# The limitations of correlation-based inference in complex virus-microbe communities

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#### 1 Abstract

Microbes are present in high abundances in the environment and in human-associated microbiomes, often exceeding one million per milliliter. Viruses of microbes are present in even higher abundances and are important in shaping microbial populations, communities, and ecosystems. Given the relative specificity of viral infection, it is essential to identify the functional linkages between viruses and their microbial hosts, particularly given dynamic changes in virus and host abundances. Multiple approaches have been proposed to infer infection networks from time-series of in situ communities, among which correlation-based approaches have emerged as the de facto standard. In this work, we evaluate the accuracy of correlation-based inference methods using an in silico approach. In doing so, we compare predicted networks to actual networks to assess the self-consistency of correlation-based 11 inference. At odds with assumptions underlying its widespread use, we find that correlation is a poor predictor of interactions in the context of viral infection and lysis of microbial 13 hosts. The failure to predict interactions holds for methods which leverage product-moment, time-lagged, and relative-abundance based correlations. In closing, we discuss alternative 15 inference methods, particularly model-based methods, as a means to infer interactions in complex microbial communities with viruses.

# 18 2 Importance

Inferring interactions from population time-series is an active and ongoing area of research. It is relevant across many biological systems – in particular in virus-microbe communities, but also in gene regulatory networks, neural networks, and ecological communities broadly. Correlation-based inference – using correlations to predict interactions – is
widespread. However, it is well known that "correlation does not imply causation". Despite
this, many studies apply correlation-based inference methods to experimental time-series
without first assessing the potential scope for accurate inference. Here, we find that several
correlation-based inference methods fail to recover interactions within *in silico* virus-microbe
communities, raising questions on their relevance when applied *in situ*.

## 28 3 Introduction

Viruses of microbes are ubiquitous and highly diverse in marine, soil, and humanassociated environments. Viruses interact with their microbial hosts in many ways. For
example, they can transfer genes between microbial hosts [1, 2], alter host physiology and
metabolism [3, 4], and redirect the flow of organic matter in food webs through cell lysis [5, 6].
Viruses are important parts of microbial communities, and characterizing the interactions
between viruses and their microbial hosts is critical for understanding microbial community
structure and ecosystem function [5, 7, 8, 9].

A key step in characterizing virus-microbe interactions is determining which viruses can infect which microbes. Viruses are known to be relatively specific but not exclusive in their microbial host range. Individual viruses may infect multiple strains of an isolated microbe or they may infect across genera as part of complex virus-microbe interaction networks [10, 11]. For example, cyanophage can infect both *Prochlorococcus* and *Synechococcus* which are two distinct genera of marine cyanobacteria [12]. However, knowledge of viral host range remains limited because existing experimental methods for directly testing for viral infection are generally not applicable to an entire *in situ* community. Culture-based methods such

as plaque assays are useful for checking for viral infection at the strain level and permit
high confidence in their results, but they are not broadly applicable as many viruses and
microbes are difficult or currently impossible to isolate and culture [1]. Partially cultureindependent methods, such as viral tagging [13, 14] and digital PCR [15], overcome some of
these hurdles but only for particular targetable viruses and microbes. Similarly, single-cell
genome analysis is able to link individual viruses to microbial hosts [16, 17, 18] but for a
relatively small number of cells.

Viral metagenomics offers an alternate route for probing virus-microbe interactions for
entire in situ communities, bypassing culturing altogether [19, 20, 21]. The viral sequences
obtained from metagenomes can be analyzed directly using bioinformatics-based methods

to predict microbial hosts [22, 23] although such methods may only be appropriate for a subset of viruses (phages and archaeal viruses but not eukaryotic viruses) and putative hosts (prokaryotes but not eukaryotes). Alternatively, metagenomic sampling of a community *over time* can provide estimates of the changing abundances of viral and microbial populations at high time- and taxonomic- resolution. Once these high-resolution time-series are obtained,

they can be used to predict virus-microbe interactions using a variety of statistical and

mathematical inference methods (see reviews [24, 25, 26, 27, 28]).

Correlation and correlation-based methods are among the most widely used network inference methods for microbial communities [25]. For example, Extended Local Similarity Analysis (eLSA) is a correlation-based method which allows for both local and time-lagged correlations [29, 30, 31] and has been used to infer interaction networks in communities of marine bacteria [32, 33]; bacteria and phytoplankton [34, 35]; bacteria and viruses [36]; and bacteria, viruses, and protists [37, 38]. In addition, several correlation-based methods have been developed to address challenges associated with the compositional nature of '-omics' datasets [39, 25], including Sparse Correlations for Compositional data (SparCC) [40].

