

A cascade of destabilizations: combining *Wolbachia* and Allee effects to eradicate insect pests

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Summary

1. The management of insect pests has long been dominated by the use of chemical insecticides, with the aim of instantaneously killing enough individuals to limit their damage. To minimize unwanted consequences, environmen-

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tally friendly approaches propose biological controls that take advantage of intrinsic demographic processes to eliminate pest populations.

2. We address the feasibility of a novel pest management strategy based on the release of insects infected with *Wolbachia*, which causes cytoplasmic incompatibilities in its host population, into a population with a pre-existing Allee effect. Successful invasion of *Wolbachia* leads to transient declines in population size, and this can theoretically trigger extinction if the population is brought below its Allee threshold.
3. We developed a stochastic population model that accounts for *Wolbachia*-induced cytoplasmic incompatibilities in addition to an Allee effect arising from mating failures at low population densities. Using our model, we identify conditions under which cytoplasmic incompatibilities and Allee effects successfully interact to drive insect pest populations toward extinction. Based on our results, we delineate control strategies based on introductions of *Wolbachia*-infected insects.
4. We extend this analysis to evaluate control strategies that implement successive introductions of two incompatible *Wolbachia* strains. Additionally, we consider methods that combine *Wolbachia* invasion with mating disruption tactics that enhance the pre-existing Allee effect.
5. We demonstrate that *Wolbachia*-induced cytoplasmic incompatibility and the Allee effect act independently from one another: the Allee effect does not modify the *Wolbachia*-invasion threshold, and cytoplasmic incompatibilities only have a marginal effect on the Allee threshold. However, the interaction

of these two processes can drive even large populations to extinction. The success of this method is amplified by the introduction of multiple *Wolbachia* cytotypes as well as the addition of mating disruption.

6. Our study provides novel and translational ideas for the use of cytoplasmic incompatibility and the Allee effect to eradicate insect pests. More generally, it points to the importance of transient dynamics, and the relevance of manipulating a cascade of destabilizations for pest management.

Keywords

biological control; cytoplasmic incompatibility; eradication; extinction; mating disruption; transient dynamics.

Introduction

1 Although most insect species provide crucial ecosystem services (Losey & Vaughan
2 (2006)), a minority of taxa that we consider pests ($\sim 1\%$) have an overwhelming
3 influence on the development of population management in theory and in practice.
4 Among the various environmentally friendly approaches that have been envisaged
5 to control invasive species, we focus on a research avenue that proposes the ex-
6 ploitation of Allee effects, i.e., the decrease in survival or reproduction at small
7 population sizes and the consequent reduction in population growth (Liebhold &
8 Bascompte (2003); Liebhold & Tobin (2008)). The central ideas surrounding these
9 methods are twofold: management tactics could be combined in order to (1) reduce
10 a population size down below the Allee threshold – the population size at which

11 the *per capita* growth rate decreases (a “weak” Allee effect) or becomes negative (a
12 “strong” Allee effect) – which, in turn, increases the probability of stochastic extinc-
13 tion, and/or (2) amplify the mechanisms underpinning a pre-existing Allee effect
14 to increase the Allee threshold itself (Liebhold *et al.* (2016); Tobin *et al.* (2011);
15 Suckling *et al.* (2012)). Capitalizing on Allee effects to manage undesirable species
16 is particularly advantageous because it drives populations into extinction vortexes
17 without needing to eliminate every last individual.

18 The idea of using Allee effects to eradicate insect pests and the subsequent de-
19 velopment of theoretical models originate from population management of various
20 insect species including the Oriental fruit fly, Indian meal moth, almond moths, and
21 arguably most importantly the Gypsy moth *Lymantria dispar* (Beroza & Knipling
22 (1972); Knipling (1970); Steiner *et al.* (1970); Sower & Whitmer (1977)). Control
23 methods centered on the usage of biological controls as alternatives to pesticides
24 have long been recognized as desirable (e.g. Knipling (1955); Baumhover (1955)),
25 and they have been successfully used to control populations with pre-existing Allee
26 effects. The Gypsy moth, for example, is an invasive forest pest in North America
27 that triggered a major containment program to slow the spread toward the west-
28 ern United States (Sharov *et al.* (2002a); Liebhold *et al.* (1992)). It is one of the
29 few insect species for which both a component (mate-finding) and demographic
30 Allee effect has been explicitly identified (Tobin *et al.* (2013, 2007); Johnson *et al.*
31 (2006)). Mating disruption has been a major tactic used to control newly estab-
32 lished low-density populations along the invasion front, with evidence supporting
33 that it is more efficient as well as economically cheaper than classic treatments with
34 the pesticide *Bacillus thuringiensis* (Sharov *et al.* (2002a,b)). This highlights the
35 potential benefits of identifying other invasive pest species that have pre-existing

36 Allee effects and determining whether environmentally desirable forms of control
37 may similarly be effective.

38 Several recent theoretical developments have focused on taking advantage of
39 Allee effects to promote pest eradication (e.g. Boukal & Berec (2009); Liebhold
40 & Bascompte (2003); Blackwood *et al.* (2012); Yamanaka & Liebhold (2009)).
41 These models capture the underlying population dynamics of a pest and evaluate
42 the success of population management tactics such as culling, release of sterile
43 males, and mating disruption to determine whether these methods can create or
44 enhance pre-existing Allee effects (Fauvergue (2013) provides a comprehensive re-
45 view). While there is evidence that such population management strategies will
46 be successful for populations with pre-existing Allee effects, the range of species
47 that might benefit from these tactics may be much greater than currently known.
48 In a meta-analysis focused on the presence of Allee effects in natural animal pop-
49 ulations (Kramer *et al.* (2009)), terrestrial arthropods were found associated with
50 the largest number of studies (22) and the highest proportion (77%) exhibiting an
51 Allee effect. Mating failure at low density appeared as the most frequent mecha-
52 nism. Additionally, Fauvergue (2013) found evidence supporting the presence of
53 mate-finding Allee effects in 19 out of 34 published studies that investigated the
54 interplay of population size and mating success in insects. Indirectly, the central
55 role of Allee effects in insect population dynamics is supported by the efficiency of
56 eradication programs based on the disruption of reproduction. Pest management
57 based on the reduction of mating success via mass trapping, mating disruption
58 with sex pheromones, or the release of sterile males has indeed proved successful
59 in several instances (Knipling (1955); Baumhover (1955); Suckling *et al.* (2014,
60 2012); Krafur (1998)).

