

1 Humans use minimum cost movements in a whole-body 2 task

3 Lijia Liu^{c,b}, Dana Ballard^c

4 *^aDepartment of Computer Science, The University of Texas at Austin, Austin, TX,
5 78712, US*

6 *^bLead Contact*

7 *^cCorresponding Author Email: lijialiu@cs.utexas.edu*

8 **Abstract**

9 Humans have elegant bodies that allow gymnastics, piano playing, and
10 tool use, but understanding how they do this in detail is difficult because
11 their musculoskeletal systems are extraordinarily complicated. Nonetheless,
12 although movements can be very individuated, some common movements like
13 walking and reaching can be stereotypical, with the movement cost a major
14 factor. A recent study has extended these observations by showing that in
15 an arbitrary set of whole-body movements used to trace large-scale closed
16 curves, near-identical posture changes were chosen across different subjects,
17 both in the average trajectories of the body's limbs and in variations within
18 trajectories. The commonality of that result motivates explanations for this
19 generality. One could be that humans also choose trajectories that are eco-
20 nomical in energetic cost. To test this hypothesis, we situate the tracing
21 data within a fifty degree of freedom dynamic model of the human skele-
22 ton that allows the computation of movement cost. Comparing the model
23 movement cost data from nominal tracings against various perturbed trac-

24 ings shows that the latter are more energetically expensive, inferring that
25 the original traces were chosen on the basis of minimum cost. Moreover,
26 the computational approach used to establish minimum cost principle sug-
27 gests a refinement of what is known about cortical movement representations.

28

29 **Keywords:** Posture analysis, whole body movement, virtual tracing, kine-
30 matic representation, movement variation costs

31 **Author Summary**

32 Although motor cortical areas have been extensively studied, their basic
33 response properties are still only partially understood, and it remains con-
34 troversial whether neural activity relates to muscle commands or to abstract
35 movement features. We provide a new perspective of how movements may
36 be resented in the brain by showing that humans chose trajectories with
37 minimum energy cost while accomplishing goal-directed tasks. Furthermore,
38 most of the current neural control studies are experimental. Our compu-
39 tational methodology coupled with a minimum energy principle suggests a
40 refinement of the brain’s storage of remembered movements.

41 **1. Introduction**

42 Advances in the speed of computing and novel formulations of the dy-
43 namic equations of motion have engendered a new methodology for un-
44 derstanding human movement fundamentals. Large-scale human musculo-
45 skeletal models have be built with the objective of understanding human
46 real time goal-oriented behaviors [1, 2]. These newer models linearize the
47 dynamic equations and use feed-forward integrations that are much better
48 conditioned than previous methods.

49 However, including all the complexity of the human musculoskeletal sys-
50 tem, with over 600 muscles controlling a complex skeletal system with over
51 300 degrees of freedom can be daunting, espicially if the goal is to generate
52 movements as compared to analyze their properties. Moreover, to achieve its

53 control complexity, the brain coordinates several cooperating neural subsys-
54 tems. In addition to its vast cortical motor memory system, the forebrain
55 coordinates specialized subsystems such as the Basal Ganglia, and Thalamus
56 and Cerebellum in realizing continuous real-time movement [3]. The upshot
57 is that research progress tends to be specialized [4] and there are many open
58 problems [5].

59 In the face of these challenges, one modeling route is to forego the level
60 of detail that includes muscles and model more abstract versions of the hu-
61 man system that still use multiple degrees of freedom but summarize mus-
62 cle effects through joint torques. The computation of the dynamics of such
63 multi-jointed systems recently has also experienced significant advances. The
64 foremost of these, use a kinematic plan to directly integrate the dynamic
65 equations. Several different systems exist, such as MuJoCo, Bullet, Havok,
66 ODE and PhysX, but an evaluation by [6] found them roughly comparable
67 in capability, and only MuJoCo [7] has been applied to human modeling.

68 Thus there is a place for a exclusively human movement based model
69 that could be used to inform laboratory experiments[8], clinical studies e.g
70 [9] and also verify experiments that have only qualitative results[10, 11]. Our
71 human dynamic model has a singular focus on human movement modeling
72 and features a unique approach to integrating the dynamic equations. We
73 have developed a direct dynamics integration method to extract torques from
74 human subjects in real time [12, 13, 14] based on a unifying spring constraint
75 formalism.

76 Our focus is the principles behind *large-scale arbitrary movements*, partic-
77 ularly with respect to variations between different subjects. Thus we eschew
78 common movements such as reaching and walking [15, 16, 17] and also stud-
79 ies of small-scale grasping movements [18, 19]. Another peripheral issue is for
80 us that that many movement tasks can have objectives that discourage low
81 energetic solutions but can be readily analyzed with decision-making tech-
82 niques [20, 21, 22, 23] that focus on repeatability; movements are committed
83 to memory with precedence based on the probability of use.

84 Our experimental setting starts with measuring the kinematics of a move-
85 ment. The model divides anatomical parts into discrete segments that have
86 their own inertia and are interconnected to other segments by joints that
87 are mostly rotary. Thus a movement can be described as the time course
88 of the coordinates of the joints. The The model’s state is indexed by fifty
89 three-dimensional coordinates of a motion capture suit. The time course
90 of these coordinates provides an equivalent representation of a movement’s
91 kinematics.

92 To refer to the kinematics at a specific time we use the term *posture*.
93 Classically, posture classically is used for particular poses such as sitting or
94 standing, but we use it for arbitrary body orientations.

95 Although computing torques in the inverse dynamics using kinematics is
96 an advance, the study of the kinematics of arbitrary body movements is in
97 itself challenging to study owing to their variation. Bernstein’s famous well-
98 known phrase characterizing repeated movements in terms of “repetition

99 without repetition,” emphasizes that repeated movements are never exactly
100 the same [24]. However repeated movement variations are never completely
101 random. Informed by task goals, subjects can shape the variations in different
102 parts of the body by co-contracting muscles to achieve desired dynamics in
103 different sections of a trajectory [25]. Thus in looking for regularities in
104 movements one has to deal with both that the trajectories will vary owing
105 to muscle co-contraction and that the amount of co-contraction itself can be
106 modulated throughout the movements.

107 These variations, we developed specialized aggregation methods for data
108 analysis that extracted similarities of posture sequences in the face of kine-
109 matic variations [26]. The task studied had subjects tracing large-scale three
110 dimensional curves in virtual reality that required a series of whole-body
111 movement sequences. Subjects could freely choose their starting posture
112 and also were given no instructions as to how to comport themselves during
113 the tracing process. Their postures were continuously recorded using the
114 motion-capture system.

115 The main result was that although the locations tracing data exhibits pos-
116 ture variations, both in repeated of a single subject and in trials by different
117 subjects, the average postures show marked regularities in several aspects of
118 the data that was subject to analysis. A t-test between a proximal relative
119 posture and distal relative posture showed that the difference is significant
120 at the 0.0001 level. Also, the variances in the subjects’ postures were cor-
121 related. If at a point on the curve the variance of a trace calculated from a

122 subject was relatively large, the average of the variance of all the repeated
123 trials from all subjects would be relatively large also.

124 The obvious inference from all the observed common movements is that
125 energetic cost should be similar and moreover, these observations arise from
126 a minimum cost principle. To test this hypothesis we computed the cost
127 of dynamic models of different subjects' curve traces and compared these
128 results with the cost of tracing under two different perturbations. In one,
129 the trajectories' cost were computed with small perturbations in the model
130 kinematic positions. In the other the original curves that were displaced in
131 five-centimeter increments. The result of both of these comparisons was that
132 the means the energetic cost of were higher than those of the original curve.
133 These results strongly suggest that that movements can be selected on the
134 basis of predicted minimum cost.

