattern early in the cell cycle suggests a different mechanism of volume growth regulation as cells
 145 re-enter interphase.

146 Volume growth rate fluctuations decrease with cell volume

147 Our analyses so far indicate that, excluding the initial (Figure 3) stage of the cell cycle, volume growth is
 148 close to exponential for a wide range of volumes (Figure 2a), at a rate that changes with cell-cycle phase
 149 (Figure 2c). Next, since little is known about the cell-specific variations around this average behavior, we
 150 set out to evaluate the fluctuations, focusing on the range of volumes for which the growth behavior is
 151 well characterized (cells between 1800 and 3100 μm^3 (Figure 2a) and starting 1.3 hrs after birth (Figure
 152 3a)). Volume growth is the result of the combination of both biosynthetic pathways that act over the cell
 153 cycle and homeostatic pathways (that typically act at shorter time scales) that maintain a balance of
 154 cellular osmosis, hydrostatic pressure, and density^{1,31}. We first quantified the variability of volume growth
 155 by the variance of the specific growth rate, proxied by $\frac{1}{V} \frac{dV}{dt}$, for cells grouped in different volume bins, and
 156 we found that this variance decreases rapidly with the time scale dt over which one takes the discrete
 157 derivative (Figure 4a). Hence, the time scale must be specified for a meaningful comparison of the size of
 158 such fluctuations (for example across different studies). Moreover, we found that, at fixed time scale of
 159 the discrete derivative, the variance in specific growth rate decreases with volume (Figure 4a). Since this
 160 observation was robust across all derivative time scales, we decided to focus on the fluctuations of growth
 161 rate measured at the shortest accessible time scale (50 minutes, corresponding to 5 frames). When we



