censcyt: censored covariates in differential abundance analysis in cytometry

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Abstract

Innovations in single cell technologies have lead to a flurry of datasets and computational tools to process and interpret them, including analyses of cell composition changes and transition in cell states. The *diffcyt* workflow for differential discovery in cytometry data consist of several steps, including preprocessing, cell population identification and differential testing for an association with a binary or continuous covariate. However, the commonly measured quantity of survival time in clinical studies often results in a censored covariate where classical differential testing is inapplicable. To overcome this limitation, multiple methods to directly include censored covariates in differential abundance analysis were examined with the use of simulation studies and a case study. Results show high error control and decent sensitivity for a subset of the methods. The tested methods are implemented in the R package censcyt as an extension of diffcyt and are available at 10 https://github.com/retogerber/censcyt. Methods for the direct inclusion of a censored variable as a 11 predictor in GLMMs are a valid alternative to classical survival analysis methods, such as the Cox proportional 12 hazard model, while allowing for more flexibility in the differential analysis. 13

Background

Flow and mass cytometry are techniques to measure the presence of fluorochromes or isotopes conjugated to 15 antibodies that are bound to specific cellular components at single cell resolution. Although cytometry can be 16 considered an established method, recent developments enable the measurement of ever more markers 17 simultaneously, resulting in a high-dimensional view for each cell [1,2]. Although the number of measured 18 features per cell is still much lower than in other single cell methods, such as single-cell RNA sequencing 19 (scRNA-seq), the throughput is typically much higher with thousands of cells per second [1,2]. An additional 20 benefit of cytometry compared to scRNA-seq is the measurement at the protein level instead at the RNA level 21 (since correlations between protein and mRNA expression can be low [3,4]), although new cytometry-by-seq 22 approaches (e.g. Cite-seq [5] and REAP-seq [6]) allow the simultaneous measurement of transcript and protein 23 expression. The antibodies used in cytometry experiments are often chosen to discriminate several cell types by 24 leveraging the biological knowledge about their protein expression (e.g. T-cells can be distinguished from other 25 lymphocytes by the amount of CD3 they express). After obtaining the raw marker intensities per cell and 26 preprocessing (including some or all of: Compensation, Quality assessment, Normalization, De-Barcoding, 27 Filtering, Transformation [7, 8]), the first step is to discern cell populations. The historical approach for this 28 clustering is manual gating, which requires an expert to choose thresholds of marker intensities to obtain (known) 29 cell populations. Challenges around manual gating include lack of reproducibility and impracticability for high 30 dimensionality [9,10], which is why modern approaches try to overcome these limitations by either automatically 31 choosing the best threshold to separate subpopulations (e.g. with *flowDensity* [11]), or by clustering cells using 32 techniques such as *FlowSOM* (using a self organizing map) [12], *flowMeans* (k-means with cluster merging) [13] 33 or *PhenoGraph* (based on a nearest-neighbor graph) [14]. Alternatives that do not strictly involve clustering 34 include classifying cells based on an annotated reference dataset (e.g., linear discriminant analysis [15]). 35

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After clustering or cell type assignment, the processed data contains a subpopulation label for each cell. The two classical analyses that can be performed are differential abundance (DA) and differential state analysis (DS) [16]. In DA, the (perhaps normalized) relative proportion of cells in a subpopulation per sample is tested for an association with additional information about the sample (e.g. control vs. treatment). The input data consists of a *cluster* × *sample* matrix of cell population abundances. In contrast, DS analyses organize the single cell data into (*cluster-marker*) × *sample* matrices, typically summarizing each subpopulation per sample with median marker expression; afterward, the summary is modeled against sample-wise annotations for the association testing.

The R [17] package diffcyt [18] provides a framework for DA and DS for flow and mass cytometry. After preprocessing of the raw data, FlowSOM is (by default) used to (over)cluster cells into many small clusters representing potential rare cell populations [18]. DA can then be performed with well-known count-based methods voom [19], edgeR [20] or Generalized Linear Mixed Models (GLMM). Alternatives for differential discovery include, among others, citrus (overclustering, building of hierarchy, model selection and regularizations to get associations) [21], cydar (differential abundance on hypersphere counts, testing with Generalized Linear Models) [22], CellCnn (convolutional neural networks) [23] and MASC (Mixed-effects modeling of Associations of Single Cells) [24]. An important distinction is that, with citrus and CellCnn on one side and diffcyt and MASC(and cydar) on the other, the association testing is "reversed": for diffcyt, the cell population (relative) abundances are represented in the statistical model as the response, whereas in citrus and CellCnn, the abundances are treated as a covariate. The reversed approach allows for more flexibility in the experimental setup since it allows to include additional covariates, such as batch or age, to be directly adjusted for [16], and diffcyt was shown to compare favourably in terms of sensitivity and specificity across several test cases [18].

Cytometry samples from clinical studies often contain additional patient data, such as treatment group (e.g., control vs. treated), age or survival time. DA with a binary variable (e.g. control vs. treated) can be seen as the "classical" case in cytometry. Of particular interest is whether a cell subpopulation is more abundant in one experimental condition compared to the other, which could be indicative of the effectiveness of a treatment. If an association with a continuous variable (e.g. age) is of interest, the modeling and testing are similar to the binary case and often the same methods can be used, since linear models underpin the statistical framework. If a time-to-event variable (e.g. time to an event, such as death or recurrence of disease) is considered, there is a need to use different methods altogether. The problem with time-to-event variables is a purely practical one caused by events that are "censored", i.e., they are not fully observed but only a minimum (or maximum) is known.

An example of cytometry data of a clinical study can be found in the FlowCAP IV (Flow Cytometry: Critical Assessment of Population Identification Methods) challenge [25]. 13 marker intensities of PBMC samples of 383 patients linked to time to progression to AIDS from HIV+ were measured with flow cytometry, with the objective to find cellular correlates that predict survival [25]. At the time, the two best performing methods, (*FloReMi* [26] and *flowDensity/flowType/RchyOptimyx*), both relied on classical survival analysis methods in the association testing step, such as the Cox proportional hazard model [27], where the censored variable is modeled as the response and the subpopulation abundance as the predictor.