61 In this article, we investigate *Wolbachia*-induced cytoplasmic incompatibility
62 (CI) as a novel method for triggering reproductive failures and consequently bring-
63 ing a pest population below its Allee threshold. *Wolbachia* are endosymbiotic bac-
64 teria that infect at least 20% of all insect species and up to two thirds in some esti-
65 mations (Hilgenboecker *et al.* (2008)). It has various effects on its insect hosts, the
66 most widespread and prominent being cytoplasmic incompatibility (Stouthamer
67 *et al.* (1999)). Under CI, matings between an infected male and a female that
68 is either uninfected or infected with an incompatible cytotype result in offspring
69 mortality during embryonic development. Fitness advantages of infected females
70 as well as maternal inheritance are key features that promote invasion of *Wolbachia*
71 into a host population: above a threshold frequency, a given *Wolbachia* strain is
72 expected to invade until near-fixation (Barton & Turelli (2011); Hancock *et al.*
73 (2011); Caspari & Watson (1959); Hoffmann & Turelli (1997); Turelli & Hoffmann
74 (1991)). As a result of the associated CI and subsequent reduction in reproductive
75 rate, *Wolbachia* invasion via the release of infected hosts is a candidate biological
76 control agent against arthropod pests (Bourtzis (2008)).

77 In practice, there are multiple ways to implement a management strategy cen-
78 tered on inducing CIs via introduction of *Wolbachia*. For example, similar to the
79 use of “Sterile Insect Technique” (SIT), males bearing a *Wolbachia* strain incom-
80 patible with that of the target population can be released in large numbers. CIs
81 arising from the mating of females and infected males would substantially limit
82 the total offspring in the subsequent generation, resulting in a decrease in overall
83 population growth rate and thereby increasing the possibility of local population
84 extinction (Laven (1967); Zabalou *et al.* (2004); Atyame *et al.* (2015)). Incom-
85 patible males can be obtained via transfection, even between completely different

86 species of host insects (e.g. Braig *et al.* (1994)). At the population level, the
87 underpinnings for mass-releases of incompatible males do not depart from that of
88 SIT, for which interactions with the Allee effect have already been thoroughly an-
89 alyzed (Boukal & Berec (2009); Yamanaka & Liebhold (2009); Fauvergue (2013);
90 Barclay & Mackauer (1980); Barclay (1982); Berec *et al.* (2016); Lewis & Van
91 Den Driessche (1993)).

92 An alternative management tactic using CI relies on the inoculation of a rela-
93 tively small number of insects of both sexes with a *Wolbachia* strain incompatible
94 with that of the target population. This method is investigated in the theoretic-
95 al model introduced in Dobson *et al.* (2002), which combines insect population
96 dynamics with releases of individuals infected with *Wolbachia*. During a successful
97 invasion of *Wolbachia*, a transient reduction in the insect population size occurs.
98 This decline results from the temporary increase in the fraction of incompatible
99 matings, which peaks in the midst of the invasion process. Therefore, systematic
100 introductions of different *Wolbachia* cytotypes could be applied to artificially sus-
101 tain an unstable coexistence of multiple incompatible infections within an insect
102 population, allowing the population size to be reduced and maintained at low levels
103 (Dobson *et al.* (2002)).

104 Our goal is to determine when the latter implementation of *Wolbachia* in-
105 troductions can drive a population to extinction in the presence of Allee effects.
106 Specifically, we derive a theoretical model built upon Dobson *et al.*'s (2002) ap-
107 proach of CI management that additionally accounts for Allee effects as well as
108 environmental and demographic stochasticity. We also consider mating disruption
109 in our model as a potential complementary tactic. We use this model to address
110 three primary questions: (1) What is the influence of Allee effects present within

111 a host population on *Wolbachia* invasion dynamics? (2) What is the influence of
112 cytoplasmic incompatibility on the demographic Allee effect? (3) What is the in-
113 fluence of a combination of *Wolbachia*-induced CI, Allee effects, mating disruption,
114 and stochasticity on the probability of host extinction?

115 **Methods**

116 **Population model**

117 In this section, we first introduce a model that considers the population dynamics
118 in the absence of individuals infected with *Wolbachia*. Our model extends the
119 framework introduced by Dobson *et al.* (2002) by accounting for pre-existing Allee
120 effects, the release of pheromone sources as a method of mating disruption, as well
121 as both demographic and environmental stochasticity.