135 The human system has a broad dichotomy into a lateral system, which
136 includes the cortical component commands, and a medial system, which in-
137 cludes the vestibular component commands. Crudely one can think of the
138 lateral system as handling predictions and the medial feedback system han-
139 dling feedback.

140 Our model also exhibits these two components. The dynamic calculations
141 are good enough to handle the majority of the torques required, but various
142 inaccuracies in the model require a residual torques such as that would be
143 produced by a vestibular system. Our residual system, is unsophisticated as
144 described in the sections section, but it does an essential job in achieving

145 balance.

146 **2. Background**

147 A general principle of human movement is that our nervous system prefers
148 trajectories that are economical in energetic cost [27, 28]. It has been estab-
149 lished for decades and has been well studied. For example, in locomotion,
150 there are a number of experiments showing that humans' walking speed [29],
151 step frequency/length [30, 31, 32, 33, 34, 35, 36], and step width [37, 38]
152 are all corresponding with the minimum metabolic cost, e.g., energetic cost
153 exhibits a U-shaped dependence on step frequency while walking at a con-
154 stant speed and the minimum of the U-shape curve is consistent with the
155 self-selected or preferred walking frequency [17, 34]. Furthermore, new ev-
156 idences [39, 40, 41] show the nervous system can adapt preferred gaits to
157 minimize energetic cost.

158 In the past, a common way to address this minimization principle was to
159 conduct experiments comparing walking or running with many other strange
160 and unpractised gaits [42, 43]. Nowadays, there are three commonly used
161 methods to study energy optimization.

162 The most straightforward and frequently used method is to measure the
163 metabolic cost, e.g., subjects breath through a mouthpiece to measure rates
164 of oxygen consumption (VO_2). For example, subjects were required to walk
165 under different circumstances, and the results showed that the metabolic cost
166 was minimum while subjects walked at the condition which was "comfort-

167 able” for them [29, 30, 31, 32, 39, 40, 41].

168 Measure the changes in muscle coactivation and stiffness using Elec-
169 tromyographic (EMG) is considered a common way to reflect metabolic
170 changes. An experiment [44] proved that that subjects’ metabolic cost re-
171 duced during the learning process of arm reaching tasks, and their muscle
172 activities and coactivation would parallel changes in metabolic power.

173 The third method is to build a mechanics-based model and determine if
174 the predicted minimum mechanical cost correlates with people’s preferences.
175 A basic understanding of trajectory choice can be obtained by calculating
176 energy cost by using minimal dynamic models, such as two-link or three-link
177 arm models [45, 46], inverted pendulum walking models [33, 34, 35, 37],
178 bounce running models [47]. For example, use the inverted pendulum model
179 to predict the optimal step length and compare it with the subjects’ real step
180 length. However, most of the experiments used two-dimensional models and
181 studied human part-body motions in the sagittal plane, such as study leg
182 motions using an inverted pendulum model or arm reaching using a 3-link
183 model.

184 We conducted a whole-body virtual tracing experiment showing that both
185 the movement’s posture trajectories and its kinematic variations showed
186 striking commonalities across subjects [26]. One possible principle of ex-
187 planations for this generality could be that humans choose trajectories that
188 are economical in energetic cost. To prove it, we need to compute the cost of
189 virtual tracing movements. One possible way is to use the VO2 method to

190 measure the metabolic cost or use the EMG to measure the muscle coactivation
191 and stiffness directly. However, subjects had already worn a VR helmet
192 on their face during the tracing tasks. Besides, motion-capture suits covered
193 their whole body thus there was no exposed skin for EMG electrodes.
194 Another possible way is to build a dynamic model. However, as we mentioned
195 above, those models were built to simulate part of the human body
196 in two-dimensions.

197 We further searched for methods to build a dynamic bipedal robot by
198 modeling the whole body with a skeleton of rigid segments connected with
199 joints. The simplest bipedal robot uses three links to represent the torso and
200 two legs in the sagittal plane [48, 49]. Five-link biped robots extend the
201 model using two links to represent each leg [50, 51, 52, 53], while seven-link
202 biped robots further extend it by adding feet to it [54, 55]. Those models
203 have three different states: (1) open-linked – one foot on the ground, (2)
204 closed-linked – two feet on the ground, (3) both feet in the air. Each state
205 corresponds to a different set of motion equations. Most researchers use
206 open-linked models [48, 49, 50, 51, 52, 53, 54, 55]. They assumed that once
207 one foot laid to the ground, the other foot would be lifted immediately.

208 Recently, 3D modeling of biped robots [56, 57] have been developed as
209 well, however, they are still not sophisticated enough compared with a real
210 human body. A real human can be considered as a 21 hierarchical link
211 humanoid robot with 48 degrees of freedom. Furthermore, because the model
212 can have three different states during a continuous motion, such as running,

213 it is hard for an optimization algorithm performing gradient descent for three
214 different sets of equations that happened alternatively. Therefore, The third
215 method described above is also not fitting for our problem: 1) it is too
216 expensive to build a dynamic model with motion equations, 2) it is too hard
217 to use an optimization algorithm to predict the optimal path for a model
218 with three different dynamic states that happened alternatively.

219 We developed a novel way to compute the energy cost of human move-
220 ments by building a dynamic human model [58, 13] on the top of a physical
221 engine – Open Dynamic Engine (ODE)¹. This dynamic human model works
222 as follows:

- 223 1. Forward kinematics: it simulates human motion by following the mo-
224 tion capture data
- 225 2. Inverse kinematics: it calls the ODE built-in functions to compute the
226 joint angles and joint angular velocities at each frame.
- 227 3. Forward dynamics: it simulates human motion based on the computed
228 joint properties.
- 229 4. Inverse dynamics: it calls the ODE built-in function to compute the
230 required joint torques.

231 At each joint, instantaneous power was computed from the product of
232 net joint torque and joint angular velocity. The work performed at each

¹<https://www.ode.org/>

233 joint were determined by numerically integrating the instantaneous powers
234 over the entire tracing task. In this way, given motion capture data, we
235 can compute the energy cost without building a humanoid biped robot with
236 motion equations. The validation of this dynamic model has been proved in
237 the paper [58, 13].

238 While doing the virtual tracing experiment, subjects could freely choose
239 their starting posture and were given no instructions on how to comport
240 themselves during the tracing process. Therefore, we could consider subjects
241 traced curves under the conditions which were "comfortable" for them. Ac-
242 cording to the previous experiments [29, 30, 31, 32, 39, 40, 41], the metabolic
243 cost of movements with those trajectories should be the minimum. To prove
244 it, we perturbed the trajectories and computed the energy cost. As expected,
245 the metabolic cost increased more or less.

246 **3. Results**

247 Using the kinematic data from [26], we scaled the dynamic model to each
248 of the nine subjects and had the models trace of the nine curves of variations
249 of difficulty that are shown in Fig. 1. The energy cost of tracing paths showed
250 marked regularities in the following aspects of the data that was subject to
251 analysis:

- 252 1. Analysis of the joints' power while tracing path1 across different sub-
253 jects showed that although the absolute cost of the movements may