Meanwhile, the performant frameworks for cytometry analysis that have been shown to perform well with completely observed data (e.g. *diffcyt* [18]) cannot directly handle censored data; in particular, a censored predictor should not be treated as fully observed, since it can lead to a bias [28]. Removing incomplete samples can be a workaround, but is inefficient for high censoring rates and might lead to a bias as well [29]. Thus, the goal of this work is to investigate how to best directly include a *censored predictor* in the modeling framework, which itself is an under-researched area compared to survival response models. The following are noteworthy: Rigobon *et al.* described basic issues that arise from censored covariates [28]; Tsimikas *et al.* developed a method based on estimating functions for generalized linear models [30]; Taylor *et al.* described two methods based on multiple imputation [31]; Qian *et al.* developed a threshold regression approach [32]; Atem *et al.* developed methods based on multiple imputation in a bootstrapping setup [33].

In the following, we describe an extension to the linear model approach to DA in *diffcyt* that allows to directly include random right censored time-to-event variables as a covariate using methods based on multiple imputation. More specifically, risk set imputation and Kaplan-Meier imputation (imputation based on the Kaplan-Meier estimator of the survival function) from Taylor *et al.* [31] and the conditional multiple imputation (imputation based on the mean residual life) from Atem *et al.* [33] are included. A simulation framework was

developed to evaluate basic properties of the model as well as differential discovery performance in the context of cytometry. The dataset from the FlowCAP IV challenge was re-analysed according to the *diffcyt* workflow with the censoring-specific methods to highlight real world applicability.

Results

In order to test the performance of the included methods that handle censored covariates, two simulation studies were performed, the first exploring basic properties in a simplistic model and the second embedded in the situation of differential discovery performance when considering a single cell dataset with multiple subpopulations.

Basic simulations

In the basic simulation, counts (Y_j) for a sample j were modeled as binomially distributed with a GLMM association with two covariates, one censored (T_j) and the other binary (Z_j) , via a logit link function with regression coefficients β :

$$Y_{j}|n_{j}, p_{j} \sim Bin(n_{j}, p_{j})$$

$$logit(p_{j}) = \beta_{0} + \beta_{1}T_{j} + \beta_{2}Z_{j} + R_{j}$$

$$n_{j} \sim U(1e4, 1e5)$$

$$T_{j} \sim Weibull(\lambda_{t}, \kappa_{t})$$

$$R_{j} \sim N(0, \sigma^{2})$$
(1)

where R_j represents an observation-level random effect to model overdispersion and n_j is the total number of cells in a sample. For further details, see the Methods section.

Results of the basic simulations are shown in Figure 1 for three different censoring rates (30%, 50%, 70%) for 102 a sample size of 100 with 100 repetitions per condition. Four different evaluation criteria are considered: raw bias 103 $(RB = E(\hat{\beta}_1) - \beta_1)$, coverage rate (CR, proportion of confidence intervals that contain the true value), confidence 104 interval (CI) width and root mean squared error (RMSE = $\sqrt{E((\hat{\beta}_1 - \beta_1)^2))}$. For a multiple imputation method 105 to be considered "randomization-valid", it should have no bias and a CR close to the specified proportion (in this 106 case, 0.95) [34]. If a method is randomization-valid, the average width of the CI is another important criterion 107 that represents statistical efficiency. On the other hand, the RMSE is an indicator of the precision of the 108 estimation as it combines the variance and the bias (RMSE = Var $(\hat{\beta}_1)$ + Bias $(\hat{\beta}_1)^2$). For increasing censoring 109 rates, the RB (top row in Figure 1) for methods Kaplan-Meier imputation (km), Kaplan-Meier imputation with 110 an exponential survival function tail (kme), risk set imputation (rs) and predictive mean matching (pmm)111 increases slightly, while for the other methods, it remains constant although the RMSE increases. In particular, 112 the RB for those four methods is positive under all conditions, indicating overestimation. This observed bias is 113 quite consistent across different simulation conditions (See Supplementary Figures S1-S4) although only for a low 114 regression coefficient of the censored covariate does it become pronounced (Supplementary Figure S2). The CR 115 (second row in Figure 1) is for all methods close to the expected value of 0.95 and taken together with the RB (in 116 general close to zero) confirms the randomization-validness of the methods under most of the tested simulation 117 conditions. The CI width (third row in Figure 1) for km, kme and rs has a nearly equal spread across all 118 conditions while for the remaining methods, it increases with increasing censoring rate. Since the RMSE (bottom 119 row in Figure 1) is a combination of the variance and the bias of an estimate it summarizes row 1 and 3 of Figure 120 1. So even though the estimates from km, kme and rs are slightly biased, their RMSE is lower compared to the 121 other methods since their variance is lower. 122 123

The distribution of p-values under the null simulation is for a low censoring rate uniform for all methods except mrl whose distribution is shifted towards 1 (Figure 2). For increasing sample sizes, the p-value distributions of all methods (except cc) shift towards 1, suggesting they become more conservative. The distribution for cc on the other hand shifts slightly towards 0.

Taken together, these results show that no tested methods stand out as being uniformly underperforming, but none is remarkably outperforming compared to its competing methods.