Regardless of the particular details of these methods, all correlation-based inference opro erates on the same core assumption: that interacting populations trend together (are cor-

related) and that non-interacting populations do not trend together (are not correlated). Particular correlation-based methods may relax or augment this assumption. For example, with eLSA the trends may be time-lagged [29, 30, 31]; with simple rank correlations the trends may be non-parametric; and with compositional methods like SparCC the trends may occur between ratios of relative abundances [40]. In communities with only a few pop-75 ulations and simple interactions, population trends may indeed be indicative of ecological mechanism. In these contexts, some correlation-based methods have been shown to recapitulate microbe-microbe interactions with limited success [25]. Typically however the challenge of inferring interaction networks applies to diverse communities and complex ecological interactions. Microbial communities often have dozens, hundreds, or more distinct populations, each of which may interact with many other populations through nonlinear mechanisms such 81 as viral lysis, as well as be influenced by fluctuating abiotic drivers. In these contexts, the relationship between correlation and ecological mechanism is poorly understood. Often correlations do not have a simple mechanistic interpretation, a well-known adage ("correlation does not imply causation") that is often disregarded. Despite the challenge of interpretation, correlation-based inference methods are widely 86 used with in situ datasets [29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 25, 40]. Benchmarking inferred networks – connecting correlations to specific ecological mechanisms – is difficult. In the context of lytic infections of environmental microbes by viruses, there is (usually) no existing "gold standard" interaction network for which to validate inferred interactions. Therefore, in this work, we take an *in silico* approach to assess the accuracy of correlationbased inference. To do this, we simulate virus-microbe community dynamics with an interaction network which is prescribed a priori and use it to benchmark inferred networks. Several existing studies have applied similar in silico approaches in the case of both microbe-microbe and microbe-virus interactions and found that simple Pearson correlation [41, 39] and several correlation-based methods [25] either fail or are inconsistent in recapitulating interaction networks. Here, we provide an in-depth assessment of the potential for correlation-based

inference in diverse communities of microbes and viruses. As we show, correlation-based inference fails to recapitulate virus-microbe interactions and performs worse in more diverse communities. The failure of correlation-based inference in this context raises concerns over its use in inferring microbe-parasite interactions as well as microbe-predator and microbe-microbe interactions more broadly.

#### 4 Methods

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## 4.1 Dynamical model of a virus-microbe community

We model the ecological dynamics of a virus-microbe community with a system of nonlinear differential equations:

$$\dot{H}_{i} = r_{i}H_{i}\left(1 - \frac{\sum_{i'}^{N_{H}} a_{ii'}H_{i'}}{K}\right) - H_{i}\sum_{j}^{N_{V}} M_{ij}\phi_{ij}V_{j}$$

$$(1)$$

$$\dot{V}_{j} = V_{j} \sum_{i}^{N_{H}} M_{ij} \phi_{ij} \beta_{ij} H_{i} - \underbrace{m_{j} V_{j}}_{\text{viral decay}}$$
(2)

our purposes, a "population" is a group of microbes or viruses with identical life history 110 traits, that is microbes or viruses which occupy the same functional niche. 111 In the absence of viruses, the microbial hosts undergo logistic growth with growth rates 112  $r_i$ . The microbial hosts have a community-wide carrying capacity K, and they compete 113 with each other for resources both inter- and intra-specifically with competition strength 114  $a_{ii'}$ . Each microbial host can be infected and lysed by a subset of viruses determined by the 115 interaction terms  $M_{ij}$ . If microbial host i can be infected by virus j,  $M_{ij} = 1$ ; otherwise 116  $M_{ij} = 0$ . The collection of all the interaction terms is the interaction network represented by matrix M of size  $N_H$  by  $N_V$ . The adsorption rates  $\phi_{ij}$  denote how frequently microbial

where  $H_i$  and  $V_j$  refer to the population density of microbial host i and virus j respectively.

There are  $N_H$  different microbial host populations and  $N_V$  different virus populations. For

host i is infected by virus j.

Each virus j's population grows from infecting and lysing their hosts. The rate of virus j's growth is determined by its host-specific adsorption rate  $\phi_{ij}$  and host-specific burst size  $\beta_{ij}$ , which is the number of new virions per infected host cell. The quantity  $\tilde{M}_{ij} = M_{ij}\phi_{ij}\beta_{ij}$  is the effective interaction strength between virus j and host i, and the collection of all the interaction strengths is the weighted interaction network  $\tilde{\mathbf{M}}$ . Finally, the viruses decay at rates  $m_j$ .

## 6 4.2 Generating interaction networks and characterizing network structure

Virus-microbe interaction networks, denoted  $\mathbf{M}$ , are represented as bipartite networks or matrices of size  $N_H$  by  $N_V$  where  $N_H$  is the number of microbial host populations and  $N_V$  is the number of virus populations. The element  $N_{ij}$  is 1 if microbe population i and virus population j interact and 0 otherwise. In this paper, we consider only square networks  $N_V = N_V = N_V$  although the analysis is easily extended to rectangular networks. We consider three network sizes N = 10, 25, 50.

For each network size N, we generate an ensemble of networks varying in nestedness and modularity (Fig 1). We first generate the maximally nested (Fig 1A) and maximally modular (Fig 1B) networks of size N using the BiMat Matlab package [42]. In order to achieve maximal nestedness and modularity, the network fill F (fraction of interacting pairs) is fixed at F = 0.55 for the nested networks and F = 0.5 for the modular networks. For the modular networks, the number of modules is set to 2, 5, and 10 for the three network sizes respectively.