122 Similar to Dobson *et al.* (2002), we considered populations such that the dy-
123 namics can be modeled in discrete time with non-overlapping generations. The
124 model explicitly tracks the total population size at each time t , given by N_t , and
125 also tracks the distribution of infected and uninfected individuals within the pop-
126 ulation. We note that our population model can be expressed in terms of either
127 census size or density. However, we are considering a theoretical population not pa-
128 rameterized to any specific species; for simplicity we hereafter refer to our model
129 in terms of size. While the population size can take on non-integer population
130 sizes, the stochastic model forces integer population sizes. We assume that each
131 time step can be broken into two stages: the first (at time $t + 0.5$) captures repro-
132 duction, and the second (at time $t + 1$) captures density dependent survivorship

133 of offspring to adults. The total number of offspring is given by

$$N_{t+0.5} = mN_t g_1(N_t) g_2(N_t), \quad (1)$$

134 where m is the maximum *per capita* fecundity. $g_1(N_t)$ captures a component Allee
135 effect that results from the failure to find mates at low densities such that

$$g_1(N_t) = \frac{N_t}{N_t + \theta}. \quad (2)$$

136 Here, θ measures the strength of the Allee effect; a convenient interpretation of
137 this term is that θ represents the population size at which half of the females
138 successfully mate. Therefore, we use the quantity θ/K as an indicator of the
139 intensity of the Allee effect. Further, $g_2(N_t)$ in Eqn. 1 captures the decline in
140 fecundity resulting from techniques to control populations via mating disruption.
141 We assume that pheromones are maintained in the population at a fixed number
142 P ; given that there are F_t females in the population, only a fraction $F_t/(F_t + P)$
143 males successfully find a mate (Fauvergue (2013)). We assume a 50:50 sex ratio
144 (i.e. $F_t = N_t/2$) so that

$$g_2(N_t) = \frac{N_t}{N_t + 2P} \quad (3)$$

145 Finally, we assume that survivorship of offspring to adults is density dependent so
146 that

$$N_{t+1} = N_{t+0.5}S_N \quad (4)$$

147 where

$$S_N = \frac{S_0}{1 + (\alpha N_{t+0.5})^\gamma} \quad (5)$$

148 where the constant α is related to the carrying capacity, γ is related to intraspecific
149 competition, and S_0 is survivorship in the absence of intraspecific competition
150 (Slatkin & Smith (1979)).

151 This modeling framework allows us to analytically determine the equilibrium
152 values of carrying capacity K as well as the Allee threshold. Equilibrium values
153 in discrete time models occur when $N_{t+1} = N_t$, and the equilibria N_-^* and N_+^* of
154 our model are given by the roots of the polynomial

$$m\alpha(N^*)^2 + (1 - mS_0)N^* + \theta = 0 \quad (6)$$

155 under the simplifying assumption $\gamma = 1$. This expression is used to analytically
156 determine the carrying capacity and the Allee threshold. In the Supplementary
157 Information (S3), we show this explicitly and also perform a basic stability analysis.
158 In all simulations, we choose our parameters to guarantee the existence of a stable
159 equilibrium corresponding to the carrying capacity (see Supplement). Importantly,
160 this formulation captures two general properties that are integral to insect species

161 that we are considering: there is a carrying capacity and a strong Allee effect for
162 sufficiently high θ . We therefore conjecture that an alternative form form density
163 dependence that captures these properties will have qualitatively similar results.

164 Based on this deterministic framework, we build in environmental and demo-
165 graphic stochasticity. Environmental stochasticity results from variation in the
166 population's fecundity between years Melbourne & Hastings (2008). Therefore,
167 we adapt the methods of Schoener *et al.* (2003) and account for environmental
168 stochasticity by rewriting Eqn. 1 as

$$N_{t+0.5} = m_e N_t g_1(N_t) g_2(N_t), \quad (7)$$

169 where the fecundity m_e is drawn at each generation from a normal distribu-
170 tion with mean m (that is truncated so that $m \geq 0$) and a standard deviation
171 σ . We fix the standard deviation so that it is the square root of the mean; how-
172 ever, a sensitivity analysis of the magnitude of the standard deviation is provided
173 in the Supplementary Information. This analysis shows that the the extinction
174 probability increases as the magnitude of the variability increases.

175 Demographic stochasticity, on the other hand, results from variation in fecun-
176 dity at the individual level Melbourne & Hastings (2008). Note that the total
177 number of individuals that successfully reproduce is given by

$$B = N_t g_1(N_t) g_2(N_t). \quad (8)$$

178 We assume that each of these individuals at a given time t reproduces with
179 fecundity m_e (as described above), and the total number of eggs produced is a
180 Poisson random variable Melbourne & Hastings (2008). Since the sum of inde-
181 pendent Poisson random variables is also a Poisson random variable, the total
182 offspring of all adults at is

$$N_{t+0.5} \sim \text{Poisson}(m_e B) \quad (9)$$

183 Finally, we include stochasticity in density dependent survivorship following
184 Melbourne & Hastings (2008). Given that S_N (as defined in Eqn. 5) is the proba-
185 bility that offspring survive to adults, we assume that survivorship is a binomially
186 distributed so that

$$N_{t+1} \sim \text{Binomial}(N_{t+0.5}, S_N). \quad (10)$$

187 Infection dynamics

188 We consider the infection dynamics of up to two different cytotypes of *Wolbachia*
189 (referred to as cytotypes X and Y) and denote the number of uninfected individuals
190 as W . Note that all variables and parameters with subscripts X (or Y) are related
191 to cytotype X (or Y). This model is adapted from Dobson *et al.* (2002); therefore,
192 we use similar notation throughout.

193 In the presence of a single cytotype of *Wolbachia*, there are only unidirec-
194 tional cytoplasmic incompatibilities (CI); in contrast, in the presence of multiple

195 cytotypes there may be bi-directional CI. We first introduce the case of a single
196 cytotype and then extend the model to include two cytotypes. Below we describe
197 the mathematical formulation of the infection dynamics, and Table summarizes
198 the processes involved.