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Simulations modeled from real data

Figure 3 depicts a schematic of the simulation procedure for the multiple cell population scenario. Based on a real dataset clustered into cell populations (e.g. data from FlowCAP IV clustered with *FlowSOM*; Figure 3a), a Dirichlet-multinomial (DM) distribution is fit to the *cluster* × *sample* matrix of abundances (Figure 3b). To insert a known association, the obtained concentration parameters $\boldsymbol{\alpha} = (\alpha_1 \dots \alpha_K) \in R_+^K$ are then adjusted to include an association with a continuous (and later, censored) and a binary variable (Figure 3c, Eqn.3). The sizes (second parameter of DM) are kept the same. For subpopulation $i \in \{1...K\}$ and sample $j \in \{1...N\}$ the counts of a sample $Y_j \in N^K$ with size $n_j \in N$ are distributed according to

$$Y_j \sim DM(n_j, A_j) \tag{2}$$

with the matrix of concentration parameters $A = (\boldsymbol{\alpha}_1^T \dots \boldsymbol{\alpha}_N^T) \in R_+^{K \times N}$ dependent among others on the continuous covariate T_j and the binary covariate Z_j :

$$A_{ij} = logit^{-1}(\beta_{0i} + \beta_{1i}T_j + \beta_{2i}Z_j) \times \sum_{l=1}^{K} A_{lj}$$
(3)

where the β parameters are the regression coefficients. A new dataset is then simulated with the adjusted parameters (Figure 3d). For further details, see the Methods section.

When two covariates are present, one option is to test for an association of the cell population abundance ¹⁴¹ with the censored covariate (i.e. by testing if the regression coefficient of the censored covariate $\beta_1 = 0$ in Eqn. 3) ¹⁴² while also accounting for the binary covariate. In Figure 4 the TPR-FDR (true positive rate versus achieved false ¹⁴³ discovery rate) curves for the detection of true association between cell population abundance and survival time ¹⁴⁴ are shown for three different censoring rates and four different sample sizes. ¹⁴⁵

The method *GLMM* is the generalized linear mixed model method from *diffcyt* using the true (but 146 unobserved) survival times and is included as a control, since it represents the maximum performance that could 147 be achieved if the data were fully observed. It is not dependent on the censoring rate, so it can also be seen as a 148 qualitative comparison of the simulation variability for a given sample size. *pmm*, on the other hand, can be 149 considered to be a quasi-negative control, since it treats censored values as missing (leading to increased 150 uncertainty about the data); thus, it highlights the gain in information from including censored values versus 151 treating them as missing. In contrast, cc only keeps the "best" samples (the ones that are observed), which leads 152 to more certainty about the data (at the cost of less data and potentially biased estimates). 153

Not surprisingly, lower sample sizes and increased censoring rates result in lower sensitivity. For a censoring 154 rate of 30%, the differences in performance between the methods are minimal, independent of the sample size. 155 For high censoring rates (70%), the differences between the methods are more prominent but decrease again for 156 large sample sizes (400). pmm has overall the lowest sensitivity and poor error control; this is especially 157 pronounced at high censoring rates leading to TPR-FDR curves with high FDR at low TPR. On the other hand, 158 cc shows moderate sensitivity but the error control is poor for both high censoring rates and small sample sizes. 159 rs, km, kme have in general a moderate sensitivity and good error control while mrl has good sensitivity and 160 decent error control. Especially for high censoring rates, mrl outperforms other methods in terms of TPR. 161

To summarize: The censoring-specific methods have in general good error control, but especially for high censoring rates, result in lower sensitivity at a given p-value threshold (e.g. 0.05) than cc (which has poor error control).

The second option is to test for the association between the binary covariate and the cell population 165 abundance (i.e. by testing if the regression coefficient of the binary covariate $\beta_2 = 0$ in Eqn. 3), in the presence 166 of a censored covariate. The TPR-FDR curves in this scenario (Figure 5) show clear differences compared to the 167 testing for the association with the censored variable. GLMM is again the unrealistic control while ncGLMM is 168 based on *GLMM*, but excludes the censored covariate in differential testing. It could therefore be seen as the 169 ad-hoc solution when a censored covariate is present but not of interest and one decides to neglect the possible 170 effect of the second covariate on the response. Two main differences compared to the association testing of the 171 censored covariate is that cc and mrl have low sensitivity, even lower than pmm in many cases. The best 172 performing methods are km, kme and rs, which often have similar sensitivity and error control. In many cases, 173

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they have a higher sensitivity than ncGLMM indicating that there is a benefit of accounting for the censored covariate instead of discarding it. Comparing the error control between Figure 4 and 5 shows that in the binary covariate association testing, the error control of the censoring-specific methods is often closer to its expected values than in the censored covariate association testing.

An alternative simulation scenario with only one censored covariate was modeled as well to compare censored-covariate methods with the Cox proportional hazard model [27] (by maintaining the simulated associations, but switching the response and the covariate in the statistical model). The results indicate similar performance in terms of specificity and error control for the Cox proportional hazards model and the censored covariate regression models (Supplementary Figure S5).

Case study

To illustrate the use of models with censored covariates in differential discovery analysis, the FlowCAP IV dataset was reanalysed. A total of 766 flow cytometry PBMC samples linked to time to progression to AIDS from HIV+ of 383 patients (two per patient, one stimulated, one unstimulated) were available. For each sample, 13 marker intensities (IFN γ , TNF α , CD4, CD27, CD107-A, CD154, CD3, CCR7, IL2, CD8, CD57, CD45RO and V-Amine/CD14) together with channels FSC-A, FSC-H and SSC-A were measured. Of the 383 available survival times, 79 were observed, resulting in a censoring rate of 79% [25].

Preprocessing was performed according to the *FloReMi* pipeline: quality control, removal of margin events, doublet removal, compensation, logicle transformation and selection of alive T-cells [26].