To generate networks that vary in nestedness and modularity, we perform the following "rewiring" procedure. Beginning with the maximally nested or maximally modular network, we randomly select an interacting virus-microbe pair  $(M_{ij} = 1)$  and a non-interacting virus-microbe pair  $(M_{i'j'} = 0)$  and exchange their values. We do not allow exchanges that would result in an all-zero row or column, as that would isolate the microbe or virus population from the rest of the community. We continue the random selection of pairs without

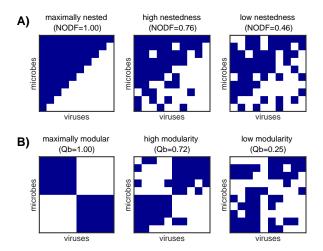


Figure 1: Example interaction networks characterized by A) nestedness and B) modularity. The networks shown here have size N = 10 and fill A) F = 0.55 and B) F = 0.5. Within each network, rows represent microbe populations and columns represent virus populations, while navy squares indicate interaction  $(M_{ij} = 1)$ . Networks were generated according to §4.2. Nestedness (NODF) and modularity  $(Q_b)$  were measured with the BiMat package and are arranged in their most nested or most modular forms [42].

replacement until the desired nestedness or modularity has been achieved. To calculate nestedness and modularity, we use the default algorithms in the BiMat Matlab package. The
nestedness metric used is NODF [43], and the algorithm used to calculate modularity is
AdaptiveBRIM [44]. The modularity is additionally normalized according to a maximum
theoretical modularity as detailed in [45].

## 4.3 Choosing life history traits for coexistence

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The life history traits for a given interaction network are chosen to ensure that all microbial host and virus populations can coexist, adapted from [46].

First we sample target fixed point densities  $H_i^*$  and  $V_j^*$  for each microbial host and virus population. In addition we sample adsorption rates  $\phi_{ij}$  and burst sizes  $\beta_{ij}$ . All of these parameters are independently and randomly sampled from uniform distributions with biologically feasible ranges specified in Table 1. We use a fixed carrying capacity density  $K = 10^6$  cells/mL for all parameter sets.

Next we sample microbe-microbe competition terms  $a_{ii'}$ . We introduce an additional

Table 1: Sampling ranges for parameters in the virus-microbe dynamical model (Eqns 1 and 2).

	parameter	sampling range	units
$H_i^*$	host $i$ target steady-state density	$10^3 - 10^4$	cells/mL
$V_j^*$	virus $j$ target steady-state density	$10^6 - 10^7$	virions/mL
K	community-wide host carrying capacity	$10^{6}$	$\rm cells/mL$
$\phi_{ij}$	adsorption rate of virus $j$ into host $i$	$10^{-7} - 10^{-6}$	$\mathrm{mL}/\left(\mathrm{virion}\cdot\mathrm{day}\right)$
$\beta_{ij}$	burst size of virus $j$ per host $i$	10 - 100	virions/cell
$H_i^{0*}$	host $i$ target steady-state density	$10^3 - 10^6$	cells/mL
	in the absence of viruses		
$a_{ii'}$	competitive effect of host $i'$ on host $i$	0 - 1	

constraint that microbial populations should coexist in the absence of all viruses. To this end, we sample target virus-free fixed point densities  $H_i^{0*}$  from a uniform distribution with a range specified in Table 1. After sampling, the  $H_i^{0*}$  remain fixed. According to Eqn 1, coexistence in the virus-free setting is satisfied when

$$K = \sum_{i'}^{N_H} a_{ii'} H_{i'}^{0*} \tag{3}$$

for each microbial host i. To start, we set all intraspecific competition to one  $(a_{ii} = 1)$  and all interspecific competition to zero  $(a_{ii'} = 0 \text{ for } i' \neq i)$ . Then for each microbial host i we randomly choose an index  $k \neq i$  and sample  $a_{ik}$  uniformly between zero and one. If the updated sum in Eqn 3 does not exceed the carrying capacity K, we repeat for a new index k. Once the carrying capacity is exceeded, we adjust the most recent  $a_{ik}$  so that Eqn 3 is satisfied exactly.

Finally, the viral decay rates  $m_j$  and host growth rates  $r_i$  are computed from the fixed point versions of Eqns 1 and 2:

$$m_j = \sum_{i}^{N_H} M_{ij} \phi_{ij} \beta_{ij} H_i^* \tag{4}$$

$$r_{i} = \left(\sum_{j}^{N_{V}} M_{ij} \phi_{ij} V_{j}^{*}\right) / \left(1 - \frac{\sum_{i'}^{N_{H}} a_{ii'} H_{i'}^{*}}{K}\right)$$
 (5)

## 4.4 Simulating and sampling time-series

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We use Matlab's native ODE45 function to numerically simulate the virus-microbe dy-174 namical model specified in §4.1 with interaction network and life history traits generated as 175 described in  $\S4.2$  and  $\S4.3$ . We use a relative error tolerance of  $10^{-8}$ . Initial conditions are 176 chosen by perturbing the fixed point densities  $H_i^*$  and  $V_j^*$  by a multiplicative factor  $\delta$  where 177 the sign of  $\delta$  is chosen randomly for each microbial host and virus population. We note that 178  $\delta$  can be used to tune the amount of variability in the simulated time-series (see Fig S1). 179 After simulating virus and microbe time-series, we sample the time-series at regularly 180 spaced sample times (every 2 hours) for a fixed duration (200 hours, or 100 samples). There-181 fore, for each virus and each microbe in the community we take S samples at times  $t_1, \ldots, t_S$ . 182 We use the same sampling frequency and the same S for each inference method, except for 183 time-delayed correlation (see §4.5).