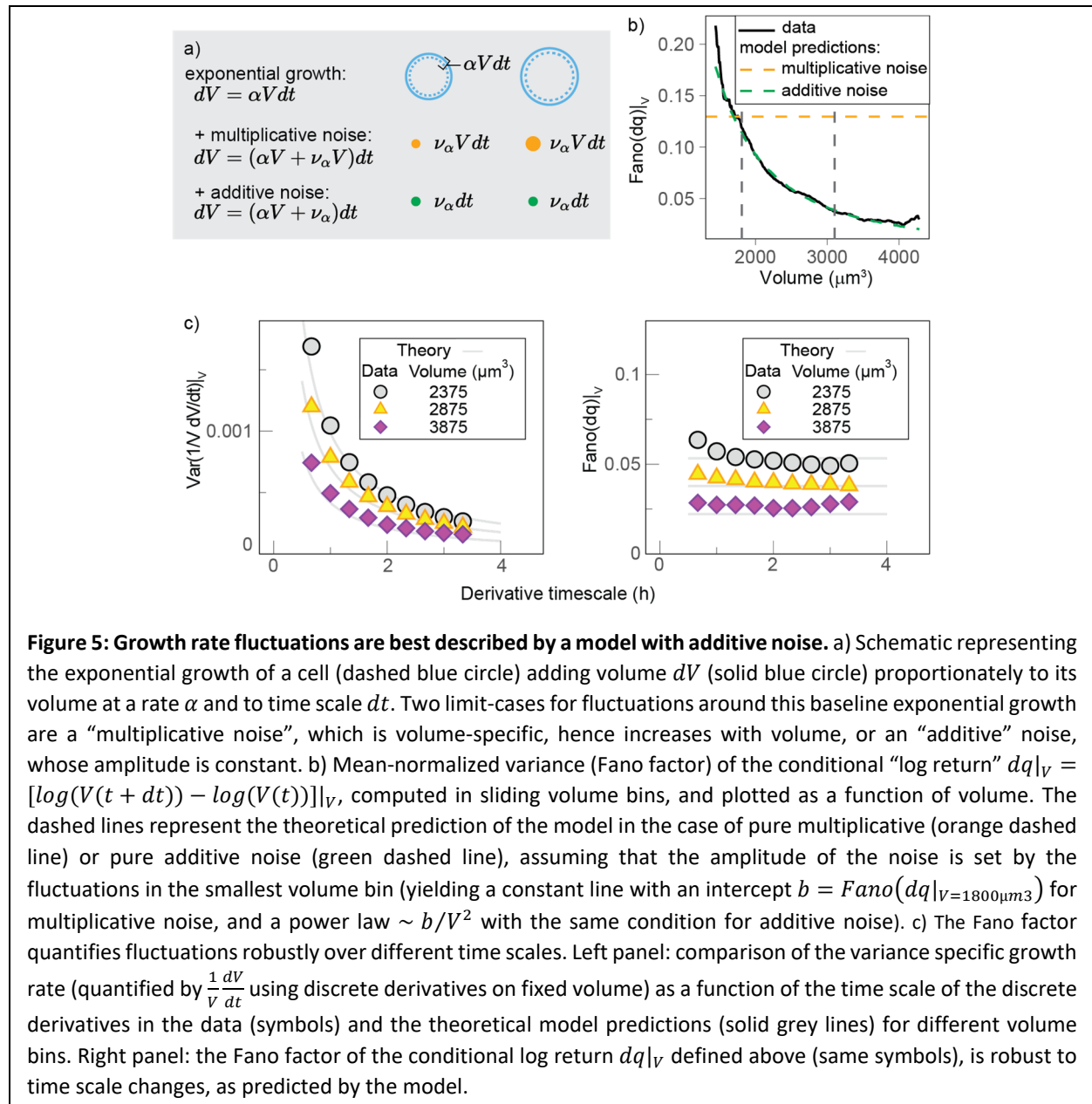


Figure 5: Growth rate fluctuations are best described by a model with additive noise. a) Schematic representing the exponential growth of a cell (dashed blue circle) adding volume dV (solid blue circle) proportionately to its volume at a rate α and to time scale dt . Two limit-cases for fluctuations around this baseline exponential growth are a “multiplicative noise”, which is volume-specific, hence increases with volume, or an “additive” noise, whose amplitude is constant. b) Mean-normalized variance (Fano factor) of the conditional “log return” $dq|_V = [\log(V(t + dt)) - \log(V(t))]|_V$, computed in sliding volume bins, and plotted as a function of volume. The dashed lines represent the theoretical prediction of the model in the case of pure multiplicative (orange dashed line) or pure additive noise (green dashed line), assuming that the amplitude of the noise is set by the fluctuations in the smallest volume bin (yielding a constant line with an intercept $b = \text{Fano}(dq|_{V=1800\mu\text{m}^3})$ for multiplicative noise, and a power law $\sim b/V^2$ with the same condition for additive noise). c) The Fano factor quantifies fluctuations robustly over different time scales. Left panel: comparison of the variance specific growth rate (quantified by $\frac{1}{V} \frac{dV}{dt}$ using discrete derivatives on fixed volume) as a function of the time scale of the discrete derivatives in the data (symbols) and the theoretical model predictions (solid grey lines) for different volume bins. Right panel: the Fano factor of the conditional log return $dq|_V$ defined above (same symbols), is robust to time scale changes, as predicted by the model.

162 plotted together the mean and standard deviation of specific growth rate as a function of volume (Figure
 163 4b), the standard deviation of specific growth rate clearly decreases with volume.

164 **Growth rate fluctuations are dominated by constant noise**

165

166 To better understand the observation of a reduction in growth rate noise with cell volume, we used a
 167 stochastic mathematical model describing cell growth (see Annex and Figure 4a). This model has a long
 168 history of applications outside biology^{32,33}, but it was recently proposed by Pirjol and colleagues³⁴ in the
 169 context of cell growth. The model considers that cells on average grow exponentially and describes
 170 fluctuations around this mean growth as noise:

171
$$\frac{dV}{dt} = \alpha V + \nu_\alpha V^\gamma$$

172 where α is the mean specific growth rate and v_α is a white noise term. Thanks to the factor V^γ , this
173 equation interpolates the limit cases of “additive” (constant amplitude) fluctuations and “multiplicative”
174 (volume-specific) fluctuations (Figure 5a, see also Annex A for details and Figure 5 – supplement 1a for
175 the validation with simulations). Central to the model is the definition of the parameter γ ($0 \leq \gamma \leq 1$) which
176 sets the relative weight between two kinds of noise: when $\gamma = 1$, we obtain: $dV/dt = (\alpha + v_\alpha)V$, and
177 the model describes multiplicative specific growth rate fluctuations (Figure 5b). These fluctuations are
178 symmetric with respect to a reference specific growth rate α , hence they can be interpreted as emerging
179 from noise in biosynthetic rates (e.g., surface synthesis, protein synthesis, etc.). When $\gamma = 0$, the model
180 describes an additive noise of constant amplitude, acting symmetrically on growth speed dV/dt , which
181 can be interpreted as resulting from any of the homeostatic processes that contribute to setting steady
182 state volume (e.g. homeostatic constraints of biophysical origin on hydrostatic pressure, osmotic
183 pressure, etc.). The model also allows for intermediate values of γ , which would effectively describe the
184 combined presence of additive and multiplicative noise sources on growth.