199

200 *One cytotype*

201 To capture the *Wolbachia* dynamics, we first determine the proportions of
202 infected and uninfected individuals in our population at time t . For example, if
203 there are W_t uninfected individuals and X_t infected with cytotype X then, under
204 the assumption that there is a 50:50 sex ratio, the fraction of females infected with
205 cytotype X at $t + 0.5$ is given by

$$a_t = \frac{X_t}{W_t + X_t},$$

206 Similarly, we find the fraction i_t of all males that are infected (where $i_t = a_t$),
207 the fraction q_t of all males that are uninfected, and the fraction c_t of all females
208 that are uninfected (again note that $q_t = c_t$).

209 Based on the proportions of uninfected and infected individuals in the popula-
210 tion, we can now determine the fraction of offspring that are infected. Crosses be-
211 tween pairs with an infected female suffer a fecundity loss due to infection ($1 - F_X$),
212 where F_X is the probability of mating success in these mating couples. Vertical
213 transmission of *Wolbachia* occurs maternally and we assume that transmission is
214 successful with probability $(1 - \mu_X)$, where μ_X is the probability of transmission
215 failure. In the instance of *Wolbachia*-induced CIs, crosses between infected females

216 and uninfected males in addition to crosses between infected males and infected
217 females give rise to infected offspring. The proportion of viable offspring that are
218 infected with cytotype X after reproduction (i.e. at time $t + 0.5$) is therefore given
219 by

$$x_{t+0.5} = a_t (1 - \mu_X) F_X (i_t + q_t) \quad (11)$$

220 where a lowercase x is used to denote proportion rather than number. Second,
221 we can identify the proportion of viable offspring that are uninfected ($w_{t+0.5}$).
222 Uninfected individuals can arise from crosses between both uninfected females and
223 uninfected males. Further, matings between both infected females and infected
224 males can have viable uninfected offspring. This results from failure to vertically
225 transmit *Wolbachia* to their offspring (i.e. with probability μ_X). When one type
226 of *Wolbachia* is present within a population, then only unidirectional cytoplasmic
227 incompatibility (CI) is possible. This type of CI occurs through matings between
228 infected males and uninfected females. Therefore, we assume that pairings between
229 infected males and uninfected females undergo CI and a fraction H_X survives.
230 The proportion of viable offspring that are not infected with *Wolbachia* following
231 reproduction is given by

$$w_{t+0.5} = (\mu_X F_X a_t + c_t) \times (i_t H_X + q_t) \quad (12)$$

232 Notice that due to cytoplasmic incompatibilities and the fecundity cost due to

233 infection with *Wolbachia*, the fraction of the total population that successfully
234 reproduces ($x_{t+0.5} + w_{t+0.5}$) is less than one. Therefore, the total number of offspring
235 as governed by Eqn. 1 can be rewritten as

$$N_{t+0.5} = m (x_{t+0.5} + w_{t+0.5}) N_t g_1(N_t) g_2(N_t). \quad (13)$$

236 In other words, the product $g_1(N_t)g_2(N_t)$ captures the total fraction of adults
237 at time t who successfully find a mate, and the sum $x_{t+0.5} + w_{t+0.5}$ is the fraction of
238 all offspring that are viable. Finally, as described in the previous section, density
239 dependent mortality limits the total number of adults at time $t + 1$ (Eqn. 5).

240

241 Our parameterization of the population model is based on both the parame-
242 terization used in Dobson *et al.* (2002) and common ranges for insect populations.
243 For example, as noted by Dobson *et al.* (2002) the basic reproductive rate in in-
244 sect species to which sterile insect technique has been employed ranges from 1-11
245 (Davidson (1974)) which is consistent with our parameterization (e.g. see Fig. 2).
246 Additionally, several parameters vary for our analysis including the strength of the
247 Allee effect, θ , and the initial population size. However, we note that our results
248 are intended to assess the general qualitative behavior of *Wolbachia* introductions
249 and consequently the actual implementation of such management tactics would
250 require a detailed analysis and parameterization specific to the target population
251 and cytotype.

252 *Two cytotypes*

253 In addition to releasing a single cytotype of *Wolbachia*, we consider a scenario

254 in which a second cytotype is introduced. When two cytotypes of *Wolbachia* are
255 present within a population, bidirectional CI occurs when a male with one cytotype
256 mates with a female infected with an incompatible *Wolbachia* cytotype. Similar
257 to the previous section, we assume that a fraction H_X (or H_Y , depending on the
258 infection type of the male and female) of offspring survives.

259 Therefore, in the presence of two strains we rewrite Eqn. 11 as

$$x_{t+0.5} = a_t (1 - \mu_X) F_X (i_t + j_t H_Y + q_t) \quad (14)$$

260 where j_t is the fraction of males infected with cytotype Y . Similarly, the
261 proportion of viable offspring infected with cytotype Y following reproduction is
262 given by

$$y_{t+0.5} = b_t (1 - \mu_Y) F_Y (i_t H_X + j_t + q_t). \quad (15)$$

263 where b_t is the fraction of females infected with cytotype Y . The proportion of
264 viable uninfected offspring is now given by

$$w_{t+0.5} = (\mu_X F_X a_t + \mu_Y F_Y b_t + c_t) (i_t H_X + j_t H_Y + q_t) \quad (16)$$

265 Our parameterization for the infection dynamics is based on values that are
266 reasonable for *Wolbachia* cytotypes (Hoffmann & Turelli (1997); Dobson *et al.*
267 (2002); Charlat *et al.* (2005)). In the main text, we assume that fecundity loss,

268 transmission failure, and survival of CI are equal between all cytotypes. However,
269 the Supplementary Information provides an analysis of the dynamics when the
270 introduced cytotypes are not identical. Although our sensitivity analysis indicates
271 that the qualitative results are robust to differences between cytotypes, practical
272 implementation of such methods would require further analyses that are specific
273 to cytotypes.