#### 185 4.5 Standard and time-delayed Pearson correlation networks

We assume S regularly spaced sample times  $t_1, \ldots, t_S$  for each host type  $H_i$  and each virus type  $V_j$ . The samples are log-transformed, that is  $h_i(t_k) = \log_{10} H_i(t_k)$  and  $v_j(t_k) = \log_{10} V_j(t_k)$  for each sampled time-point  $t_k$ . The standard Pearson correlation coefficient between host i and virus j is then

$$r_{ij} = \frac{\sum_{k=1}^{S} (h_i(t_k) - \bar{h}_i) (v_j(t_k) - \bar{v}_j)}{\sqrt{\sum_{k=1}^{S} (h_i(t_k) - \bar{h}_i)^2} \sqrt{\sum_{k=1}^{S} (v_j(t_k) - \bar{v}_j)^2}}$$
(6)

where  $\bar{h}_i = \frac{1}{S} \sum_{k=1}^{S} h_i(t_k)$  and  $\bar{v}_j = \frac{1}{S} \sum_{k=1}^{S} v_j(t_k)$  are the sample means. The correlation coefficients for all virus-host pairs are represented as a bipartite matrix  $\mathbf{R}$  of size  $N_H \times N_V$  analogous to the interaction network (see §4.2).

Time-delayed correlations are computed by sampling the virus time-series later in time.

Each virus-host pair may have a unique time-delay  $\tau_{ij}$ . For example, if host i is sampled at times  $t_1, \ldots, t_S$  then virus j is sampled at times  $t_1 + \tau_{ij}, \ldots, t_S + \tau_{ij}$ . We keep the number of samples S fixed, and consequently allow virus j to be sampled beyond the final sample time  $t_S$  of the hosts. The time-delayed Pearson correlation coefficient is

$$r_{ij}^{\tau} = \frac{\sum_{k=1}^{S} \left( h_i(t_k) - \bar{h}_i \right) \left( v_j(t_k + \tau_{ij}) - \bar{v}_j^{\tau_{ij}} \right)}{\sqrt{\sum_{k=1}^{S} \left( h_i(t_k) - \bar{h}_i \right)^2} \sqrt{\sum_{k=1}^{S} \left( v_j(t_k + \tau_{ij}) - \bar{v}_j^{\tau_{ij}} \right)^2}}$$
(7)

where  $\bar{v}_j^{\tau_{ij}} = \frac{1}{S} \sum_{k=1}^{S} v_j(t_k + \tau_{ij})$  is the mean of the time-delayed virus sample. As before, the correlation coefficients for all virus-host pairs is a bipartite matrix  $\mathbf{R}^{\tau}$  of size  $N_H \times N_V$ . Pearson correlation coefficients, as specified above, were computed using Matlab's native corr function with type="pearson". Alternate correlation types including Spearman and Kendall are also supported by the corr function and are utilized in the SI.

#### 203 4.6 eLSA networks

Extended Local Similarity Analysis (eLSA) is a correlation-based inference method which 204 is widely used with in situ time-series of complex microbial communities (e.g. [32, 33, 34, 205 35, 36, 37, 38). eLSA attempts to detect local correlations, that is, time-series which trend 206 together for only a portion of the sample period. In addition, eLSA allows for time-delayed 207 correlations (as described in the previous section §4.5). To this end, a "local similarity" 208 (LS) score is computed for each pair of time-series. The LS score is analogous to computing the Pearson correlation for all possible subsections of the two time-series, with offsets up to 210 a pre-decided length, and keeping the maximum absolute correlation. As an example, two 211 time-series may trend strongly during the first half of the sample period but not during the 212 second. For such a pair of time-series, the Pearson correlation would be low, but the LS 213 score would be high. 214 To compute the LS score, the two time-series are first transformed to have normal distri-215 butions (we note that such a transformation is non-stationary and thus may induce spurious 216 correlations). The LS score is the maximal sum of the product of the entries across all possible subsections, normalized by the time-series length. If a pre-defined delay is specified, the subsections are additionally offset from one another from zero up to to the delay amount [29, 30, 31].

We applied eLSA to our simulated time-series data. We used samples of all  $N_H$  host types 221 and all  $N_V$  virus types with S regularly spaced sample times  $t_1, \ldots, t_S$  as input. We used the 222 1sa-compute.py Python script and set parameters to specify the number of sampled points 223 (spotNum=S), number of replicates (repNum=1), number of bootstraps (b=0), and number 224 of permutations (x=1). All other parameters were left with their default settings including 225 the maximum allowed time delay (delayLimit=3). The lsa-compute.py script computes 226 eLSA scores between all virus-host, host-host, and virus-virus pairs. We selected only the 227 virus-host eLSA scores and arranged them in a bipartite matrix of size  $N_H \times N_V$  analogous 228 to the interaction network (see §4.2). We used a custom Matlab script write\_elsa.m to 229 generate .csv data files in the format specified by the eLSA documentation. We used a 230 custom bash script elsa\_compute\_all.sh to run the eLSA analysis on the ensemble of 231 virus-microbe communities. Finally, we used a custom Matlab script read\_elsa.m to import 232 the results into Matlab for scoring (see §4.8). 233