185
186 An important prediction of the model (Figure 5c) is that the mean-normalized variance of growth rate
187 (Fano factor) is not dependent on the time scale of the derivative. When we examined our experimental
188 data, we observed that this was indeed the case (Figure 5c). The Fano factor of growth rate depends on
189 γ , the mean specific growth rate α and constant σ as follows:

$$190 \quad Fano(dq)|_V = \frac{\sigma^2 V^{2(\gamma-1)}}{\alpha_q(V)}$$

191
192 Where α_q is the average log-return rate at fixed volume (see Annex A), and $dq|_V = [\log(V(t + dt) -$
193 $\log(V))]|_V$ is a conditional “log return” and quantifies the volume specific growth rate in presence of
194 possible multiplicative fluctuations. Importantly, other estimators of variability such as variance, SD or CV,
195 do depend on the time scale of the discrete derivatives, in both the model and the data.

196
197 We thus decided to use the Fano factor to estimate the variability in growth rate. We found that the Fano
198 Factor of volume growth rate (measured using the “log return” dq) decreased rapidly with volume and
199 that the trends measured in G1 and S-G2 overlapped well (Figure 5 - Supplement 1b). This suggests that
200 the decay in volume growth rate variability is independent of cell cycle phase. We thus pooled the two
201 phases together and compared the decay with our model predictions (Figure 5b). Comparison of the
202 model predictions for the two extreme cases ($\gamma = 1$ and $\gamma = 0$) and the data shows that the data are very
203 close to a scenario where $\gamma \approx 0$ and fluctuations are almost entirely additive (Figure 5b). Such additive
204 noise could arise from an extrinsic additive noise on volume measurement. To estimate the noise on
205 volume measurement, we compared the fluctuations on the detrended volume curves of cells and
206 neighboring background areas of same surface area (Figure 5 - supplement 2 a-b). The standard deviation
207 of the fluctuations was 3-4 times larger for cells than for the background areas, showing that fluctuations
208 of biological origins dominate over fluctuations of technical noise origin (Figure 5 -supplement 2c). Overall,
209 this analysis suggests that volume fluctuations come principally from a constant-amplitude noise in
210 instantaneously added volume, rather than from random variations of the volume-specific growth rate
211 itself.

212 213 **Discussion:**

214 Most recent studies on animal cell growth have considered the patterns of dry or buoyant mass^{9,35}. Here,
215 we complement this knowledge by providing a high-throughput dataset of volume trajectories in adhering
216 cells allowing us to compare the behavior of these different parameters.

217 During the first 1.3 hours following birth, volume growth displays a pattern where cells of intermediate
218 and small volume grow slowly while larger cells at birth lose volume. This is very different from the mass
219 pattern previously reported²³. Our volume curves exclude the period in mitosis where cell volume
220 transiently increase by 10-30% in volume called the mitotic volume overshoot^{11,24}. This 1.3 hours period
221 also spans both the period of completion of cytokinesis and the beginning of G1 phase, ruling out the
222 hypothesis that mitosis alone determines the early volume growth pattern. Our observations instead
223 point to the existence of a previously overlooked period early in the cell cycle where several processes
224 such as the change in cell shape due to the spreading of the cell after mitotic rounding^{36,37}, and the onset
225 of cell growth in early G1 compete to determining volume growth. It is possible that this period also
226 corresponds to an extreme case where volume and mass growth rate, which are very different during
227 mitosis^{11,23,24}, progressively ‘tune in’ before they reach a common steady behavior.

228 Throughout the cell cycle, mass growth has been reported to oscillate periodically in HeLa cells^{9,35}. We
229 find that this phenomenon is not simply reflected by cell volume. We found some oscillations in volume
230 growth speed as a function of time from birth only in birth volume outliers, and not matching the period
231 and amplitude of the mass oscillations reported previously.^{9,35} (Figure 3 – Supplement 1). This finding
232 suggests that dry-mass biosynthesis and volume growth, while being interrelated, can be independent in
233 specific phases of the cell cycle. Studies combining measurements of cell shape, mass, and volume at high
234 time resolution will be particularly important to clarify the complex interplay between these parameters.