FlowSOM was used for clustering with all marker intensities except FSC-C, FSC-H and SSC-A. The number 193 of clusters was set to 400 and additionally, the metaclustering step was performed to obtain different 194 subpopulation resolutions. The differential testing was then performed for a number of clusters of 20, 50, 100 and 195 400. The covariates were the survival time and the condition (stimulated or unstimulated) of the sample. Two 196 random effects were modeled, one on a per sample level and the other on a per patient level. The three main 197 methods (rs. km, mrl; number of imputations equal to 200) plus the complete case analysis were applied. An 198 illustration of how the association between the survival time and the abundance for a cell population looks like is 199 shown in Figure 6. At the top is a cluster with small adjusted p-value while the cluster in the bottom has a high 200 adjusted p-value (as evaluated by mrl). No immediate association is visible, which could have various 201 explanations, including high censoring rate, overdispersion, weak association. 202

Although no ground truth is established (i.e. which cell belongs to which cell population and which (if any) cell population is DA), a comparison to results from other methods (i.e. the original FlowCAP IV submissions) still gives insights into differential discovery performance. For the differential testing, the proportion of significant clusters for multiple cut offs differed substantially (Table 1). In general, the proportion of significant clusters is higher for a lower number of total clusters. While rs and km did not detect any DA clusters, cc found a large proportion of clusters to be significant and mrl has intermediate detection rates.

Based on the proportion of detected clusters (Table 1), a level of 100 clusters was deemed to have a good balance between precision (cell population sizes) and sensitivity (proportion of detected clusters). A closer look at the (unadjusted) p-values of those clusters (at a level of 100 clusters) revealed similarities between the methods: 6 clusters were found in the 10 clusters with lowest (unadjusted) p-value for at least 3 methods. The adjusted p-values for rs and km are much higher than any reasonable significance level, however, cc and mrl have clusters that are differentially abundant. For cc, the proportion of significant clusters seems to be rather high ($\sim 50\%$), which is not unexpected given the poor error control observed in the simulations.

Comparing the marker expressions of those "top" 6 clusters (Supplementary Figure S6) with the discovered subpopulations in the FlowCAP IV challenge reveals some similarities. For example, Cluster 9 matches the described population of CD3+ CD4- CD14/VIVID+ CD57- cells [25]) and cluster 38 is similar to the CD4-CD27- CD107a- CD154- CD45RO- population described in FloReMi [26].

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Number of cluster	cut off	mrl	\mathbf{rs}	$\rm km$	cc
20	0.01	0.35	0	0	0.75
	0.05	0.55	0	0	0.9
	0.1	0.85	0	0.1	0.95
50	0.01	0.3	0	0	0.62
	0.05	0.48	0	0	0.76
	0.1	0.66	0	0	0.8
100	0.01	0.13	0	0	0.49
	0.05	0.32	0	0	0.59
	0.1	0.39	0	0	0.64
400	0.01	0	0	0	0.0575
	0.05	0	0	0	0.07
	0.1	0	0	0	0.108

Table 1. Proportion of significant clusters for different total number of clusters (20, 50, 100, 200) for different significant levels (0.01, 0.05, 0.1) after multiple hypothesis correction in the case study. cc: complete case analysis, km Kaplan-Meier imputation, mrl mean residual life imputation (conditional multiple imputation), rs Risk set imputation.

Discussion

In differential abundance analysis with a variable subject to censoring, existing methods make use of classical 221 survival analysis methods, such as the Cox proportional hazard model. In particular, this would model the 222 observed cell population abundances as predictors. The use of a reversed approach (cell population abundance as 223 response), however, has the benefit to directly include confounders such as batch or age. The problem is that this 224 reversed approach leads to a censored predictor, which renders standard differential abundance analysis 225 inapplicable. A workaround to this issue is the use of multiple imputation, where the imputation step is 226 specifically designed to handle censored values. Simulation studies indicate that in general, there is a gain by 227 including the censored data instead of discarding samples (complete case analysis; cc) or treating censored values 228 as missing (predictive mean matching; *pmm*). 229

More specifically, the basic simulations revealed consistent but slightly biased parameter estimation for the 230 related methods rs, km and kme, and the simulations modeled from real data showed similar or increased 231 performance in terms of sensitivity compared to cc but with better error control. Parameter estimation with mrl 232 on the other hand was unbiased in the basic simulation, but the coverage rate was higher than expected, which 233 typically leads to conservative performance. In the simulations modeled from real data, the conservative 234 performance of mrl was apparent for low FDR, while the TPR was often (especially for higher censoring rates) 235 higher than for other methods. In the case study (no ground truth), only mrl and cc were able to detect 236 differentially abundant cell (sub) populations although especially for cc, the number of detected clusters was high, 237 which could indicate many false positives. But since for mrl the FDR was in the simulations in general very low, 238 this could mean that indeed many clusters are differentially abundant or alternatively, the real data is 230 substantially different in structure compared to the simulations. For example, the simulations assumed a missing 240 data mechanism that is missing-completely-at-random (MCAR), which might not be given in this case. 241 Especially for cc, a missing data mechanism different from MCAR could be a problem since it is known to be 242 biased under this condition. On the other hand, mrl (and rs and km) should be able to handle certain 243 missing-at-random (MAR) cases [33], although this was not directly confirmed here. 244

The methods considered for direct inclusion of a censored covariate all rely on multiple imputation, which has 245 the advantage of high interpretability since the underlying statistical models are classical GLMMs. A 246 disadvantage are high computing costs caused by the need for repeated imputations (e.g. for high censoring rates, 247 runtimes of 1 h instead of 1 min); runtimes can be nonetheless reduced through parallelization. The resolution at 248 which to analyze is another issue, since a high number of clusters may reduce the statistical power imposed by 249 multiple hypothesis correction, while associations with rare cell populations might be overlooked for a low total 250 number of clusters. If a hierarchical structure of the cell populations is available (e.g. via metaclustering in 251 FlowSOM), tree-based aggregated hypothesis testing methods (e.g. treeclimbR [35]) could increase differential 252

discovery performance. Additional improvements of the differential discovery performance could be achieved by the use of a different analysis method such as edgeR or voom, which were shown to have increased performance compared to GLMM [18]. A further issue is of general nature: testing the association with a continuous (censored) covariate requires larger sample sizes compared to the testing with a binary covariate, although this nonetheless also depends on the dispersion and the strength of the association. 254

Conclusion

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Statistical modeling with a high proportion of censored data is always challenging, but even more so in DA settings with often overdispersed data and the need for multiple hypothesis testing correction. Nonetheless, we showed that including censored variables as a predictor in GLMMs results in high error control and decent sensitivity for a subset of the tested methods. Compared to classical survival analysis methods, such as the Cox proportional hazard model, higher flexibility in testing is provided, reflecting the need in typical experimental and clinical setups.