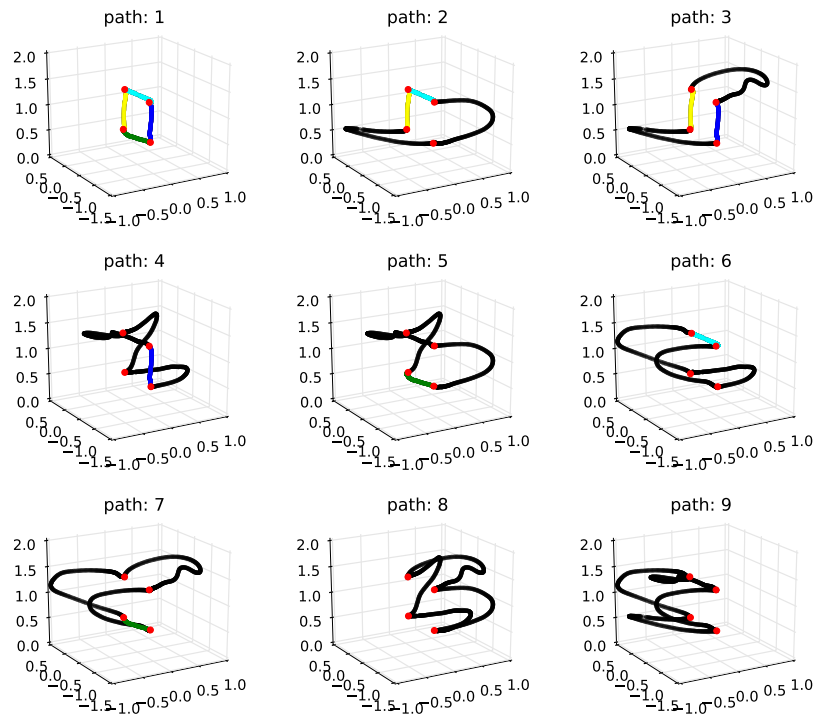


Figure 1: **The nine 3-dimensional paths in the virtual environment that were used in the experiment.** They are ordered by their complexity. For reference, colors denote common segments and points. For the subjects, the paths were all rendered in black, The scale is in meters.

- 254 vary between subjects, the cost is qualitatively very similar. (See sec-
255 tion 3.1, Figure 2);
- 256 2. The computation of average energy cost while tracing path1 showed the
257 corresponding residual forces were relatively small. (See section 3.1,
258 Figure 3 and Figure 4);

- 259 3. The costs of tracing each path by each subject, normalized by body
260 weight, are very similar and scaled with the length of the paths. (See
261 section 3.2 and Figure 5);
- 262 4. Although there are variations in the cost across the repeated traces,
263 the cost of using the perturbed model parameters is significantly higher
264 than the original. (See section 3.2, Figure 6 Figure 7, and Figure 8);
- 265 5. The increment of energy cost while using perturbed model parameters
266 distributes more on the joints' cost than on the residual component.
267 (See section 3.2 and Figure 9);

268 3.1. *Energy cost analysis of tracing path1*

269 **The mean of total power across different participants.** As an initial
270 analysis we established the variations in the costs of each curve exhibited
271 by different subjects. A representative result is shown in Fig.2 for path
272 one. The plot shows the total power at each frame for each subject. The
273 trace reveals that the subjects have to put more effort into the trace at the
274 same times. Thus although the absolute cost of the movements may vary
275 between subjects, the costs during the traces are qualitatively very similar.
276 [58] showed different participants used similar postures sequence while tracing
277 the same curves from kinematic perspective. Here, the similarity of the power
278 pattern along frames across different subjects reinforces this conclusion from
279 a dynamics perspective[59].

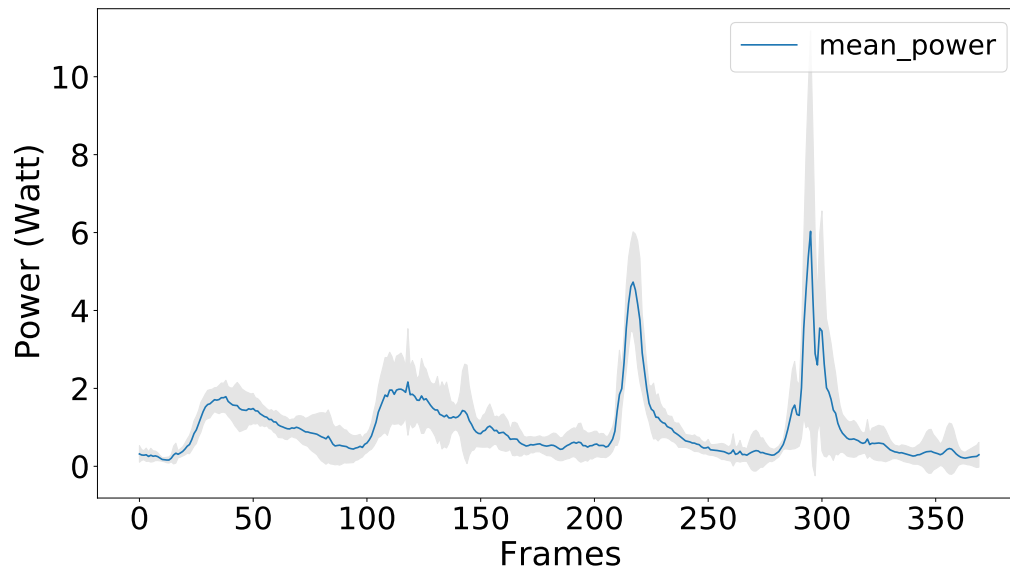


Figure 2: **The mean of power at each frame** Nine subjects traced path1 for 5 times each. The plot shows the joints' power at each frame of different subjects. The small standard deviation means that different subjects had similarity power patterns while tracing the same curve, which shows that the curve has points of difficulty in tracing shared by the subjects. Psth 1 is a simple so it can be traced quickly, but the observation of correlated effort representative of patterns in tracing other curves

280 **Average energy cost of five repetitions.** Although there are qualitative
281 similarities in the difficult points on the curve, the totalled costs of the traces
282 differ across different subjects. This result is shown in in Fig. 3, which
283 reports the cost per subject. The total energy of joints including the residual
284 components is shown in blue and the residual component is shown separately
285 in orange. When reporting the costs of the traces, we always use the total
286 cost shown here in blue, which includes the orange residual component.

287 As shown in Fig. 3, the by far largest cost of the tracing movement is
288 the component owing to the joint torques that are producing the kinetic

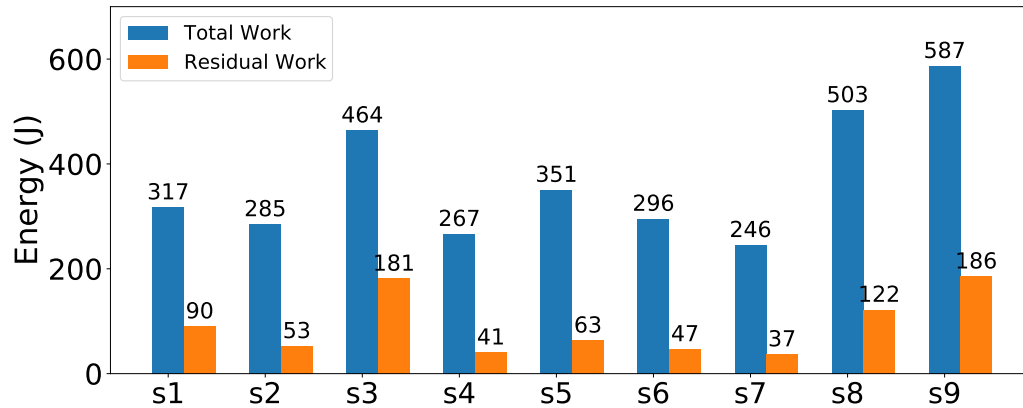


Figure 3: **The average energy cost of tracing and residual force component** Each subject traced path1 with 5 repeats. The horizontal labels indicate the corresponding subjects, e.g. "S1" represents the subject1. The total cost is shown in blue and the portion of that cost due to residual forces is shown in orange, A low cost in residual forces usually signifies that the dynamic model is a good match for that subject's kinematic data.

289 trajectories, and the additional cost of the residual from the inverse dynamic
290 calculation is small. In the human system, this residual is most prominently
291 due to the vestibular system but just how the vestibular connects to the
292 muscular system is not modeled by the human dynamic model. Instead we
293 implemented a provisional system of torques referred to a coordinate system
294 positioned and the center of mass to maintain balance.