#### 234 4.7 SparCC networks

Sparse Correlations for Compositional data (SparCC) is a correlation-based inference 235 method for use with compositional time-series data. This is relevant for '-omics' data in which 236 abundances are typically relative. It is well known that compositional data pose challenges for 237 standard statistics, including Pearson and other types of correlation. Because the data sum 238 to one, individual time-series are not independent. This biases correlations to be negative 239 regardless the trend between the underlying absolute abundances. SparCC estimates the 240 Pearson correlation between two time-series while taking into account these compositional 241 dependencies. In particular, SparCC computes the variance of the log-transformed ratio of two time-series, and compares this quantity to the variances of the individual log-transformed time-series. SparCC assumes sparsity in the correlation matrix but is robust to violations of this assumption [40].

We applied SparCC to our simulated time-series data as a means to evaluate correlationbased inference in a scenario in which underlying viral and microbial densities can be measured only relatively. Given samples at S regularly spaced sample times  $t_1, \ldots, t_S$ , we first normalized the  $N_H$  host types and  $N_V$  virus types at each sample time  $t_k$  by

$$\mathcal{N}_{H,k} = \sum_{i=1}^{N_H} H_i(t_k) \tag{8}$$

250 for the hosts and by

$$\mathcal{N}_{V,k} = \sum_{j=1}^{N_V} V_j(t_k) \tag{9}$$

for the viruses. We used the normalized  $N_H$  host and  $N_V$  virus samples as input for the SparCC computation using the SparCC.py script. All parameters were left with their default settings. We used a custom Matlab script write\_sparcc.m to generate .csv data files in the format specified by the SparCC documentation. We used a custom bash script sparcc\_compute\_all.sh to run the SparCC analysis on the ensemble of virus-microbe communities. Finally, we used a custom Matlab script read\_sparcc.m to import the results into Matlab for scoring (see §4.8).

#### 8 4.8 Scoring correlation network accuracy

To evaluate how well the Pearson correlation, eLSA, or SparCC (collectively referred to as "correlation") network  $\mathbf{R}$  recapitulates the original interaction network  $\tilde{\mathbf{M}}$ , we compute the receiving operator curve (ROC). First, we binarize the interaction network  $\tilde{\mathbf{M}}$  so that it is a boolean network  $\mathbf{M}$  of zeros (non-interactions) and ones (interactions). Then we choose a threshold of interaction c between the minimum and maximum attainable values of the correlation network  $\mathbf{R}$ ; for Pearson correlation these are -1 and +1. Correlations in  $\mathbf{R}$  that are greater than or equal to c are categorized as interactions (ones), while those that are less are non-interactions (zeros). The true positive (TP) count is the number of interactions in

M correctly predicted by the thresholded correlation network  $\mathbf{R_c}$ . The false positive (FP) count is the number of non-interactions in M incorrectly predicted by  $R_c$ . The TP and FP 268 counts are normalized by the number of interactions and non-interactions in M to obtain 269 the true positive rate (TPR) and false positive rate (FPR). TPR and FPR are computed for 270 all thresholds c to obtain the receiver operator curve (ROC). 271 The overall "score" of the correlation network  $\mathbf{R}$  is the area under the curve (AUC). A 272 perfect prediction results in AUC=1, since for some threshold TPR=1 and FPR=0. Ran-273 dom predictions result in AUC=1/2, since TPR=FPR across all possible thresholds. AUC 274 values which are less than 1/2 indicate a misclassification of "interaction", that is, catego-275 rizing interactions and non-interactions in the opposite way would have resulted in a better 276 prediction of  $\tilde{\mathbf{M}}$ . 277

#### 278 5 Results

## 279 5.1 Standard Pearson correlation

We calculated the standard Pearson correlation networks for an ensemble in silico commu-280 nities that varied in network size and network structure. For each network size N = 10, 25, 50,281 we generated 20 unique interaction networks. 10 of the networks were generated so that they 282 were distributed along a range of nestedness values, and the other 10 were generated so that 283 they were distributed along a range of modularity values (see §4.2). For each interaction 284 network, a single set of life history traits were generated to ensure coexistence using biologi-285 cally feasible ranges according to §4.3. The mechanistic model for the community dynamics 286 is described in §4.1. Time-series were simulated according to §4.4 with  $\delta = 0.3$ , that is, the 287 initial conditions were the fixed point values perturbed by 30% (for additional values of  $\delta$  see 288 Fig S5 in the SI). For  $\delta = 0.3$ , the mean coefficient of variation was 12% for host time-series 280 and 4% for virus time-series (see Fig S1 in the SI). The time-series were sampled during the 290 transient dynamics to represent in situ communities which are likely perturbed from equilibrium due to changing environmental conditions and intrinsic feedback. We sampled the time-series every 2 hours for 200 hours, that is, we took 100 samples (for additional sample frequencies see Fig S7 in the SI).

For each *in silico* community, we calculated the standard Pearson correlation network as described in §4.5. Two example *in silico* communities of size N=10 are shown in Fig 2 with their simulated time-series, log-transformed samples, and resulting correlation networks. The correlation networks were scored against the original interaction networks by computing AUC as described in §4.8. The procedure for computing AUC is shown in Fig 3 for the two example *in silico* communities.