The tested methods were implemented in R and are submitted to Bioconductor. The source code is also available on GitHub https://github.com/retogerber/censcyt. Scripts for reproducing results and figures can be found on https://github.com/retogerber/censcyt_paper_scripts.

Methods

Censoring

The data mechanism for simulating censored data is based on the one described in Atem *et al.* [33]. The variable X to be censored is drawn from a Weibull distribution with scale λ_x and shape κ_x with the following parameterization:

$$f(x) = \frac{\kappa}{\lambda} \left(\frac{x}{\lambda}\right)^{\kappa-1} e^{-\left(\frac{x}{\lambda}\right)^{\kappa}}$$

with the scale parameter $\lambda > 0$, the shape parameter $\kappa > 0$ and $x \ge 0$. A second variable C that corresponds to the censoring time is also drawn from a Weibull distribution, but with different shape and scale parameters. The observed value T is then the minimum of X and C. In summary:

$$X \sim Weibull(\lambda_t, \kappa_t)$$
$$C \sim Weibull(\lambda_c, \kappa_c)$$
$$T = Min(X, C)$$

The parameters of the Weibull distributions are derived from the FlowCAP IV dataset [25]. More precisely λ_x and κ_x are obtained by fitting a Weibull distribution on the full dataset (taking into account censoring), while λ_c is from fitting only on the censored samples. κ_c is then calculated by first defining the desired censoring rate and then solving for κ_c (by calculating the probability $P(C < X) = \int_0^\infty \int_0^x f(c)f(x)dcdx$, which can be seen as the expected censoring rate, for different values of κ_c).

Single cluster simulation

For the basic simulations with only a single cluster, the counts Y_j (number of cells) with $j \in 1..N$ was sampled from a generalized linear mixed model with a logit link function where the response (the number of cells) followed a binomial distribution (Eqn. 1) where T_j follows a Weibull distribution with parameter as described above estimated from the FlowCAP IV dataset [25], the regression coefficients were set to $b_0 = -2$, $b_1 = -0.0001$ and $b_2 = 1, Z_j \in \{0, 1\}$ is a binary covariate with balanced groups, R_j is an observation level random effect to model overdispersion distributed according to a standard normal distribution ($\sigma^2 = 1$) and n_j is the sample size distributed according to a uniform distribution with a minimum limit of 10'000 and a maximum limit of 100'000.

Multiple cluster simulation

The matrix of counts $Y \in \mathbb{R}^{K \times N}$ for K clusters (cell populations) and N samples follows a Dirichlet-Multinomial (DM) distribution (Eqn. 2) for $j \in \{1...N\}$ where n_j is the total number of cells in sample j and $A = (\boldsymbol{\alpha}_1^T \dots \boldsymbol{\alpha}_N^T) \in \mathbb{R}^{K \times N}$ with $A_{ij} > 0$ for $i \in \{1...N\}$ are the concentration parameters dependent on covariates T_j and Z_j . Additionally $Y_j = (Y_{1j} \dots Y_{kj}), T_j \sim Weibull(\lambda, \kappa)$ and $Z_j \in \{0, 1\}$ is a binary variable with balanced groups. The proportions of cells in cluster i in sample j is simply

$$\pi_{ij} = \frac{Y_{ij}}{\sum_{l=1}^{K} Y_{lj}}$$

An association for cluster i is then assumed to be the following:

$$E(\pi_{ij}|T_j, Z_j) = logit^{-1}(\beta_{0i} + \beta_{1i}T_j + \beta_{2i}Z_j)$$
(4)

with an intercept β_{0i} , a slope β_{1i} for T_j and a slope β_{2i} for Z_j . The β 's are therefore fixed for a cluster but are different between clusters. The covariates T_j and Z_j are specific for a sample but not a cluster. The proportions π_j for sample j follow a Dirichlet distribution, meaning the π_{ij} themselves follow a Beta distribution with mean

$$E(\pi_{ij}|T_j, Z_j) = \frac{A_{ij}}{\sum_{l=1}^{K} A_{lj}}$$
(5)

This allows to combine Eqn. 4 and Eqn. 5 leading to Eqn. 6 (which is the same as Eqn. 3):

$$A_{ij} = logit^{-1}(\beta_{0i} + \beta_{1i}T_j + \beta_{2i}Z_j)A_{\bullet j}$$

$$\tag{6}$$

with the sum of the concentration parameters for a sample $A_{\bullet j} = \sum_{l=1}^{K} A_{lj}$. This means that $A_{ij}|T_j, Z_j$ is 289 dependent on six values: the intercept β_{0i} , the first slope β_{1i} , the continuous covariate T_j , the second slope β_{2i} , 290 the binary covariate Z_j and $A_{\bullet j}$. Since the sum $A_{\bullet j}$ depends on all A_{ij} for a given sample j, this means that in 291 order to keep this sum equal across samples, for every A_{ij} that is increased with increasing T_j there has to be an 292 A_{ij} that decreases the same amount. Because of the non-linearity of the logit function this could lead to a very 293 weak association of the second A_{ii} (which would not strictly follow the logit relationship). The strategy is 294 therefore to allow small discrepancies of the sum $A_{\bullet i}$ in order to get the specified associations. To decrease the 295 variation of the sum $A_{\bullet i}$ two clusters of similar proportion are chosen. To obtain β_0 and β_1 , the desired minimum / maximum mean proportion π_{ij} for $max(T_j)$ is determined and then Z_j is set to zero to solve for β_0 297 and β_1 . This will result in a sum $A_{\bullet j}$ that is exactly the same at $T_j = 0$ and $T_j = max(T_j)$. All sums $A_{\bullet j}$ in 298 between will slightly deviate but this deviation is too small to detect under the simulation conditions considered 299 here. To obtain β_{2i} , a difference of the mean abundance at $T_i = 0$ is specified, which then allows to calculate β_{2i} . 300 In short, the β 's are calculated by specifying border constraints, consisting of maximum differences in the mean 301 abundance dependent on the covariates. Because it was observed that the spread of the simulated data was 302 higher than in the real dataset, the concentration parameters were multiplied by a factor of five (keeping the 303 expected counts per cluster the same) to reduce the variance of counts. 304

Multiple imputation

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The goal of multiple imputation is not to replace the missing or censored values by estimates but rather to find a parameter estimate of the statistical model being tested that is *unbiased and confidence valid* [34].