295 **Residual forces.** All our subjects' costs are derived from the same inverse
296 dynamics technique [14], which combines measured kinematics and external
297 forces to calculate net joint torques in a rigid body linked segment model. A
298 feature of the dynamic method is that it can reduce potential errors, both
299 in the matches of the motion capture suit and the model. Analogous to

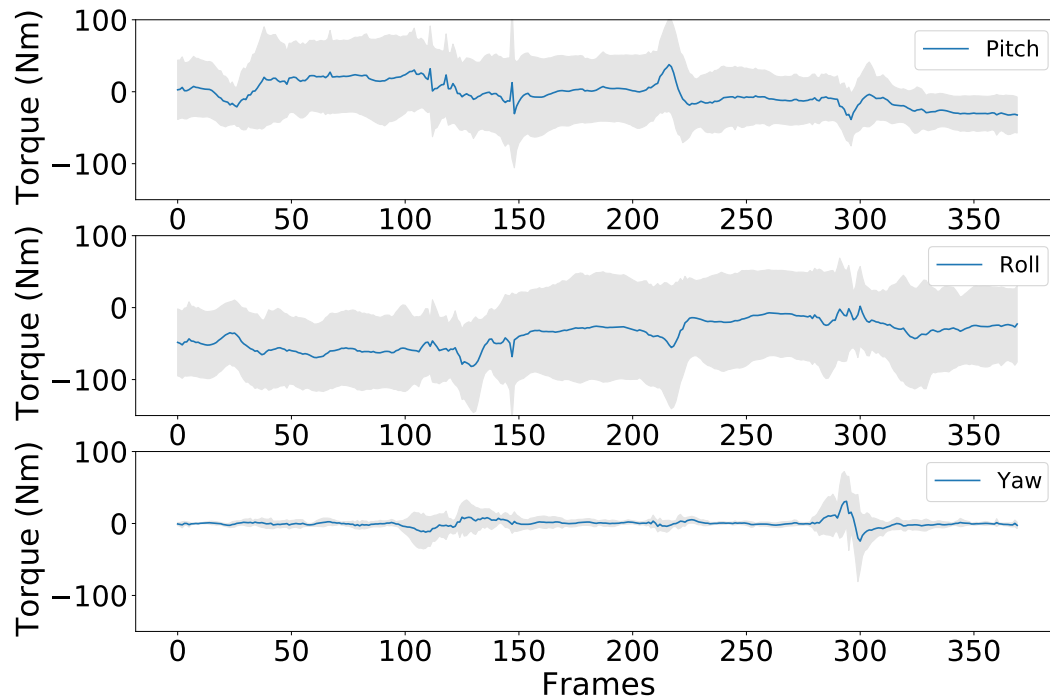


Figure 4: **Residual torque magnitudes**Errors in the calculation of joint torques from the inverse dynamics require additional torques for stabilization. Shown here are the magnitude of the torques seen in pitch roll and yaw axes.)

300 the human body's ligament structure to join joints, some leeway is allowed
301 in the ,model joints in the integration process.Nonetheless, even after these
302 adjustments, some errors remain. In the model, the main source of the
303 residual forces is usually attributable inaccuracies in the matches between the
304 motion capture suit makers and their match with their corresponding points
305 on the model. This is commonly resolved by introducing 'residual forces'
306 which compensate for this problem . This resolution with a dichotomy of
307 forces is analogous to the human system which combines feed forward lateral

308 pathway forces with medial pathway feedback forces

309 The temporal cost of such residuals is small as shown in Fig. 4 that
310 shows the distributions of magnitudes of the residual in orange in Fig. 4
311 for the nine subjects tracing tracing path1. For the model, roll torques
312 are the largest, with pitch torques second. Both pitch and roll torques can
313 exploit the background of the skeleton's inverse pendulum construction for
314 walking. These torques are necessary, but their magnitudes are small and not
315 a factor in distinguishing original and perturbed costs. Residual torques are
316 applied at the figure's waist, which is next to the center of mass. The small
317 magnitudes measured for the residual, together the observation that that
318 the residual is similar for the original and perturbed paths argues against
319 the residual torques being a factor in the analysis.

320 *3.2. Energy cost analysis of tracing individual paths*

321 **Energy cost of tracing nine paths.** Although there are similar costs per
322 subject in the sample trace, this arrangement does not carry over to the
323 comparison between paths, which has larger differences. We hypothesized
324 that the cost should scale as the length of the path a as shown in Figure 5,
325 which shows the average energetic cost of tracing the nine different paths.
326 The paths differ in tracing cost, but the costs of tracing each path by each
327 subject, normalized by body weight, are very similar and scale with the
328 length of the paths.

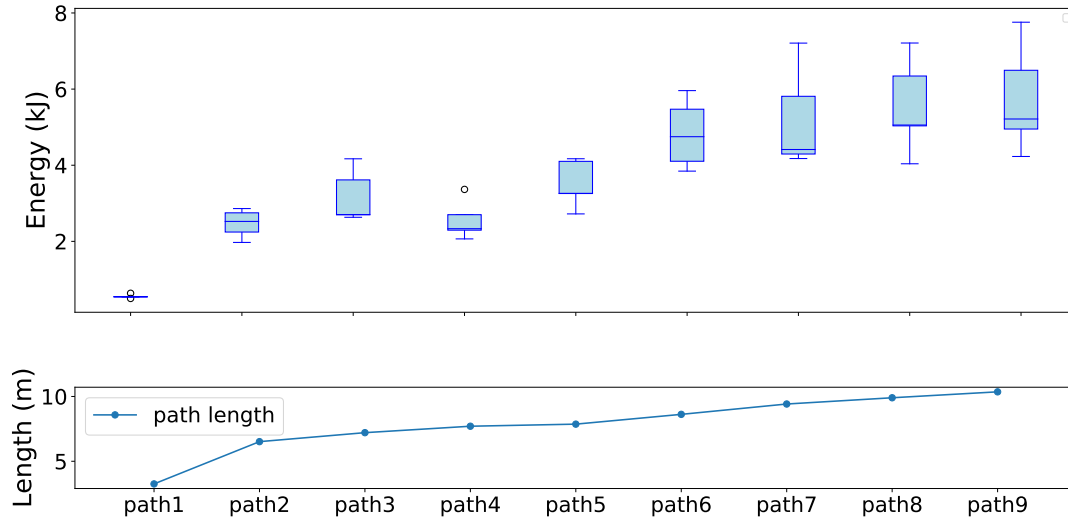


Figure 5: **Cost of tracing** These results portray the possibility that the costs vary across the best-fit five subjects. The statistics show that each path traced has a unique cost that distinguishes it from the rest.

329 **Tracing perturbation.** Given these regularities, The next step is to evalu-
330 ate the significance of perturbations in the tracing protocol. The hypothesis
331 is that if the tracing postures are chosen to be of minimum energy, chang-
332 ing the configuration away from the original tracing situation should incur
333 a cost, and that is what happens. The first perturbation tests changes in
334 model marker parameters. A marker is changed by a small delta and this is
335 a constraint that is satisfied while the model traces the paths. The way this
336 is implemented is to have the model's right finger follow the cue on the curve
337 as before. The kinematics is as it was for the unperturbed trace, but there
338 is enough freedom so that the dynamics can adapt to follow the perturbed

339 trace.