AUC values for all in silico communities are shown in Fig 4. Across varying network 301 size and network structure, AUC is approximately 1/2 implying that standard Pearson cor-302 relation networks lack predictive power. Similar results were found when varying the initial 303 condition perturbation  $\delta$  (Fig S4) and the sampling frequency (Fig S7). There are some cases 304 for the smaller networks (N = 10) where AUC does deviate from 1/2 although these devi-305 ations are small ( $\approx \pm 10\%$ ). Interestingly these deviations tend to be negative indicating a 306 misclassification of the interaction condition, that is, negative correlations are slightly better 307 predictors of interaction than positive correlations. Overall however, the deviations disap-308 pear for larger networks (N = 50) implying that they are exceptions rather than the norm. 309 We completed identical analyses for additional correlation metrics in particular Spearman correlation and Kendall correlation (see Fig S2 in SI). We found similar results reinforc-311 ing our conclusion that simple correlations between time-series are poor predictors of the 312 underlying interaction network. 313

# 5.2 Time-delayed Pearson correlation

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Given the results of the previous section §5.1 – that standard correlations do not recapitulate interactions – we computed time-delayed correlation networks for the same ensemble
of *in silico* communities. The addition of time-delays to standard correlation approaches
is motivated by a large body of theoretical work on predator-prey dynamics, where both
predator and prey populations oscillate but with a phase delay between them [47]. Similar
results hold for the phase delay in simple phage-bacteria dynamics [48]. Time-delayed corre-

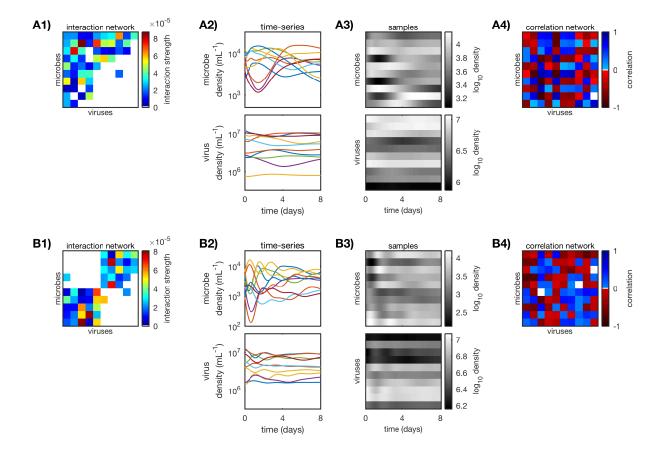


Figure 2: Calculating standard Pearson correlation networks for two *in silico* A) nested and B) modular communities (N=10). A1-B1) Original weighted interaction networks, generated as described in §4.2 and §4.3. A2-B2) Simulated time-series of the virus-microbe dynamical system as described in §4.4 ( $\delta=0.3$ ). A3-B3) Log-transformed samples, sampled every 2 hours for 200 hours from the simulated time-series. A4-B4) Pearson correlation networks, calculated from log-transformed samples as described in §4.5.

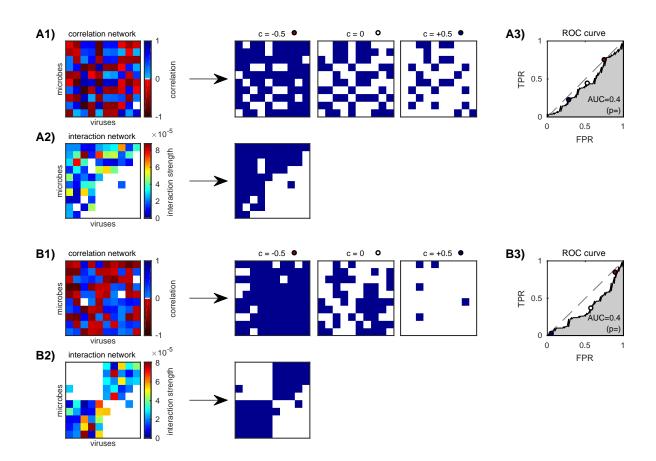


Figure 3: Scoring correlation network accuracy of the two in silico A) nested and B) modular communities (N = 10, see Fig 2) as described in §4.8. A1-B1) Correlation networks are binarized according to thresholds c between -1 and +1, three of which are shown here (c = -0.5, 0, and 0.5). A2-B2) Original interaction networks are also binarized. A3-B3) True positive rate (TPR) versus false positive rate (FPR) of the binarized correlation networks for each threshold c. Three example thresholds (c = -0.5, 0, and 0.5) are marked (red, white, and blue circles). The "non-discrimination" line (grey dashed) is where TPR = FPR. The AUC or area under the ROC curve is a measure of relative TPR to FPR over all thresholds; AUC = 1 is a perfect result. Distributions for the reported p-values are shown in the SI.

lations are the basis of several existing correlation-based inference methods including eLSA [29, 30, 31].