235 The high number of curves in our study allows us to investigate systematically the fluctuations in volume
236 growth. Our observation that volume growth shows additive fluctuations counters the idea that the
237 volume specific growth rate itself fluctuates (Figure 5a-b). What could be the origin of such additive noise
238 and how can we explain the absence of noise on the rate itself? Our controls show that extrinsic
239 experimental noise only minimally contributes to the additive fluctuations we measure (Figure 5 –
240 Supplement 2). Intrinsic sources of additive fluctuation could arise from noise in the processes involved
241 in volume regulation. Cell volume is a physical parameter that results from an equilibrium of cell
242 hydrostatic pressure, osmolarity and membrane tension at time scales of minutes to hours¹ and much less
243 understood mechanisms that maintain cell density^{31,38–41} by coupling mass and volume at timescales of
244 several hours. In a scenario where volume is unidirectionally coupled to mass, even if mass growth rate is
245 noisy⁴², one could potentially obtain an apparent⁴² volume growth rate with fluctuations that are only due
246 to the coupling. None of the available studies considering cell mass has addressed the question of whether
247 mass biosynthesis fluctuations are themselves mass specific.

248 Finally, our analysis shows that growth-rate variability, quantified by variance, SD, or CV, is strongly
249 dependent on the time scale used to evaluate discrete derivatives (Figure 4a, Figure 5c). This result poses
250 an important caveat for the quantitative comparison of growth rate fluctuations across different studies,
251 as absolute values of growth rate fluctuations evaluated in different ways and on different time scales (or
252 smoothing windows) may strongly differ. In general, single-cell growth studies are currently limited by the
253 development of theoretical tools that could quantify the contribution of the different determinants of
254 growth such as size, time, and cell cycle phase that act at different time scales. These tools, together with
255 experimental approaches that allow the combined measurement of several size parameters (mass,
256 volume) concomitantly are needed to further elucidate the growth patterns of animal cells.

257

258 **Materials and Methods**

259 **Cell line and cell culture.**

260 HeLa cells expressing hgemin-GFP were a kind gift from Buzz Baum's lab (UCL, London, United Kingdom).
261 Cells were cultured in DMEM-Glutamax media and imaged in DMEM without phenol red, supplemented
262 with Glutamax. Both media were supplemented with 10% FBS and 1 % penicillin-streptomycin.

263

264 **Volume measurement with FXm**

265 The detailed protocol for FXm is described previously²⁵ and the design is described in ref.¹⁰. Briefly,
266 measurement chambers were replicated in PDMS (crosslinker:PDMS, 1:10). To prevent leakage of the
267 fluorescent dextran from the chamber, 4mm high PDMS cubes were stuck on top of the inlets before
268 punching 2mm diameter holes for every inlet. The chamber was then irreversibly bound to the 35 mm
269 diameter glass-bottom fluorodishes[®] by plasma treatment, coated with fibronectin 50 ($\mu\text{g}.\text{mL}^{-1}$), for about
270 30 min, then rinsed and incubated in phenol-free media overnight. To ensure that cells were in a similar
271 growth phase when starting an experiment, cells were seeded at constant density ($1.9 \times 10^4 \times \text{cm}^{-2}$) two days
272 prior to the experiment. The day of the experiment, cells were detached by incubating with EDTA for 15-
273 20 min, recovered and seeded at intermediate density in the measurement chamber (Figure 1a). 4 hours
274 after seeding, the media was replaced with equilibrated media containing 1 $\text{mg}.\text{mL}^{-1}$ of 10 kDa Dextran
275 Alexa-645. Imaging started 2-4 hours after changing the media. While imaging, cells were kept at 37°C
276 with 5% CO₂ atmosphere. Imaging was performed on an inverted epifluorescence microscope (Ti inverted
277 (Nikon) or DMI8 inverted (Leica)) equipped with a LED excitation source. Images were acquired with a
278 CoolSnap HQ2 camera (Photometrics) or an ORCA-FLASH4.0 camera (Hamamatsu). Images were
279 obtained using a low magnification (10X), low numerical aperture objective (NA=0.3, phase) every 10 min
280 (FXm measurement) and 30 min (hgeminin-GFP imaging).