274 Results

275 In the following sections we first test our model against well-established results
276 related to *Wolbachia* invasion as a method of model validation, establish the re-
277 lationship between *Wolbachia* and the location of the Allee threshold, and finally
278 evaluate the potential for the release of infected insects to control a population.

279 Model validation

280 We first determine whether our model captures the same features of the important
281 earlier work (Hoffmann *et al.* (1990); Turelli & Hoffmann (1991); Hancock *et al.*
282 (2011)). Hoffmann *et al.* (1990) derived an analytic expression for the expected
283 equilibrium infection frequencies. After adjusting their notation to match ours and
284 simplifying, the equilibrium infection frequency for a single cytotype of *Wolbachia*
285 X should satisfy the equation

$$(1 - H_X)(1 - \mu_X F_X) p^2 + (F_X + H_X - 2)p + 1 - F_X(1 - \mu_X) = 0.$$

286 Their work predicts that there is an unstable equilibrium, below which the
287 invasion of the introduced cytotype is unsuccessful and above which invasion is
288 successful. This equilibrium is therefore referred to as the *Wolbachia* invasion
289 threshold. Initial infection frequencies above this value will increase until reaching
290 the higher stable equilibrium that indicates a successful invasion.

291 Following Charlat *et al.* (2005), we considered invasion dynamics by estimat-
292 ing the infection frequency at generation $t + 1$ as a function of the frequency at
293 generation t . Doing so allows us to create a simple graphical representation of
294 the stable and unstable equilibria (Fig. 1A). More specifically, we compute the
295 infection frequency between two subsequent generations with generation t on the
296 horizontal axis and generation $t + 1$ on the vertical axis. Equilibria occur when
297 this curve intersects the line $y = x$ (i.e. the infection frequency at generation
298 $t + 1$ is the same as at generation t). In addition to using our model to find these
299 equilibria, we also used the analytically derived results of Hoffmann *et al.* (1990)
300 (Fig. 1). Unless stated otherwise, the default parameter values are listed in Table
301 .

302 As evidenced by the results shown in Fig. 1, our simulation results are consis-
303 tent with the analytically derived equilibrium infection frequencies. This verifies
304 that our simulations are in line with the behavior we would expect from our model
305 and are consistent with the results observed in Charlat *et al.* (2005). However, this
306 is not surprising given that our model makes similar assumptions on the mecha-
307 nisms driving *Wolbachia* invasion dynamics (e.g. fecundity loss and cytoplasmic in-
308 compatibilities). In contrast to earlier studies, our population model is dynamically
309 different because of the inclusion of Allee effects and false pheromones. Therefore,
310 we determined the relationship between the invasion threshold and these features

311 of the model. We found that the *Wolbachia* invasion threshold is not affected by
312 Allee effects (Fig. 1B) nor by the application of false pheromones to the host in-
313 sect (not shown). This is important to note because in all of our simulations and
314 analyses, the invasion threshold does not vary as θ and P are adjusted. Finally, we
315 note that the invasion threshold is not affected by the parameter S_0 (not shown).

316 **The effect of *Wolbachia* on the Allee threshold**

317 To determine the dynamical effects of the presence of *Wolbachia* infection within a
318 population, we find the Allee threshold in insect populations both in the presence
319 and absence of infection. In this section, we ignore stochasticity as well as the
320 release of pheromones ($P = 0$). For a given initial proportion of infected indi-
321 viduals, we calculate the reproductive rate between two consecutive generations
322 (i.e. N_{t+1}/N_t) across all population sizes. The resulting reproductive rates for
323 a population exhibiting strong Allee effects (that is, the population growth rate
324 drops below one at low densities) are provided in Fig. 2. The equilibria for our
325 population model occur when $N_{t+1} = N_t$, and there are three equilibria: the first
326 corresponds to population extinction (stable), the second is the Allee threshold
327 (unstable), and finally the third is the carrying capacity (stable).

328 In addition to considering the population model in the absence of *Wolbachia*-
329 infected individuals, we calculated the reproductive rates when the initial popu-
330 lation is comprised of 10% and 50%, respectively, infected individuals (Fig. 2).
331 Given our parameterization, the frequency of infected individuals is chosen to lie
332 above the invasion threshold (which is $\sim 8.5\%$); therefore, this figure captures
333 the dynamics between two consecutive generations during the replacement process

334 when the population contains the specified distribution of infected and uninfected
335 individuals. As a consequence of cytoplasmic incompatibilities, the maximum re-
336 productive rate decreases as the proportion of infected individuals increases and
337 there is also an increase in the Allee threshold. Finally, there is a slight decrease
338 in the carrying capacity that results from the fecundity loss associated with *Wol-*
339 *bachia* infection. However, the proportion of *Wolbachia*-infected individuals has a
340 significantly smaller effect on the location of the Allee threshold than the strength
341 of the Allee effect itself (Fig. 2).

342 **Implications for population management**

343 *Deterministic results*

344 In this section, we characterize implications for population management through
345 the release of *Wolbachia*-infected individuals into an insect population. As ob-
346 served by Dobson *et al.* (2002), there should be a transient decline in the population
347 size during the replacement of uninfected hosts by *Wolbachia*-infected individuals.
348 Therefore, we find the magnitude of this decline in the presence of Allee effects
349 to determine the conditions under which the replacement process brings the pop-
350 ulation size below the Allee threshold in a deterministic setting, thereby forcing
351 extinction. This is achieved by running our model over a range of values for the
352 strength of the mate-finding Allee effect (θ) and we assume that the initial popu-
353 lation size is at its carrying capacity (which can be found explicitly, as shown in
354 the Supplementary Information).

355 We find the minimum population size (relative to K) over 50 generations fol-
356 lowing the introduction of one cytotype (Fig. 3A) and two cytotypes (Fig. 3B).