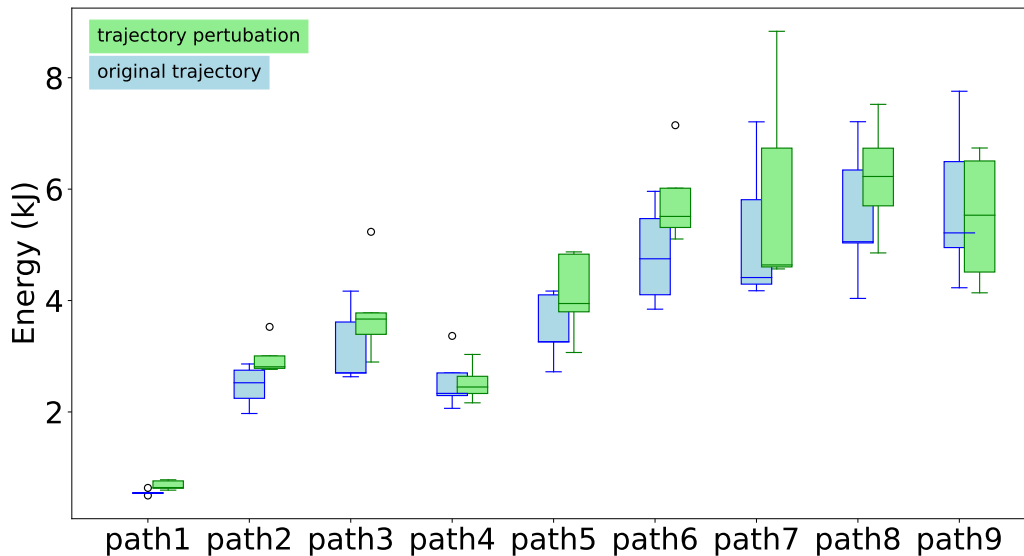


Figure 6: **Cost of tracing perturbed model** Cost of tracing each of the nine paths with a perturbations in the model elbow parameter. The elbow was moved up 3.5cm. This shows that for all the paths and the averages across subject tracers, the original path is always the least expensive. moreover the differences are highly significant

340 For in each trace the elbow marker is raised by 5 cm. The rest of the
341 system can adapt is the way dictated by the dynamic constraints. The result
342 is shown in Fig.6. Each subject traced each path five times and the resultant
343 costs are averaged. It is seen that although there are variations in the cost
344 across the repeated traces, the cost of using the perturbed model parameters
345 is significantly higher that the original. Note that outside of the changes,
346 the rest of the model solves the inverse dynamic model with the unperturbed
347 parameters, and thus the model has very large degrees of freedom at its

348 proposal.

349 In the model perturbation experiment, the system must followed the orig-
350 inal paths used in the nominal case. The second case makes adjustments in
351 the traced path. Some effects in a displaced can be intuited. For example, if
352 a subject has to reach over their head during the trace, it can be expected
353 that lowering the traced path would result in a cost savings. For this reason,
354 we chose path perturbations in the horizontal plane. Two such perturba-
355 tions were used: a 5centimeter dis[placement and a 5 centimeter rightward
356 displacement.

357 Figure 7 shows the result of averaging the traces across the displaced paths
358 averaged across each subject normalized by body weight. Each original path
359 is seen to be the lowest cost.

360 Here again the results are striking. Although there is some overlap, for all
361 curves, the originally more economical than the displacements. The observa-
362 tion that the averages of all the perturbed costs are more than the average
363 cost of their original progenitors strongly suggests that energy cost is the
364 factor in the choice of tracing postures. Figure 8 shows that all the three
365 sets of tests are significant to the $p=0.01$ level.

366 Given the dynamics dichotomy, a natural question that rises concerns
367 the magnitude of the extra torques in the perturbation cases. Are the extra
368 costs carried by the dynamic model or the residual? This is easily answered
369 by interrogating the simulation, and it turns out that that dynamics models
370 contribution is overwhelming. This is shown in Fig 9.

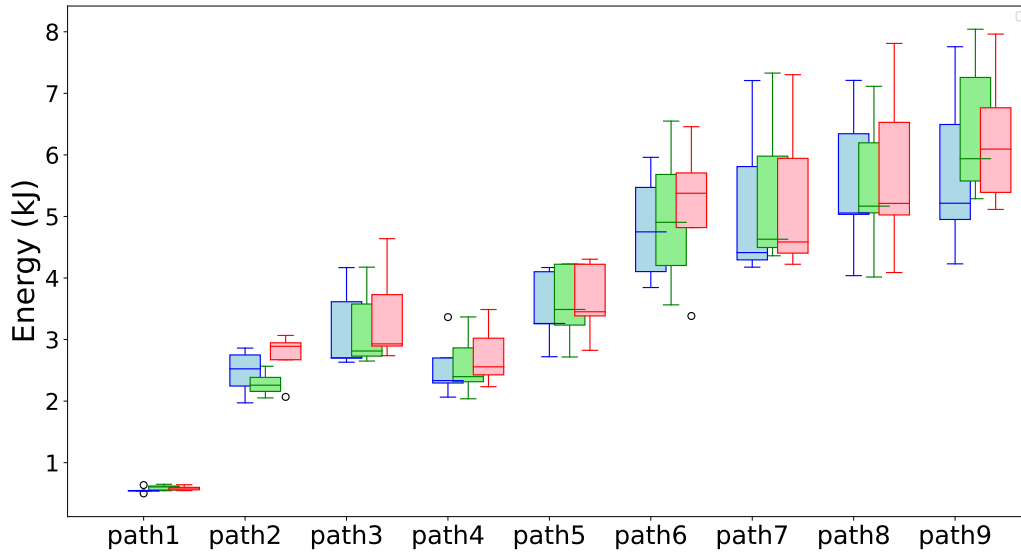
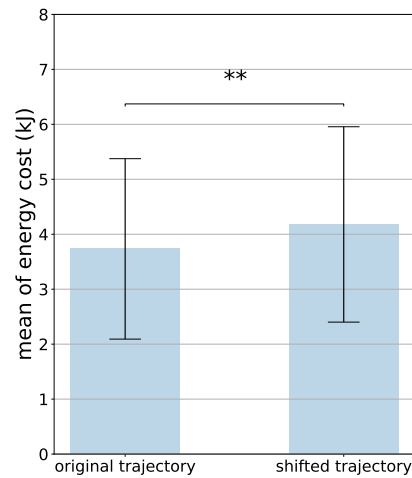
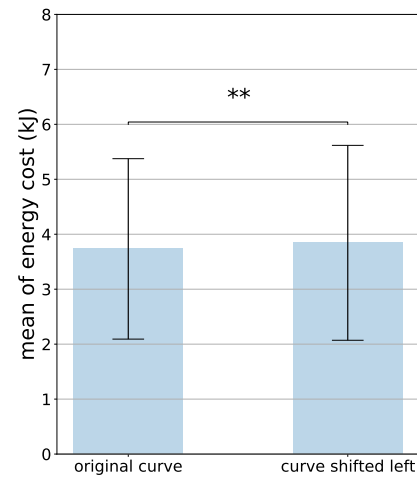


Figure 7: **Cost of tracing perturbed paths** Each of the nine paths have the two perturbations of 5 cm: left in green, right in red. This main result shows that for both averages across subject traces, the original path is always the least expensive.

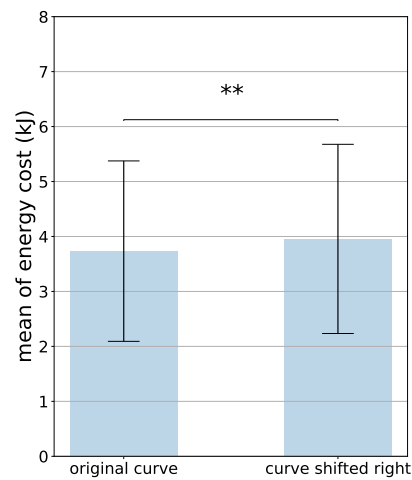
371 If the constraints on the dynamics were extremely stiff then the model
372 would have no course other than tracing an exact copy of the unconstrained
373 trajectory and let the residual torques contribute the need difference. How-
374 ever, the markers on the body for these experiments are limited to 15~18
375 of key points, leaving the extra degrees of freedom to be determined by the
376 dynamics. Moreover the torque computation, to model the reality of muscles
377 [60] uses spring constrains at each joint degree of freedom. Finally the
378 figure is forced to contact the displaced path, and the large features of the
379 movement such as footfalls are the same, leaving the dynamics to fill in the
380 rest.