281

282 **Software analysis**

283 To extract cell volume and cell cycle information from the images we used a custom-made Matlab
284 software²⁵. The software contained an image analysis algorithm previously optimized²⁵ that performed
285 successive image treatments to normalize the background intensity and correct for background
286 inhomogeneity (for example due to an inhomogeneous light source). The algorithm then segmented the
287 pillars and background to calibrate the fluorescence intensity signal using: (i) the average background
288 intensity to calculate I_{max} , (ii) the average intensity under the pillars in the chamber to calculate I_{min}
289 and (iii) the known height of the chamber. The software then segmented and tracked single cells (Figure
290 1b) throughout the duration of the movie. If a cell divided, the event was recorded and the lineage tree
291 for that cell recorded. Finally, the cell volume and hgeminin-GFP intensity was calculated over the
292 segmented area for each cell and each movie frame.

293

294 The background normalization algorithm required the user to manually set several parameters. To ensure
295 that these parameters were accurately chosen, a graphical user interface allowed the user to visualize the
296 results of the image treatment given a set value for each parameter and assess its validity. This allowed,
297 after a few trial and errors, setting a set of parameter values that were optimal for each set of movies
298 obtained in the same FXm chamber. There were four steps to set these parameter values. First, to detect
299 the pillars and later calculate I_{min} , the user manually set a threshold that segmented the pillars (from 0
300 to 1 on a normalized image), the user also set a 'distance of influence' around each pillar which consisted
301 in an area larger than the pillar where cell volume would not be calculated to prevent potential artefacts
302 of volume calculation due to a shadow caused by the pillars (see ref.²⁵). Second, to estimate the
303 background and later calculate I_{max} , the user chose a threshold to detect the cells (from 0 to 1 on a
304 normalized image) and set the parameter called 'noise factor'. Third, using these parameter values, the
305 image treatment algorithm was applied to the image. Fourth, on the resulting renormalized image, the

306 user set the parameter values for cell segmentation and tracking: the average size of the mask and the
307 threshold value to detect the cell, the maximum moving distance for a cell from one image to the next,
308 the radius around each detected cell (to prevent measuring cells that were too close to each other), the
309 minimum cell size and a parameter 'sigma' that represented a threshold allowing splitting of an object
310 into two distinct object (for example after cell division or when two cells are near each other).

311
312 During the optimization phase of the analysis pipeline, we re-ran the analysis on the same movies using
313 different parameter values to test the robustness of the volume calculated to variabilities in user-defined
314 parameter values. The volume curves obtained were very similar and indicated that errors in volume
315 measurement due to variability in parameter settings were negligible. Once the algorithm parameters
316 were set, the software processed hundreds of movies in batch mode using parallel processing to increase
317 the speed of processing. The analysis of a set of movies coming from one experiment took 2 to 3 full days
318 of computer processing. At the end of this step, we obtained hundreds of movies showing the tracking
319 results (Figure 1c). Each movie was then visually checked to correct any errors of segmentation.

320

321 **Visual assessment and manual curation of the single cell tracks**

322 Each single-cell trajectory was visually checked. We verified that the segmentation and lineage tracing
323 (recording of mitotic events) were accurate. During this manual curation, we visually assessed and noted,
324 for each cell: (i) potential frames when the volume calculated should be excluded from the analysis due
325 to a segmentation error, (ii) if the cell divided, the frame when the cell started rounding and the frame
326 when the first evidence of cytokinesis occurred. We also checked for and corrected mistakes in the lineage
327 tracking. For the first experiment analyzed, we also noted any frames where a cell was near another cell
328 (typically right after birth when the two daughter cells are near each other or later when two cells bump
329 into each other). We then checked that the presence of a neighboring cell was not affecting the volume
330 curve in an obvious way. Since it did not, we stopped tracking this information for the subsequent
331 experiments. This visual assessment and manual curation, although time consuming, was essential to our
332 analysis because it increased our confidence that any fluctuation seen on the resulting volume curve was
333 not due to an identifiable artefact. The resulting volume curves and hgeminin-GFP signal were then
334 imported into R. Using a graphical user interface, each curve was plotted and the user manually selected,
335 for each cell: (i) the beginning and the end of the mitotic volume overshoot^{10,11,24} on the volume curve and
336 (ii) the point of increase in hgeminin-GFP intensity indicating G1/S transition (Figure 1d).