357 Here, values of zero for the minimal population size indicate that the transient
358 reduction in population size brought the population below the Allee threshold,
359 therefore leading to deterministic extinction. The first cytotype is always released
360 in the second generation, and the release of the second cytotype is optimized so
361 that the release occurs in the generation that causes the largest decline in popula-
362 tion size. To implement this strategy, we assume that the longest amount of time
363 between introductions is 25 generations. While in the main text we assume that all
364 cytotypes have the same infection properties, this assumption is challenged in the
365 Supplementary Information and our qualitative results are unchanged. To ensure
366 that the introduction size is above the invasion threshold, in all simulations we
367 assume that the introduction is large enough so that the initial infection frequency
368 is 10%. This value lies just above the actual threshold of $\sim 8.5\%$ resulting from
369 our parameter values. Therefore, the number of infected individuals introduced in
370 our simulations directly depends on the current host population size.

371 To determine the success of the releases under varying reproductive rates, we
372 replicated the results for four different values of S_0 . We observe that for all repro-
373 ductive rates, the release of individuals infected with a second and incompatible
374 cytotype of *Wolbachia* leads to greater success of these methods. Importantly,
375 we also find that the effectiveness of control via releases of infected individuals
376 critically depends on the maximum reproductive rate of a population: for low re-
377 productive rates, a single release is likely more effective at driving a population to
378 extinction for smaller values of θ relative to K (as indicated in Fig. 3).

379 In contrast, however, populations with high reproductive rates are only driven
380 to extinction with unrealistically strong Allee effects, suggesting the need for com-
381plementary management tactics to successfully eliminate a pest population. While

382 low reproductive rates point to greater success of *Wolbachia* introductions, how-
383 ever, it should be noted that the maximum reproductive rates reported in the
384 caption to Fig. 3 are in the *absence* of Allee effects. As a consequence of a low
385 reproductive rate in the absence of Allee effects, strong enough Allee effects can
386 bring the overall population growth rate below one. In that case, the population
387 will be driven to extinction by Allee effects alone (this is explored further in the
388 Supplementary Information). Because we are interested in the combined effect
389 of Allee effects and *Wolbachia*-induced CI on population dynamics, we consider
390 populations with relatively high reproductive rates ($S_0 = 0.15$) for the remainder
391 of the manuscript (the Supplementary Information provides a sensitivity analysis
392 for lower reproductive rates).

393 When reproductive rates are relatively high, complimentary tactics that either
394 decreases the population size or further increases the Allee threshold may amplify
395 the effects of *Wolbachia* introductions. Therefore, we additionally consider the use
396 of mating disruption through the release of sex pheromones (P) as a supplemental
397 management tactic (see Eqns. 3 and 1). In Fig. 3C, we fix the strength of the
398 Allee effect relative to K so that a release of *Wolbachia*-infected individuals does
399 not successfully bring the population below the Allee threshold. Further, θ/K is
400 a measure for the intensity of the Allee effect – which impacts low density popu-
401 lations – so this value was chosen to be relatively small ($\theta/K = 0.1$). Of course,
402 this value would vary by species. We then calculate trajectories of the model to
403 determine the minimum population size as the number of pheromone sources is
404 varied. While the release of pheromones alone can drive the population below the
405 Allee threshold, combining both methods is significantly more effective than either
406 tactic alone (as shown in Fig. 3C).

407

408 *Stochastic results*

409 The analysis in the previous section was centered on the deterministic outcome
410 of population management strategies in the presence of Allee effects. Additionally,
411 we assumed that releases of *Wolbachia*-infected individuals as well as the imple-
412 mentation of mating disruption occurred in populations that have reached their
413 carrying capacity (i.e. as assumed in obtaining the results shown in Fig. 3). How-
414 ever, the interplay of Allee effects and stochasticity can be especially important
415 at low population sizes, when the population is at higher risk of stochastic extinc-
416 tion. Therefore, in this section we determine the ability of *Wolbachia* and mating
417 disruption to drive populations with variable initial population sizes to extinction
418 in the presence of Allee effects and stochasticity.

419 To achieve this, we determine the probability of extinction based on 500 realiza-
420 tions of the stochastic model (i.e. Eqns. 7-10) over all relevant combinations of the
421 initial population size and strength of the Allee effect (i.e. the initial population
422 size is at most at carrying capacity and the Allee threshold is below the carry-
423 ing capacity). To determine the relative roles of environmental and demographic
424 stochasticity, we simulate the model while including both types of stochasticity
425 (bottom row of Fig. 4) as well as demographic stochasticity alone (top row of Fig.
426 4). Further, we find the extinction probability under three scenarios: no intro-
427 duction of *Wolbachia*-infected individuals (first column of Fig. 4), introduction of
428 one cytotype (second column of Fig. 4), and the introduction of two incompatible
429 cytotypes (third column of Fig. 4). As in the deterministic setting, we assume
430 that the introduction of the first cytotype occurs at the second generation. When
431 two cytotypes are introduced, the generation of the second release is determined

432 in the same way as it is found in the deterministic setting: the second introduc-
433 tion is optimized for each realization so that it occurs in the generation (up to 25
434 generations) that creates the largest transient decrease in population size resulting
435 from the *Wolbachia* introduction. The generation of the second introduction (as it
436 corresponds to the third column of Fig. 4C) is usually only a small number (~ 2)
437 of generations after the first generation. However, the number of generations be-
438 tween releases increases as the strength of the mate-finding Allee effect decreases
439 (see Supplementary Information). As before, each release is implemented so that
440 the proportion of infected individuals of a given cytotype is 10% (just above the
441 invasion threshold).