(a) shifted model marker



(b) shifted curve (left)



(c) shifted curve (right)

Figure 8: Cost of tracing perturbed paths. Each of the nine paths have three perturbations. (a) Perturbed model marker. (b) curve perturbed 5 cm to the left (b) curve perturbed 5 cm to the right. This main result shows that for both averages across subject traces, the original path is always the least expensive. All three manipulations are different with a significance at the $p=0.01$ level.

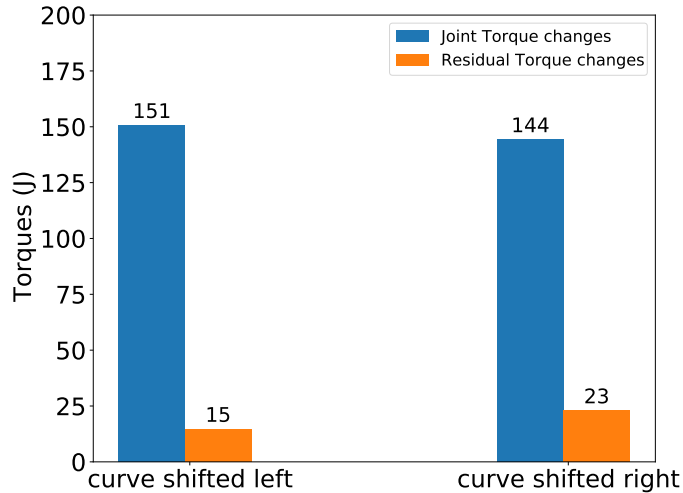


Figure 9: **Extra torques** The extra cost of tracing each of the nine paths with left and right perturbations in the traced path summed over five repeated traces

381 Discussion

382 Given that the cost of the movements is a significant fraction of a
383 human's caloric budget [61] one might expect that humans would exhibit
384 common low cost postures. This turns out to be the case for stereotypical
385 situations such as reaching or walking on a planar surface, but arbitrary whole
386 movements have been less studied so the expectations are more open. Thus it
387 was a surprise to measure arbitrary movements in a large scale tracing task
388 and find markedly common posture sequences used by all tested subjects
389 [26]. An obvious possibility for the similar posture sequences is energetic
390 cost, especially since there were no complex constraints in the movements
391 and no constraints in the time to perform the traces.

392 Our stimulations extend the kinematic finding to show that tests of hu-
393 man dynamics provide evidence that movements are chosen on the basis of

394 economic energetic costs. The initial measurements were unsurprising. The
395 cost of tracing scales monotonically with the length of a traced path as ex-
396 pected and the necessary residual forces, as would be expected from the
397 human's vestibular system and others, given that the subjects had to choose
398 their movements, were relatively small.

399 The main substantive results are that subjects' traces of each of nine space
400 paths all have minimal costs with respect to local perturbations. One manip-
401 ulation introduced perturbations in their kinematic variables. The subjects
402 traced the path but their model with small displacements in kinematic mark-
403 ers. The other experiment used local horizontal displacements of the paths.
404 Vertical were not used as they can be equivocal as the displacements can
405 interact with the different body sizes as when a short subject has to reach
406 up to an uncomfortable height. But outside of this caveat, the all the data
407 can be interpreted as the the tracing posture sequences are selected on the
408 basis of energetic cost.

409 The hypothesis that humans use minimum cost movement trajectories
410 was tested by the use of a human dynamic model that leverages a major
411 innovation in dynamics computation that allows the the recover torques
412 from kinematic data. The model also provides a fresh perspective for dif-
413 ferent interpretations of the representations of movements in the brain's mo-
414 tor cortex. The motor cortex has been extensively researched over many
415 decades [62], providing many different perspectives as to its complex struc-
416 ture [63, 64, 65, 66], and the computational modeling cannot be expected to

417 be definitive but it can endorse certain perspectives, which we attempt to
418 do here. The focus is in presenting that our model endorses the use cortical
419 area M1 as the site of kinematics representing posture changes in multiple
420 joints.

421 Our perspective follows from computer science's classic dichotomy be-
422 tween tables and functions in computation. Computing something as simple
423 as a trigonometric sine function, one has the option of pre-computing sine
424 values at some resolution and memorizing them in a table for instant access or
425 computing them on line using the slower series expansions for computing the
426 values as needed. These differences are placed in sharper relief in the human
427 system as the on-line aspect is much more formidable owing to the brains
428 relatively slow circuitry. In silicon, processing times are over million faster
429 so the trade-off tends to favor computed functions. In human biology, the
430 vastly slower computations favor pre-computation and tabular formats. the
431 torques for a posture change in an online fashion or pre-compute descriptions
432 ahead of time and save them in memory until needed. The dynamics models
433 advance informs this choice.

434 These broad computational realities favor memorization, they can also
435 evaluate the different suggestions that have been proposed for motor cor-
436 tex. The neurobiology of motor cortex also exhibits memorization but also
437 adds another between what we will call *local* and *global* memories. Local
438 memories have been the standard ever since [67] who showed body maps of
439 local movements and somatosensation. Homunculus maps reflect the reality

440 of local stimulation, but eschew any larger picture. The global picture owes
441 its discovery to [68] who show that cortical stimulation could larger posture
442 movements directed to external goals. In this organization the neural maps
443 produce cooperative movements reflective of the animal's task. This orga-
444 nization has been refined by [69]. Using larger amplitude altering current
445 stimulation, Graziano produced large global posture movements that he was
446 able to classify in into five behaviors. A similar point made with very dif-
447 ferent methodology hasbeen made by [70]. Studies using a balance platform
448 show that humans use muscles is stereo-logical groups.

449 The global movement data together with the modeling complexity of
450 generating the movements makes it likely that they are pre-computed and
451 save in motor cortex in some form, but what exactly does this take?

452 The dynamic model argues for parsing the information that can be pre-
453 planned and has reliable generality such as kinematics [71] and stiffness [72].
454 These parameters are distinguished by having to be pre-planned. Before the
455 movement, many of the details of the movement, such as surface properties
456 are not known and have to be only estimated. Thus the invariant components
457 of the movement are most likely to be memorized.

458 In contrast, the torques are likely left for the spinal cord. There are many
459 reasons for this. 1) The spinal cord contains a vast store of programmable
460 reflexes that handle the fastest responses. 2 The spinal cord integrates the
461 commands from the brain's lateral system used by the cortex and medial
462 system used by the residual torques like those from the vestibular system.

463 3) While footfalls can be estimated kinematically, the complex interactions
464 to handle the contact with an uncertain surface need use feedback loops
465 given an kinematic estimate just as a starting point. 4) Given the torques
466 are implied by the kinematics, any manipulation with a kinematic correlate
467 e.g.[73] may have kinematics as its interpretation 5) It has been shown that
468 the extraction of a movement by the cortex has its own dynamic process
469 which is not instantaneous [74], but nonetheless is decoupled with the mea-
470 sure of muscle activation with electromyography. This observation, together
471 with another that shows supplementary motor cortex is correlated with the
472 timing of movement onset [75], suggest that the motor cortecies, may be fo-
473 cused on the overhead in movement planning than the fine-grained movement
474 dynamics.