337

338 **Volume curve smoothing and calculation of growth speed**

339 To then get into the analysis of volume growth speed and growth rate fluctuations, we developed a
340 cleaning and algorithm that would filter out the clear outliers and smooth fluctuations that are within the
341 noise of our measurement technique. Several algorithms were tested, each time checking visually the
342 resulting comparison of the raw volume measurement with the smoothed, filtered curve. The final
343 algorithm selected worked in two steps. First, to filter out clear outliers, a histogram of values on sliding
344 windows of 11 frames was established and the fourth quantile (Q_4) and interquartile range (IQR) of that
345 distribution were calculated. Then, points that were above or below $Q_4 \pm 0.9 * IQR$ were removed from
346 the volume curves. Second, a smoothing algorithm based on centered averages on sliding windows of 3
347 frames was applied. To calculate growth speed ($\frac{dV}{dt}$), local robust linear fits of single cell volume curves as
348 a function of time were performed on sliding windows of 5 frames (all figures except Figure 4a where the
349 fits were performed on increasingly long windows of time). The slope coefficient of the fit corresponds to
350 the instantaneous growth speed. We compared this approach to calculating the discrete time derivative
351 by plotting the resulting local fits from both methods and visually concluded that the robust linear fit
352 method gave a more faithful representation of the curve fluctuations.

353

354 **Model and simulations**

355 The model and analytical calculations to estimate cell-to-cell variability are presented in Annex A. These
356 calculations were compared with simulations based on a discrete-time realization of the Langevin
357 equation defining the model (See Figure 5 - supplement 1).

358

359 **Statistical analysis**

360 All figure generation and statistical analysis was performed in R. Packages used were 'ggplot2', 'gridExtra',
361 and 'robustbase'.

362

363

364 **Authors contributions**

365 C.C. and M.P. designed the experiments, C.C. and M.C.L. designed the data analysis, C.C. performed
366 experiments and data analysis, M.C.L. and C.C. worked on model and simulations, and performed model-
367 related data analysis. C.C., M.C.L. and M.P. wrote the paper.

368

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377

378 **Competing interest**

379 The authors declare no financial and non-financial competing interests.

380

381 **Software**

382 Matlab® (MathWorks, Natick, Massachusetts, USA) was used for the image analysis (version R2018a or
383 later).

384

385

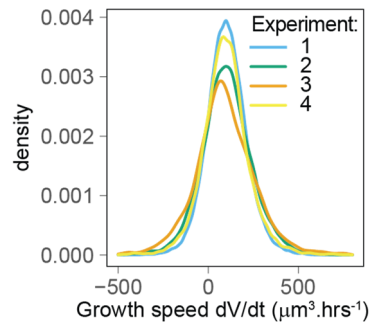
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389 **Supplementary figures**

390



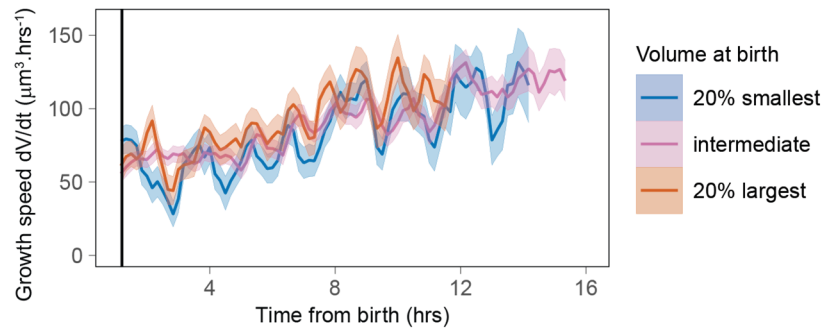
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392 **Figure 1 – supplement 1: The distributions of growth speed show good agreement between the 4 experimental**
393 **replicates.** Experiment 1: mean \pm SD=102 \pm 111 $\mu\text{m}^3 \cdot \text{h}^{-1}$, n=50708; experiment 2: mean \pm SD=102 \pm 142 $\mu\text{m}^3 \cdot \text{h}^{-1}$,
394 n=23739; experiment 3: mean \pm SD=101 \pm 196 $\mu\text{m}^3 \cdot \text{h}^{-1}$, n=4570; experiment 4: mean \pm SD=96 \pm 138 $\mu\text{m}^3 \cdot \text{h}^{-1}$, n=12494.