Multiple imputation consists of three main steps [34]: Imputation, Analysis, Pooling. In the first step, multiple 308 complete datasets are generated by replacing the incomplete values with a random draw from a set of possible 309 true values. This can, for example, be the assumed or empirical distribution of the incomplete value. In the 310 second step, each completed dataset is individually analysed, e.g. by fitting a regression model. In the third step, 311 the results from the second step are combined using Rubin's rules [36] that consider the additional variances in 312 the analysis. A slight variation is the use of *Resampling* in the first step. Before imputation, a bootstrap sample 313 is drawn, which is then the new incomplete dataset where the missing values get replaced. One of the advantages 314 of this approach: the incomplete value can be replaced by a deterministic quantity of the data (e.g. the mean), 315 which would not work in classical multiple imputation (each imputed dataset would be the same). A drawback is 316 that *Resampling* techniques are based on large-sample theory and might not work properly for small samples [29]. 317

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DA

The presented DA methods are based on the GLMM approach in *diffcyt* which consists of fitting a generalized linear mixed model with a logit link function for each cell population, testing and multiple hypothesis testing correction. When a censored covariate is present, multiple imputation is used to handle the additional uncertainties of the parameters caused by incomplete data. The imputation methods are described in the following.

In complete case analysis (*cc*, also known as listwise deletion [34]), only the observed values ($T_i | T_i < C_i$ for $j \in \{1...N\}$) are used by discarding all incomplete samples.

The risk set imputation (rs) [31] first constructs the risk set $R(T_i) = \{T_j | T_j > T_i\}$ for $j \in \{1...N\}$ for all $T_l | T_l < X_l$ with $l \in \{1...N\}$ and second, randomly draws one of those as the imputed value. If censoring depends on a covariate, the risk set is calculated as described in (Hsu et al.) [37], incorporating the idea of predictive mean matching.

The Kaplan-Meier imputation (km) [31] is similar to risk set imputation. It first constructs the risk set $R(T_l)$ for all $T_l | T_l < X_l$ and then estimates the survival function with the Kaplan-Meier estimator for each of those sets. A random event time according to the survival curve is drawn and replaces the censored value.

Conditional multiple imputation [33] (labeled here as mean residual life imputation (mrl)) is based on the mean residual life, which is the expected remaining survival time until an event happens

$$mrl(t) = E(X - t|X > t) = \frac{\int_t^\infty S(u)du}{S(t)}$$

$$\tag{7}$$

with the random variable X representing the true (unobserved) survival time and S(t) the survival function. It 335 can be used to get an estimate of how long it will take until an event happens given that the event did not 336 happen yet. Conditional single imputation [33] (Conditional multiple imputation with only one imputation) 337 imputes censored values by adding the corresponding mean residual life. First a survival curve $S(\mathbf{T})$ (using the 338 Kaplan-Meier estimator) is fitted and then the mean residual life is added to the censored value [33]. If censoring 339 depends on a covariate, $S(\mathbf{T})$ can be fitted using the Cox-proportional hazards model [27]. Mean residual life 340 imputation (Conditional multiple imputation) can not be used in the normal multiple imputation set up since all 341 imputed datasets would be the same. Instead *Resampling* is applied to first generate incomplete datasets before 342 imputation. 343

The estimation of S(T) is done without any distributional assumptions resulting in a high data dependency. 344 If the sample size is small and/or many values are censored the estimation can be drastically different from the 345 true (unobserved) survival function. Especially towards the tails, as data gets even sparser, estimation is difficult. 346 If the highest measured value is censored, $S(\mathbf{T})$ does not reach its theoretical minimum (zero). The usual way to 347 deal with this problem is to treat the maximum value as if it was observed. Another possibility is to make a 348 distributional assumption for the tail of the survival function. This was explored for the method Kaplan-Meier 349 imputation by assuming an exponential tail, which is referred to here as kme (based on [38]). 350

Unfortunately, there is no clear rule as to how many imputations are needed [34]. In general, this depends on 351 (among other things) the censoring rate; higher censoring requires more imputations. Two methods to estimate 352 the number of imputations are based on a linear rule [39] and a quadratic rule [40]. Only minor changes in the 353 results after around 50 imputations could be seen in our case leading to the use of 50 imputations as the default. 354

Case study

Following are some clarifications of the description in the main text. The raw flow cytometry data is available 356 under http://flowrepository.org/id/FR-FCM-ZZ99. The data set consists of 766 PBMC samples linked to time to progression to AIDS from HIV+ of 383 patients. For each patient, two samples (measured at the same 358 time) are available: one untreated and one treated with HIV-Gag proteins. 359

Preprocessing was performed according to the FloReMi pipeline [26]: First, quality control by removing cells 360 within a certain time sampling interval where the median FSC-A value differed dramatically from tolerable limits. 361 Then, removal of margin events by removing cells that have a minimum and maximum value for some channel. 362 Next, the selection of single cells by removing cells whose FSC-A to FSC-H ratio was larger than the median 363 ratio plus two times the standard deviation of the ratios. Next, compensation with the given spillover matrices 364