475 4. Methods

476 4.1. *Virtual tracing experiment*

477 For the original kinematic data capture we designed a virtual whole-body
478 tracing experiment to elicit natural movements under common goals [26].
479 Subjects wore a virtual-reality helmet, Oculus Rift [76], to see a virtual three
480 dimensional interior room with a dojo backdrop via stereo video. They were
481 required to trace a series of paths positioned at fixed locations in the virtual
482 environment. The movements of their bodies and variables relevant to the
483 tasks were simultaneously recorded using the PhaseSpace motion capture
484 system [77]. The WorldViz Vizard software package [78] both controlled the

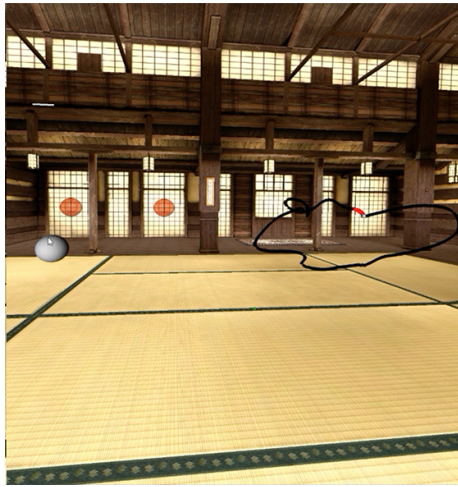
485 virtual tracing protocol and the recording of the motion capture data. Fig
486 10 shows the virtual environment setup. Fig 1 shows the nine paths that
487 subjects traced.

488 **Data pos-processing.** For some frames the motion capture system is un-
489 able to determine the 3-dimensional location of some markers, thus raw mo-
490 tion capture data usually contains some segments of signal loss (dropouts).
491 Dropouts are relatively infrequent in practice but can occur over significant
492 temporal intervals, which makes linear interpolation a poor choice for recon-
493 structing the raw motion capture data. In this experiment, trajectory-based
494 singular value threshold was implemented to reconstruct missing marker data
495 with a minimal impact on its statistical structure. The data for each subject
496 was interpolated using a separate matrix completion model.

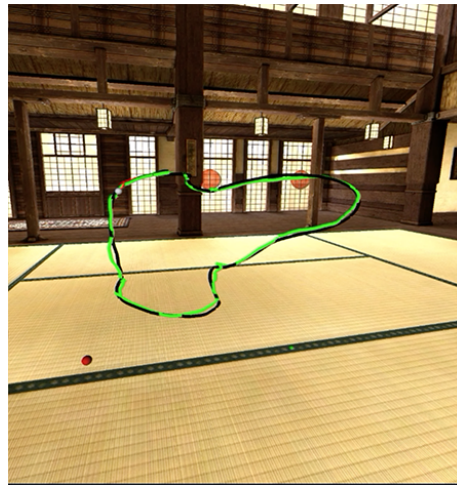
497 In addition to the data interpolation process if a participant did not trace
498 the path successfully, e.g. their index fingers were too far behind the tracing
499 points at a certain frame, or a recording of a tracing trial failed, e.g. too
500 many markers were off during a tracing which leads to a extremely large
501 joint torques, we would consider this tracing invalid and the data would not
502 be used.

503 4.2. *Human dynamic model*

504 To compute the energy cost of subjects tracing paths, we used our human
505 dynamic model [58]. By replaying the virtual tracing experiment's kinematic
506 data, we can compute can the joints' properties, e.g. torques and angles, at



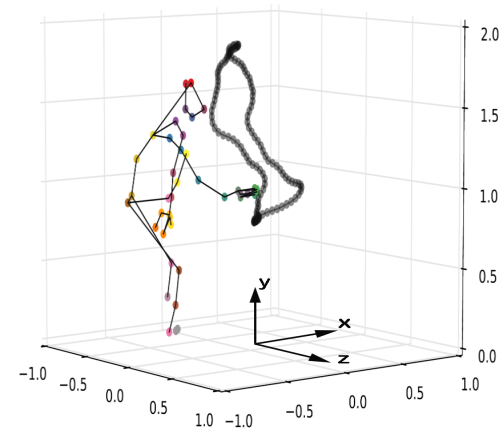
(a) Before tracing



(b) After tracing



(c) A subject doing the tracing task



(d) The skeleton plot of the subject

Figure 10: the virtual environment setup. (a) shows a full view of a path, denoted by a black path, and the starting position, denoted by a large white sphere. The small white sphere on the path at the end of a red segment is the tracing target sphere. (b) depicts the scene when a trial is finished. The green path is the actual tracing trajectory generated by a subject. (c) illustrates a subject in the act of tracing a path in the laboratory's motion capture 2 x 2 x 2 meter volume. and (d) shows the lab coordinate system. The scale on the graph is in meters. The the subject's skeleton and the traced path in the 3D space are plotted. The color dots correspond to a subset of the fifty active-pulse LED markers on the suit and the virtual-reality helmet. Related to Figure 2.

507 frame rates. The human dynamic model is built on top of the ODE physics
 508 engine [79]. It consists of a collection of rigid bodies connected by joint.
 509 Each joint connects two rigid bodies with anchor points (center of rotation)
 510 defined in the reference frame of both bodies. The locations of these anchor
 511 points determine the segment dimensions (bone lengths) of the character
 512 model. Fig. 11 shows the number of body segments and topology of the
 513 human dynamic model.

B

Joint	Part 1	Part 2	DOF/joint	Total DOF
Cervical	Head	Neck	3	3
Thoracic	Neck	Upper Torso	3	3
Lumbar	Upper Torso	Lower Torso	3	3
Sacral	Lower Torso	Pelvis	3	3
c.Clavicle	Upper Torso	c.Collar	3	6
c.Shoulder	c.Collar	c.Upper Arm	3	6
c.Elbow	c.Upper Arm	c.Lower Arm	2	4
c.Wrist	c.Lower Arm	c.Hand	2	4
c.Hip	c.Pelvis	c.Upper.Leg	3	6
c.Knee	c.Upper Leg	c.Lower Leg	2	4
c.Ankle	c.Lower Leg	c.Heel	2	4
c.Tarsal	c.Heel	c.Sesamoid	1	2

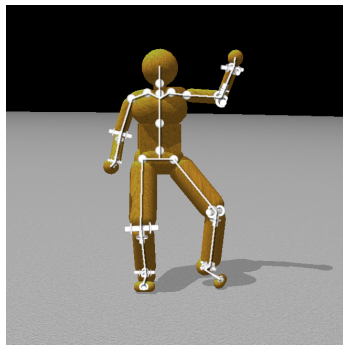


Figure 11: **The 48 internal DOF Model** A. Four ball-and-socket joints connect five body-segments along the spine from the head to the waist. Ball-and-socket joints are also used at the collar-bone, shoulder, and hip. B. A summary of the joints used in the model. c. = chiral: there are two of each of these joints (left and right). Universal joints are used at the elbows, wrists, knees, and ankles. Hinge joints connect the toes to the heels. All joints limit the range of motion to angles plausible for human movement. Our model assumes that joint DOFs summarize the effects of component muscles.

514 Fig. 12 shows a interface that allows the simulation of human movements
 515 via a multi-purpose graphical interface for analyzing movement data cap-
 516 tured through interaction with the virtual environment. With this tool, it

517 is possible to interactively fit a model to motion capture data, dynamically
 518 adjust parameters to test different effects, and visualize the results of kine-
 519 matic and dynamic analysis, such as the example in Fig 13, which shows
 520 a four stages in a tracing sequence made originally by a participant of the
 521 virtual tracing experiment and recreated by applying the inverse dynamics
 522 method using this tool.