For this analysis, we used the same ensemble of *in silico* communities (network sizes N=10,25,50 of varying nestedness and modularity), simulated time-series ( $\delta=0.3$ ; see Fig S5 in SI), and sample frequency (2 hours; see Fig S8 in SI) as before (see §5.1 for time-series). We calculated the time-delayed Pearson correlation networks as described in §4.5, where for each virus-host pair the virus time-series is sampled later in time by some

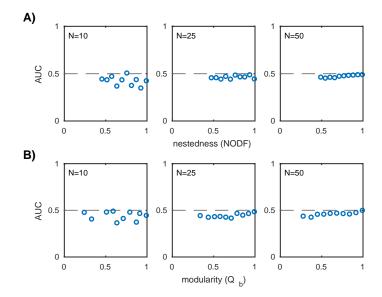


Figure 4: AUC values for standard Pearson correlation for the ensemble of A) nested and B) modular communities over three network sizes N=10,25,50 (20 communities for each network size). AUC is computed as described in §4.8. Each plotted point corresponds to a unique *in silico* community. Dashed line marks AUC=1/2 and implies the predicted network did no better than random guessing.

delay amount  $\tau_{ij}$  relative to the host time-series (for Spearman and Kendall correlation, see 328 Fig S3 in SI). Each delay is chosen such that the absolute value of the correlation for the 329 virus-host pair is maximized. Since the optimal time-delay is not known in advance, delays 330 between  $0 < \tau_{ij} < t_S/2$ , (0 hours and  $t_S/2 = 100$  hours) were considered. The number 331 of samples used to compute each correlation coefficient was kept fixed at S=100 (sample 332 duration 200 hours). Time-delayed Pearson correlation networks for the two example in 333 silico communities of size N=10 are shown in Fig 5A-B. AUC was computed as described 334 in §4.8. 335

AUC values for all *in silico* communities are shown in Fig 5C. For the small networks (N=10) there are a few particular networks which have AUC scores greater than 1/2. For the remaining small networks and the large networks (N=25,50), AUC  $\approx 1/2$  implying time-delayed Pearson correlation lacks predictive power for these networks. Similar results were found for alternate correlation metrics (Spearman and Kendall; Fig S3), initial condition perturbations  $\delta$  (Fig S5), and sampling frequencies (Fig S8). Because AUC deviates from

1/2 for only a few small networks and disappears for large networks, it should be considered
an exceptional result rather than the norm for time-delayed Pearson correlation.

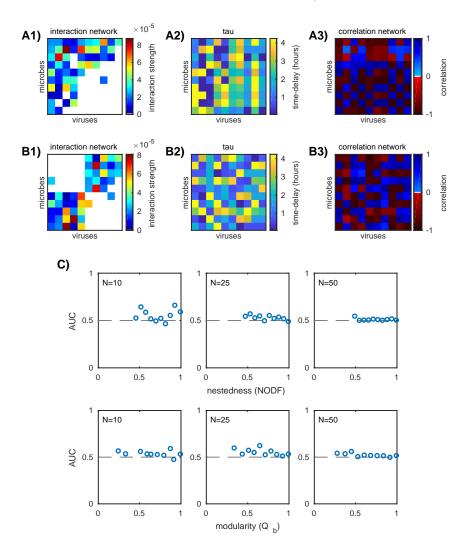


Figure 5: Performance of time-delayed Pearson correlation. A1-B1) Two example in silico interaction networks (N=10). A2-B2) Time-delays  $\tau_{ij}$  for each virus-host pair, chosen so that the absolute value of the correlation is maximized. A3-B3) Time-delayed Pearson correlation networks calculated as described in §4.5. C) AUC values for the ensemble of nested (top row) and modular (bottom row) communities over three network sizes N=10,25,50 (20 communities for each network size). Each plotted point corresponds to a unique in silico community. Dashed line marks AUC=1/2 and implies the predicted network did no better than random guessing.

# 5.3 Correlation-based methods eLSA and SparCC

We performed a similar in silico analysis using eLSA [29, 30, 31] and SparCC [40], two 345 established correlation-based inference methods which are widely used with in situ timeseries data. We used the same ensemble of in silico communities as before (network sizes N=10,25,50 of varying nestedness and modularity), along with the simulated time-series  $(\delta = 0.3; \text{ see Fig S6}), \text{ sample frequency (2 hours; see Fig S9)}$  and sample duration (200) 349 hours). We implemented eLSA and SparCC as described in §4.6 and §4.7 respectively. eLSA 350 and SparCC predicted networks for the two example in silico communities of size N=10351 are shown in Fig 6A-B. AUC was computed as before and as described in §4.8. 352 AUC values for all in silico communities are shown in Fig 6C. We see the same trends 353 as with standard correlation and time-delayed correlation (see Figs 4 and 5). Similar results 354 hold for varying values of the initial condition perturbation  $\delta$  (Fig S6) and sampling frequency 355 (Fig S9). For small networks (N=10), there are a few AUC scores which deviate weakly 356

from 1/2 ( $\approx \pm 10\%$ ). Interestingly, AUC scores for eLSA tend to be negative, implying a

misclassification of interaction. AUC converges to 1/2 as network size increases (N=25,50)

indicating that the AUC scores for small networks may themselves be spurious.