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(from the .fcs files) was applied, data was logicle transformed and alive T-cells were gated (using <i>flowDensity</i>) using channels V-Amine/CD14 and CD3 and selection of V-Amine/CD14-CD3+ population. In the differential testing a transformed survival time, according to $s_{trans} = log_e(s + 11)$ (the +11 is to obtain only positive values since the lowest survival time is -10), was used.	365 366 367 368
Acknowledgements	369
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Abbreviations	375
scRNA-seq single-cell RNA sequencing	376
DA differential abundance	377
DS differential state	378
GLMM generalized linear mixed model	379
RB raw bias	380
CR coverage rate	381
CI confidence interval RMSE root mean squared error	382
km Kaplan-Meier imputation	383 384
kme Kaplan-Meier imputation with an exponential suvival function tail	385
rs risk set imputation	386
mrl mean residual life imputation	387
pmm predictive mean matching	388
cc complete case analysis	389
DM dirichlet-multinomial distribution	390
TPR true positive rate FDR false discovery rate	391
r DR laise discovery rate	392
Availability of data and materials	393
The tested methods were implemented in R and are submitted to Bioconductor. The source code is also available	394
on GitHub https://github.com/retogerber/censcyt. Scripts for reproducing results and figures can be	395
found on https://github.com/retogerber/censcyt_paper_scripts.	396
Ethics approval and consent to participate	397
Not applicable.	398
Competing interests	399
	555
The authors declare that they have no competing interests.	400

Consent for publication

Not applicable.

Authors' contributions

RG and MDR developed methods, designed analyses, and wrote the manuscript. RG implemented methods and 404 performed analyses. All authors read and approved the final manuscript.

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Ado	ditional Files	506
Add	itional file 1 — Supplementary figures	507
Addit	ional Figures, as mentioned in the main text.	508

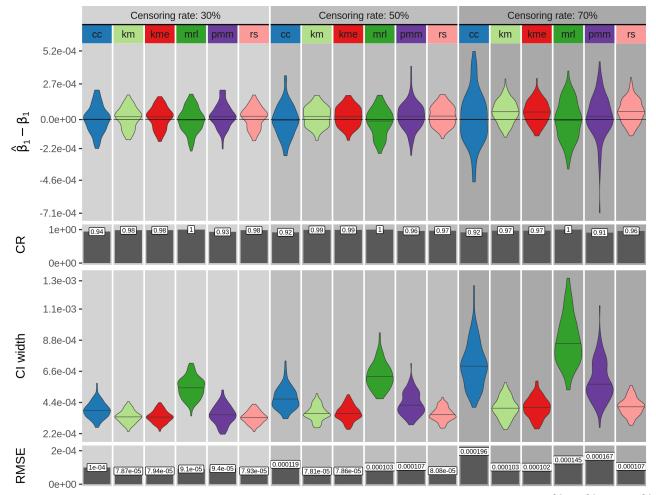


Figure 1. Single cluster simulation results for a sample size of 100 for censoring rates of 30%, 50% and 70%. Shown are four measures calculated from 100 simulation repetitions: difference of the estimated regression coefficient ($\hat{\beta}_1$) and its true value (β_1), coverage rate (CR), confidence interval (CI) width and root mean squared error (RMSE). *cc*: complete case analysis, *km*: Kaplan-Meier imputation, *kme*: Kaplan-Meier imputation with an exponential tail, *mrl*: mean residual life imputation (conditional multiple imputation), *pmm*: predictive mean matching (treating censored values as missing), *rs*: risk set imputation. Other parameter values are: true regression coefficient $\beta_1 = -1e - 4$, number of multiple imputations = 50 and the variance of the random effect = 1.

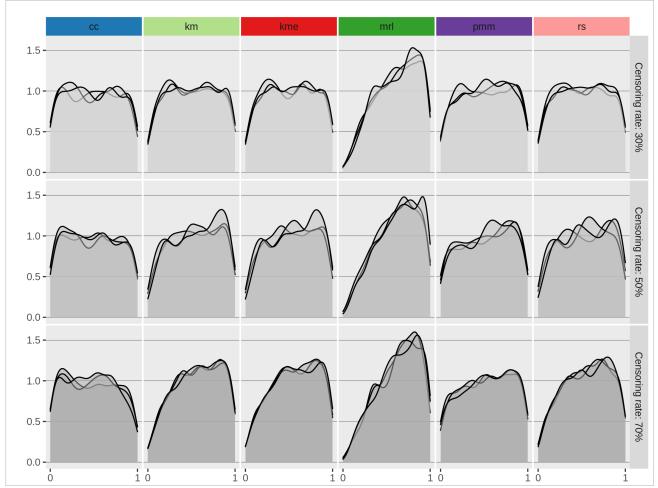


Figure 2. Single cluster simulation p-value distribution under the null model for three different censoring rates (30%, 50%, 70%). *cc*: complete case analysis, *km* Kaplan-Meier imputation, *kme* Kaplan-Meier imputation with an exponential tail, *mrl* mean residual life imputation (conditional multiple imputation), *pmm* predictive mean matching (treating censored values as missing), *rs* Risk set imputation. Each line represents 1000 repetitions.

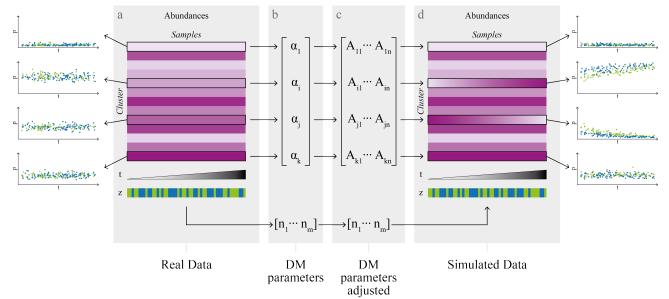


Figure 3. Simulation schema for multiple cell populations. (a) Starting with a $cluster \times sample$ matrix of abundances from a real dataset (b) a dirichlet-multinomial (DM) distribution is fitted. (c) The DM parameters are expanded and adapted to include an association of the abundances with a continuous covariate t and a binary covariate z. (d) A new dataset is simulated from the new parameters.