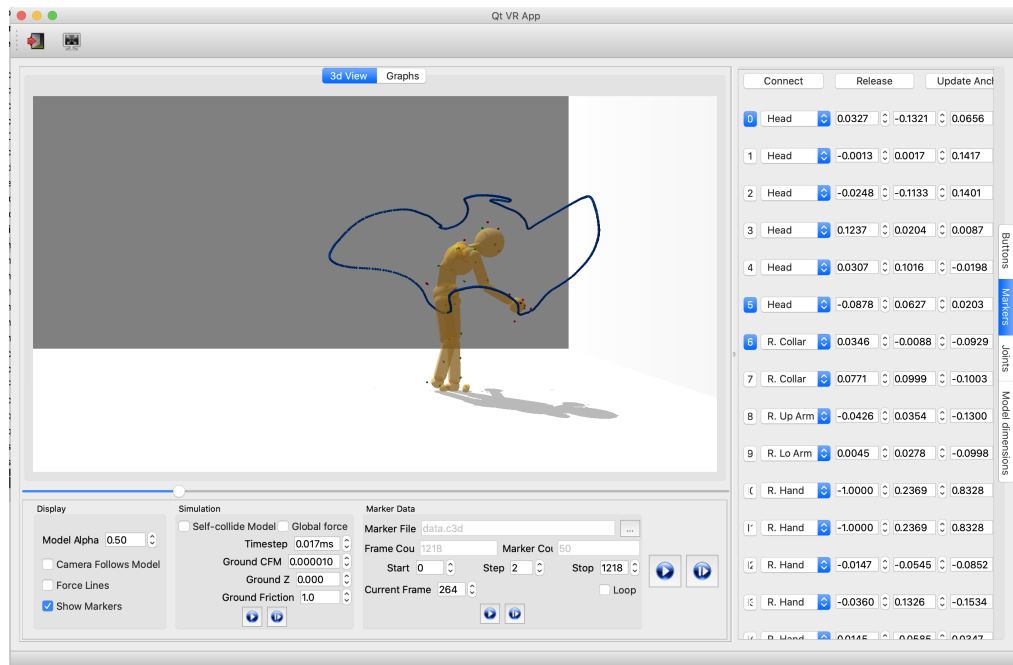


Figure 12: Our analysis tools use the physics engine to compute inverse kinematics and inverse dynamics. They also support various visualizations of relevant data and control for analyzing and producing physically-based movements. The programmed parameters of the model consist of its joints and its 3D marker positions. For example, the right column represents the positions of the markers relative to their corresponding body segments, e.g. the first row shows the information of marker1: 1) "1" represents the marker index, 2) "head" means marker 1 is attaching to the "head" body segment, 3) the remaining three float numbers are marker1's relative position.

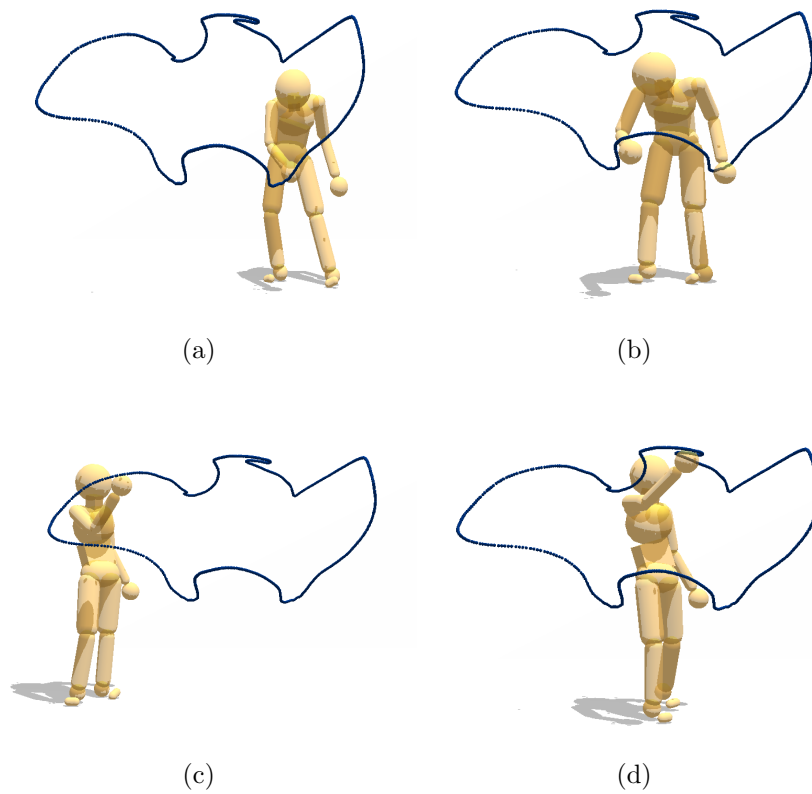


Figure 13: **Model capability illustration.** Four points in a tracing sequence reproduced with physics-engine-based inverse dynamics using recorded motion capture data from a human subject.

523 **Model fitting.** The quality of the fit between the data and the model can
524 significantly influence the energy computation. The technique for fitting
525 a model to data begins with a character model that serves as a template,
526 Fig. 12, providing the number of body segments and topology of the model.
527 We further require that labeled markers used in motion capture be assigned
528 to specific model segments. It may be straightforward to derive these using
529 a technique such as in [80, 81]. However, it is also not difficult to do by
530 hand. It would become tedious if one had to go through the process for
531 many different models. Fortunately, the motion capture suit typically puts
532 the markers on the same body segments, even if they are in slightly different
533 places and the body segments have different dimensions.

534 4.3. Energy cost computation

535 The centerpiece of the analysis depends critically on the definition of a
536 posture. At each frame, posture is defined as a vector of the joint torques
537 and angles of each of N joints ($N = 22$ in our dynamic human model). The
538 posture p at a frame is a 6n-dimensional column vector presenting the joints
539 properties of the i th participant, thus

$$\mathbf{p} = [\mathbf{j}_1, \mathbf{j}_2, \dots, \mathbf{j}_N] \quad (1)$$

$$\mathbf{j}_i = (\boldsymbol{\tau}_i, \mathbf{a}_i) \quad (2)$$

540 where $\boldsymbol{\tau}_i = (\tau_{i_x}, \tau_{i_y}, \tau_{i_z})$ and $\mathbf{a}_i = (a_{i_x}, a_{i_y}, a_{i_z})$ represents the torques and
541 angles of the i th joint at a frame respectively and $i = 1, 2, \dots, N$. For the

542 joints whttps://www.overleaf.com/project/5f25a6a28109fa0001d5233chich have
543 less than three dimensions, e.g. hinge joints, universal Joints, the values at
544 unused dimension were assigned zero.

545 The power W of i th joint at a frame t is a scale and equals to the inner
546 product of its torque $\boldsymbol{\tau}_i$ and its angular velocity $\boldsymbol{\omega}_i$, thus

$$\boldsymbol{\omega}_i(t) = \mathbf{a}_i(t) - \mathbf{a}_i(t - 1) \quad (3)$$

$$W_i(t) = \boldsymbol{\tau}_i(t) \cdot \boldsymbol{\omega}_i(t) \quad (4)$$

547 Therefore the power of a posture at frame t is presented as:

$$W(t) = \sum_{i=1}^N W_i(t)$$

548 Assuming it takes a participant T frames to trace a path, then the total
549 energy cost E of the participant tracing a path is:

$$E = \sum_{t=1}^T W(t)$$

550 The energy cost analysis is naturally organized into three separate stages.
551 Initially, we analyze the subjects energy cost and residual torques of tracing
552 path1 which is the simplest path. Next, we computed the tracing cost of all
553 nine paths. To compare the energy cost of tracing a path across subjects, we

554 computed the average energy cost for all five repeated traces of each subject.
555 Finally, we measured the tracing cost of perturbed participant's trajectories
556 and perturbed paths.

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560 **Declaration of Interests**

561 The authors have no financial or personal relationships with other people
562 or organizations that could inappropriately influence their work. The authors
563 declare no competing interests.

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