395

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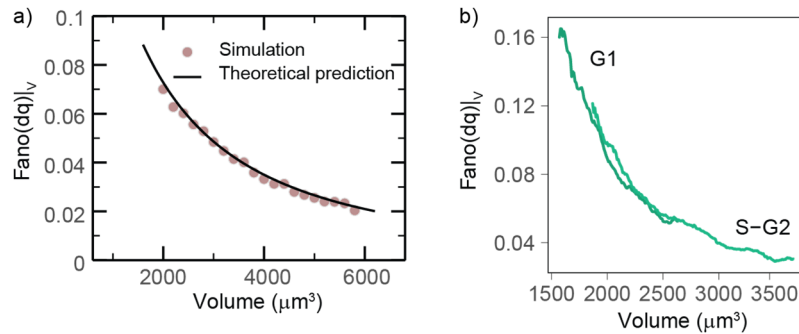
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398

399 **Figure 3 - supplement 1: Growth speed as a function of time from birth shows time periodicity for the largest and**
400 **smallest cells at birth.** The solid lines represent averages (each bin contains at least 100 cells) and the ribbon the
401 standard error on the mean. Black vertical line indicates 1.3 hours after birth (N=4).

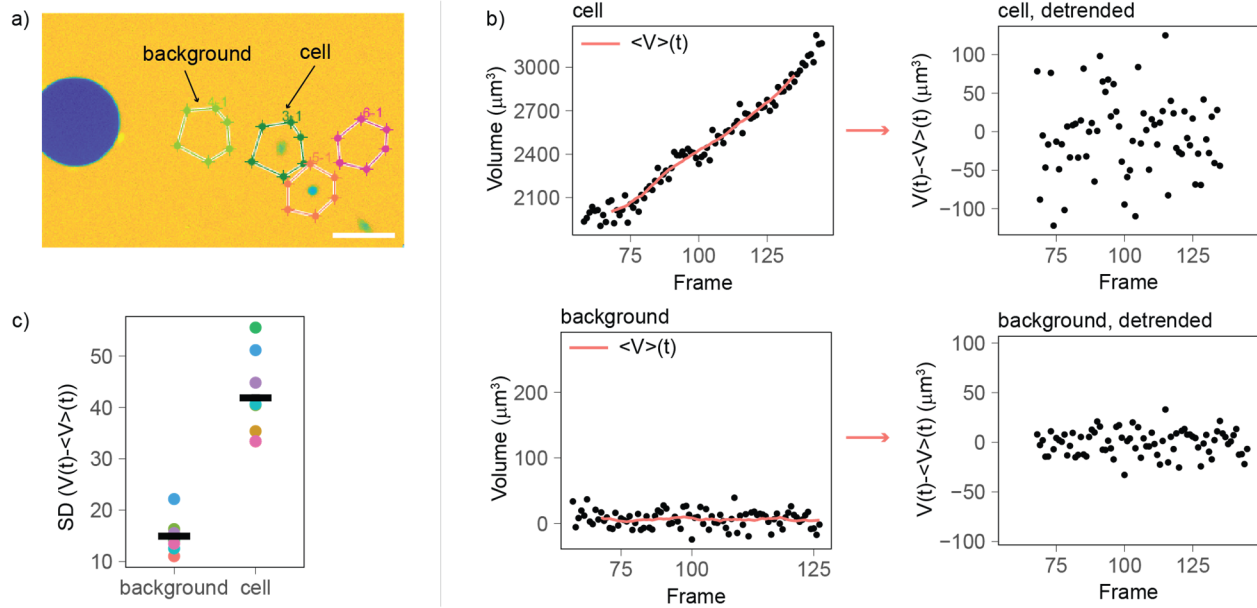
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403

404 **Figure 5 – supplement 1: Validation of the model.** a) Theoretical model predictions agree with direct simulations of
405 the stochastic exponential growth model. The plot compares simulations (circles) with the theoretical predictions
406 (lines) for the Fano factor of the conditional log return (the logarithmic size increment at fixed volume) from the
407 same set of parameters. Model parameters were derived from a fit of the pulled experimental data from
408 experiments 1 and 2 in the volume range 1800-4000 μm^3 : $\alpha=0.038.\text{h}^{-1}$, $\sigma = 41.5(\mu\text{m}^3)^{1-\nu}.\text{h}^{-(1/2)}$, $\nu=0.12$. b) Mean-
409 normalized variance (Fano factor) of the conditional “log return” $dq|_V = [\log(V(t + dt)) - \log(V(t))]|_V$,
410 computed in sliding volume bins, and plotted as a function of volume and phase.