442 Similar to the deterministic results, we find that the introduction of one cyto-
443 type of *Wolbachia* leads to a higher extinction probability of the population as the
444 strength of the Allee effects increases (as displayed in the first and second columns
445 of Fig. 4). Moreover, when the strength of the Allee effect is high, demographic
446 stochasticity does not increase extinction probability significantly as compared to
447 the deterministic setting (e.g. Fig. 3). This result follows from the relatively large
448 initial population sizes near the Allee threshold when θ is large. However, the
449 presence of environmental stochasticity increases the uncertainty in whether ex-
450 tinction will occur near the boundary between extinction and population survival
451 (as observed in the second row of Fig. 4). As described in the previous section,
452 the success of *Wolbachia* releases increases for lower reproductive rates. This find-
453 ing holds in the stochastic setting (see sensitivity analysis in the Supplementary
454 Information).

455 Interestingly, when θ is relatively high, *Wolbachia* introductions succeed in
456 driving population extinction independent of the initial population size. This

457 result has the important implication that the success of *Wolbachia* introductions
458 in driving extinction do not necessarily rely on having a pest population at the
459 initial stage, or at the front, of the invasion. Additionally, the introduction of two
460 cytotypes is much more successful than one (Fig. 4, third panel).

461 As explored in the deterministic framework, combining *Wolbachia* introduc-
462 tions with other methods that increase the Allee threshold (e.g. mating disruption)
463 will likely further increase the success of the overall management strategy. This is
464 highlighted in Fig. 4 as a result of the sensitivity of the extinction probability to
465 the combination of *Wolbachia* introduction and mating disruption. Therefore, as
466 in Fig. 3C, we fix θ relative to K at a value of 0.1. With this parameterization,
467 introductions of *Wolbachia* generally do not drive the population to extinction
468 (with the exception of small initial population sizes). Here, we again consider the
469 population dynamics under three different management programs: mating disrupt-
470 tion only (first column of Fig. 5), mating disruption and the introduction of one
471 cytotype (second column of Fig. 5), and mating disruption and the introduction
472 of two cytotypes of *Wolbachia* (third column of Fig. 5). Additionally, we note that
473 utilizing both mating disruption and CI is much more effective than using mating
474 disruption alone (Fig. 5). Additionally, it is important to note that when θ/K is
475 fixed at 0.1, releases of infected individuals in the absence of mating disruption
476 has little effect on the extinction probability. Therefore, these two methods can
477 serve as complementary tactics for pest management.

478 Discussion

479 We investigated a population management strategy that considers *Wolbachia*-
480 induced cytoplasmic incompatibility in the presence of Allee effects. In particular,
481 we developed a stochastic population model, building upon the seminal approach
482 of Dobson *et al.* (2002) and the continuously expanding body of literature inves-
483 tigating the use of Allee effects for the eradication of pest species (Liebhold &
484 Bascompte (2003); Tobin *et al.* (2011); Liebhold *et al.* (2016)). Our model demon-
485 strates that the introduction of a small number of incompatible individuals into
486 a pest population that has a strong pre-existing Allee effect can drive the pest
487 population to extinction with no further intervention. We also demonstrate that
488 extinction is possible for surprisingly large pest populations, and that combinations
489 of more than one strain of *Wolbachia* and mating disruption via sex pheromones
490 work synergistically to increase the population's extinction risk. Biological control
491 has been studied for decades as an environmentally friendly alternative to pesti-
492 cide use (e.g. Knippling (1955); Baumhover (1955)), and our study adds to this
493 work by providing insight into ways that *Wolbachia* invasions can take advantage
494 of intrinsic population processes – that is, Allee effects – to manipulate and control
495 pest populations.

496 An important first step of our modeling work was to uncover the basic inter-
497 actions between Allee effects and cytoplasmic incompatibility (CI). We show that
498 the interactions between Allee effects and CI are weak or non-existent: the *Wol-*
499 *bachia* invasion threshold does not depend on the strength of the Allee effect of its
500 insect host, and the Allee threshold has a marginal decrease in the presence of CI.
501 Therefore, invasion of a particular *Wolbachia* strain into a population only depends

502 on the critical population size (or infection frequency) above which invasion suc-
503 ceeds in a deterministic setting (Barton & Turelli (2011)). This invasion threshold
504 corresponds to a proportion of infected hosts above which infection spreads up
505 to almost fixation, and is determined by parameters such as the reduction in egg
506 hatch-rate caused by CI, the fitness costs of *Wolbachia* carriage, and the fraction
507 of offspring that inherit the bacteria from an infected mother (Turelli (1994)).
508 The invasion threshold found with our simulation model is consistent with that
509 derived analytically (Turelli & Hoffmann (1991)), and unaffected by the intensity
510 of a mate-finding Allee effect (Fig. 1B). In addition to adding validation to our
511 model, this result holds interest because many theoretical approaches of *Wolbachia*
512 invasion dynamics are purely genetic and consider changes in invasion frequency
513 without considering host population dynamics. Our result follows that of Hancock
514 *et al* (2011) in suggesting that *Wolbachia* invasion thresholds predicted analyti-
515 cally hold for closed populations, even when, as assumed here, host reproductive
516 rate is affected by both positive and negative density dependence.