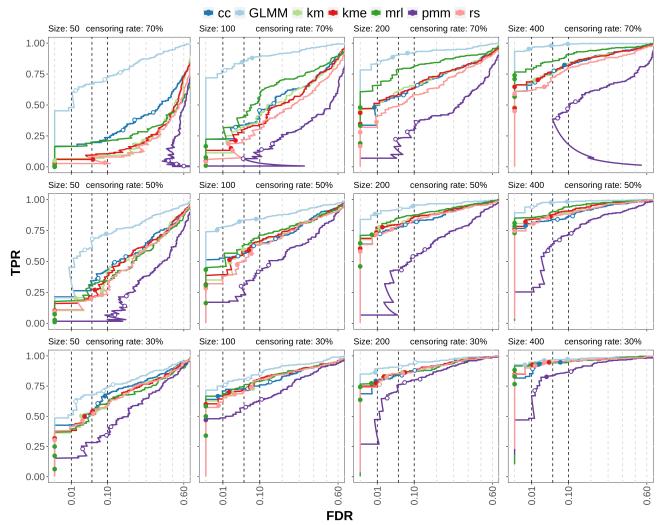


Figure 4. Multiple cluster simulation results testing for the association of the censored covariate. TPR-FDR curves for censoring rates of 30%, 50% and 70% (rows) and samples sizes of 50,100,200 (columns). The x-axis is square root transformed. *cc*: complete case analysis, *km*: Kaplan-Meier imputation, *kme*: Kaplan-Meier imputation with an exponential tail, *mrl*: mean residual life imputation (conditional multiple imputation), *pmm*: predictive mean matching (treating censored values as missing), *rs*: Risk set imputation. *GLMM* uses the (unobserved) ground truth of the survival time and can be considered to be the maximum possible performance of the other methods.

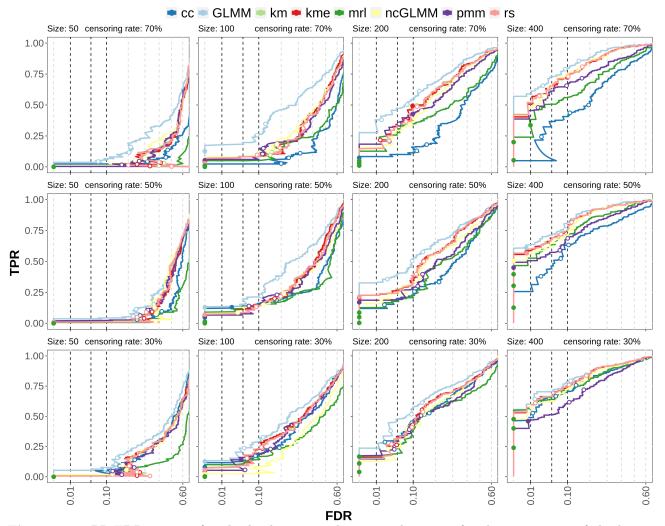


Figure 5. TPR-FDR curves of multiple cluster simulations with testing for the association of the binary covariate. Rows are censoring rates (30%, 50% and 70%) and columns are samples sizes (50, 100, 200 and 400). The x-axis is square root transformed. cc: complete case analysis, km: Kaplan-Meier imputation, kme: Kaplan-Meier imputation with an exponential tail of the survival function, mrl: mean residual life imputation (conditional multiple imputation), pmm: predictive mean matching (treating censored values as missing), rs: risk set imputation. GLMM uses the (unobserved) ground truth of the survival time and can be considered to be the maximum possible performance of the other methods. ncGLMM: same as GLMM but uses only the binary covariate for fitting and testing.

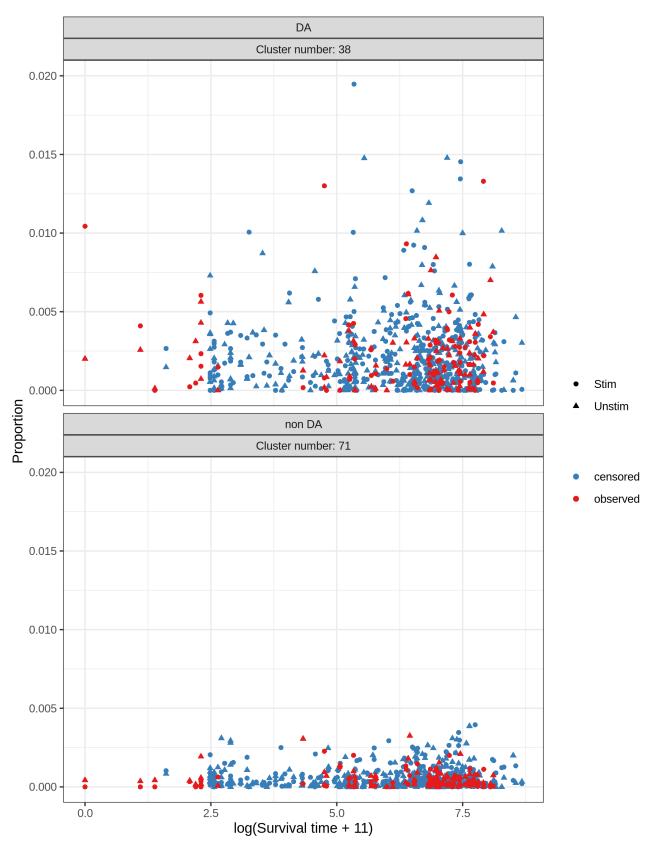


Figure 6. Association between cluster proportion and survival time for two clusters, one DA (top) and one non DA (bottom), evaluated with mrl for a total number of clusters of 100. The survival time is translated to get only positive values and then log transformed. Scaling of the axis in the upper plot removed 7 data points (0.9%).