411



412

413 **Figure 5 – supplement 2: Comparison of the volume fluctuations on cells (biological fluctuations) and background**
414 **areas (measurement noise).** a) Image of the volume analysis software showing two cells and two background areas
415 of same surface area (scale bar: 100 μm). b) Left: For each cell and background area, volume was calculated over
416 time, and an average volume trend computed over sliding windows of 10 frames was calculated (solid red line).
417 Right: the average volume trend was subtracted from the volume curve to obtain a detrended volume curve. c) To
418 estimate the volume fluctuations around the mean detrended curves, the standard deviation was calculated for each
419 cell (6 total) and corresponding background areas (colors of the dots correspond to a pair of cell and matching
420 background area). The volume fluctuations measured for cells are about 3-4 times higher than those measured on
421 background areas, suggesting that fluctuations of biological noise origin dominate of fluctuations of technical noise
422 origin.

423

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530
531

Annex A: Stochastic model of exponential growth with noise

This annex describes the stochastic growth model developed to interpret the reduction of noise on growth rate as a function of volume (Figure 5). We compared our data with the stochastic growth model of ref. 9. Following this study, we write

$$\begin{aligned} \frac{dV}{dt} &= \alpha V + \nu_\alpha V^\gamma, \\ \gamma &< 1. \end{aligned} \quad (1)$$

This model interpolates the regimes of additive noise ($\gamma = 0$) from fluctuations of biophysical nature and corresponds to purely multiplicative noise ($\gamma = 1$), corresponding to constant-amplitude fluctuations of the specific growth rate around its typical value α . The intermediate case $\gamma \in [0, 1]$ effectively describes a combination of biophysical noise and growth rate fluctuations. Here we assume that ν_α is a zero-mean delta-correlated Gaussian white noise, i.e. $\langle \nu_\alpha(t) \rangle = 0$, and $\langle \nu_\alpha(t) \nu_\alpha(t + \tau) \rangle = \sigma^2 \delta(\tau)$.

This model was used to compute analytically different proxies of growth-rate fluctuations, which we used to describe the data. We also verified the agreement of our theoretical predictions with direct numerical simulations of Eq. (1).

For $\gamma < 1$. Let us look at the expected rate and the rate of the mean in this case. Averaging Eq. (1) one has that

$$\left\langle \frac{dV}{dt} \right\rangle = \alpha \langle V(t) \rangle,$$

because the average of the noise is zero. Hence we always have that

$$\frac{1}{\langle V \rangle} \left\langle \frac{dV}{dt} \right\rangle = \alpha.$$

The growth rate in the model can be estimated as the conditional average of the speed at fixed volume. Indeed, since

$$dV = \alpha V dt + \sigma V^\gamma dW,$$

One immediately has that

$$\frac{1}{V} \left\langle \frac{dV}{dt} \right\rangle \Big|_V = \alpha.$$

Considering the variance, with some straightforward algebra one can obtain from Eq. (1) the expression

$$\text{Var} \left(\frac{dV}{dt} \right) \Big|_V = \frac{\sigma^2 V^{2\gamma}}{dt}.$$

This expression is compared to data in Figure 5c.

Defining $q = \log(V)$, one has that

$$dq = \frac{dq}{dV} dV + \frac{d^2q}{dV^2} dV^2 + \dots,$$

hence

$$dq|_V = \left(\alpha - \frac{\sigma^2}{2} V^{2\gamma-2} \right) dt + \sigma V^{\gamma-1} dW := \alpha_q dt + \sigma V^\beta dW, \quad (2)$$

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$$\alpha_q = \alpha_q(V) = \left(\alpha - \frac{\sigma^2}{2} V^{2\beta} \right) .$$

From Eq. (2), we compute with some algebra the Fano factor of dq (variance over mean), conditional to volume, which is not dependent on the time scale dt ,

$$\text{Fano}(dq)|_V = \frac{\sigma^2 V^{2\beta}}{\alpha_q(V)} .$$

This expression is compared to data in Figure 5c.