360 6 Discussion

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Using in silico virus-microbe community dynamics, we calculated correlation networks 361 among viral and microbial population time-series samples. We tested the accuracy of several 362 different types of correlation and time-delayed correlation (Pearson, Spearman, and Kendall) 363 and existing correlation-based inference methods (eLSA and SparCC). The correlation net-364 works for all of these implementations failed to effectively predict the original interaction 365 networks, as quantified by the AUC score. Failure persisted across variation in network 366 structure, network size, degree of initial condition perturbation (i.e. scaling the variability 367 of dynamics), and sampling frequency. We therefore conclude these correlation-based in-368 ference methods do not meaningfully predict interactions given this mechanistic model of

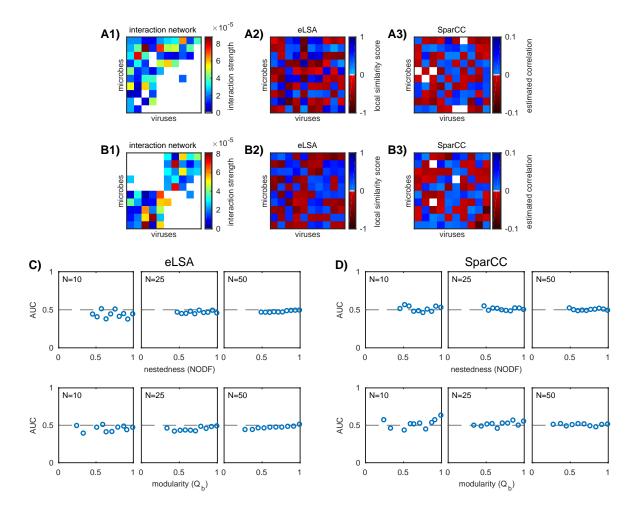


Figure 6: Performance of correlation-based inference methods eLSA and SparCC. A1-B1) Two example in silico interaction networks (N=10). A2-B2) eLSA predicted network computed as described in §4.6. A3-B3) SparCC predicted network computed as described in §4.7 (color bar adjusted for visibility). C-D) AUC values for the ensemble of nested (top row) and modular (bottom row) communities over three network sizes N=10,25,50 (20 communities for each network size). Each plotted point corresponds to a unique in silico community. Dashed line marks AUC=1/2 and implies the predicted network did no better than random guessing.

virus-microbe community dynamics.

Earlier, we stated the core assumption of correlation-based inference: that interacting populations are correlated and that non-interacting populations are not correlated. While this core assumption may sometimes hold in small microbe-only communities with simple interaction mechanisms [25], we find it does not necessarily hold in more complex virusmicrobe communities. (Each inference method also faces challenges unique to its formulation:

eLSA in particular uses a non-stationary data transformation which may induce additional spurious correlations.) We considered communities with microbes and viruses that interacted through a nonlinear mechanism (infection and lysis) across a spectrum of network sizes 378 and network structure. We found that correlation-based inference performed poorly given 379 variation in these network properties, but that there was greater variation in performance for 380 small networks. Because this variation is relatively small and disappears for larger networks, 381 successful predictions for small networks may themselves be spurious. Namely, for a small 382 network (e.g. N < 10), there is a greater probability of randomly guessing the interactions 383 correctly because the space of possible networks is smaller. 384

Our results raise concerns about the use of correlation-based methods on in situ datasets, 385 since a typical community under consideration will have dozens or more interacting strains 386 and therefore will not be in the low diversity microbe-only regime explored by [25]. Ad-387 ditional challenges such as external environmental drivers, measurement noise, and system 388 stochasticity must also be carefully considered before applying correlation-based methods to 389 in situ datasets. Although the degree of variability of dynamics had no effect on inference 390 quality here, it may also be an important consideration for both experimental design and 391 choice of inference method. For example, the model-based inference method developed by 392 [49] performs better when dynamics are highly variable. On the other hand, co-occurrence based inference methods, which require samples across space instead of time, may enable inference across different baseline environmental conditions even if the dynamics within a 395 given environment are relatively stable. 396

In light of the poor performance of correlation-based methods, we advocate for increased studies of model-based inference. Model-based inference methods operate by first assuming an underlying dynamical model for the community (such as the one used in this manuscript, Eqns 1 and 2). The dynamical model is then used to formulate an objective function for an optimization or regression problem, where the solution is the interaction network which best describes the sampled community time-series (for example, see [41, 39, 50, 49, 51, 52]).

Unlike correlation-based methods which assume that similar trends in population indicate interaction, model-based inference has the potential to be tailored to complex communities and environments while leveraging existing knowledge about ecological mechanisms. Given favorable results of *in silico* benchmarking of model-based inference methods [41, 39, 50, 49, 51, 52], it will be important to investigate the efficacy of model-based inference methods for complex microbial and viral communities in practice.

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#### 414 8 Availability of data and materials

Analysis was primarily performed in Matlab. All Matlab scripts, Matlab data files
(also available as .csv files), and custom bash scripts for implementing eLSA and SparCC
are publicly available on GitHub (https://github.com/WeitzGroup/correlation\_based\_
inference) and archived on Zenodo (DOI 10.5281/zenodo.844918). The BiMat Matlab
package [42] used for characterizing bipartite networks is available on GitHub (https://github.com/cesar7f/BiMat). The eLSA Python package [29, 30, 31] is available on Bitbucket (https://bitbucket.org/charade/elsa/wiki/Home). The SparCC Python package [40] is available on Bitbucket (https://bitbucket.org/yonatanf/sparcc).

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