517 In the presence of strong Allee effects a population below the Allee thresh-
518 old will be forced to extinction in a deterministic setting, making the Allee effect
519 a central paradigm for conservation (Deredec & Courchamp (2007); Stephens &
520 Sutherland (1999)), invasions (Taylor & Hastings (2005)), biological control in-
521 troduction (Fauvergue *et al.* (2007, 2012)), and as hypothesized in the present
522 work, eradication (Tobin *et al.* (2011)). Whether an Allee effect is weak or strong
523 (and the value of the Allee threshold in the latter case) depends on the strength
524 of the underlying component Allee effect(s) relative to other density-dependent
525 processes. Our simulations of various levels of cytoplasmic incompatibilities in
526 a population with a pre-existing mate finding Allee effect suggest that the Allee

527 threshold is much less sensitive to variations in the initial frequency of *Wolbachia*-
528 infected individuals (0-50% infected individuals) than it is to variations in the
529 mate-finding Allee effect (Fig. 2B). *Wolbachia*-induced cytoplasmic incompatibil-
530 ity does decrease population growth rate, as expected, but it has a minimal effect
531 on the extinction threshold. Therefore, *Wolbachia*-induced CI may be considered
532 a culling population management tactic where population size is temporarily de-
533 creased as a result of cytoplasmic incompatibilities (Dobson *et al.* (2002)).

534 Despite their initial apparent independence, cytoplasmic incompatibility and
535 the Allee effect yield interesting properties when acting in concert. Our first anal-
536 ysis that considered the combined occurrence of Allee effects and CI in a deter-
537 ministic context reveals that the transient decrease in population size (resulting
538 from the invasion of an incompatible *Wolbachia* strain into an uninfected insect
539 population) is large enough to trigger extinction when the reproductive rate of the
540 host species is relatively low. Extinction caused by the introduction of a single
541 *Wolbachia* cytotype in populations with higher reproductive rates is only observed
542 for very strong Allee effects (Fig. 3A). However, the strength of the Allee effect
543 required for extinction lowers for the introduction of an additional incompatible
544 *Wolbachia* strain. The resulting insect extinction probability, estimated by simu-
545 lating the model in the presence of stochasticity, confirmed the interaction between
546 the two processes. In the absence of *Wolbachia*, we determined the population's
547 extinction probability as it varies with its population size and the strength of the
548 component Allee effect (Fig 4A and 4D). Introducing infected individuals results
549 in the extinction of populations that would have persisted otherwise (i.e., a popu-
550 lation that is above its Allee threshold can be brought to extinction). Introducing
551 a second incompatible cytotype of *Wolbachia* increases CIs within the population

552 and, consequently, increases the extinction domain by reducing the severity of
553 Allee effect necessary to trigger extinction (Fig 4).

554 Nonetheless, with the exception of species with low reproductive rates, our
555 model predicts that although Allee effects and CI combine to drive populations to
556 extinction – even in surprisingly large populations – these extinctions may occur
557 for unrealistically severe Allee effects. For instance, after the introduction of two
558 incompatible *Wolbachia* strains, extinction is expected when $\theta/K \geq 0.4$; that is,
559 extinction occurs if only half of all females successfully mate when the population
560 is at 40% of the carrying capacity. Unfortunately, field estimations of mate-finding
561 Allee effects in insects are rare, but it is probable that mating failures only occur
562 at very low densities. For instance, in the Gypsy moth *Lymantria dispar*, mating
563 failures occurred below a density (estimated via the rate of male captures on sex-
564 pheromone traps) of about 4 whereas the carrying capacity was estimated around
565 800 (Tobin *et al.* (2007, 2013)), so that estimation of θ/K in this species could be
566 one or two orders of magnitude lower than that yielding extinction in our model.

567 Our results indicate that cytoplasmic incompatibility management may inde-
568 pendently be an effective strategy for populations with a mate-finding Allee ef-
569 fect and low reproductive rates. Regardless of reproductive rate, our simulations
570 demonstrate that eradication can be obtained via the combined introduction of
571 *Wolbachia*-infected individuals and sex pheromone sources into an insect popu-
572 lation subject to pre-existing Allee effect (Fig. 5). Further, eradication is not
573 restricted to small populations, but also applies to populations that have reached
574 carrying capacity. Our results support previous studies that highlight the poten-
575 tial benefit of simultaneously using multiple complementary management tactics
576 (Blackwood *et al.* (2012); Suckling *et al.* (2012)); however, the benefits and prac-

577 ticality of such methods will depend on, for example, associated economic costs
578 and available resources.

579 It is important to note that our analysis is intended to provide insight into a
580 broad variety of pest species; therefore, in an empirical context it is essential to
581 perform more thorough analyses of the dynamics of *Wolbachia* invasion and mat-
582 ing disruption using a species-specific parameterization. Additional methods for
583 controlling a pest population should also be considered. For example, the combi-
584 nation of *Wolbachia*-induced CI and mating disruption work synergistically, and
585 other methods for population control such as parasitism or predation by native
586 natural enemies may also be complementary. Additionally, while our focus was on
587 *Wolbachia*-induced CI, there is evidence that *Wolbachia* is capable of other repro-
588 ductive manipulations including male-killing (Dyer & Jaenike (2004); Richardson
589 *et al.* (2016)). This suggests the existence of additional avenues for utilizing *Wol-*
590 *bachia* in the context of pest management.

591 There is a long and prolific body of research in population dynamics that
592 focuses on understanding the mechanisms stabilizing species near their carrying
593 capacities (e.g. Hassell & May (1973); May & Anderson (1978); Bernstein (2000)).
594 More recently, global climate change and the biodiversity crisis, including popu-
595 lation declines, extinctions, or biological invasions, points towards the increasing
596 relevance of nonequilibrium ecology (Rohde (2006)) and the biology of small popu-
597 lations (Fauvergue *et al.* (2012)). Transient dynamics are increasingly emphasized
598 (Hastings (2004)) and sometimes considered in the specific context of population
599 management (Ezard *et al.* (2010); Kidd & Amarasekare (2012)). As first high-
600 lighted by Dobson *et al.* (2002), cytotype replacement which occurs in the course
601 a successful *Wolbachia* invasion yields a transient coexistence of incompatible in-

602 fections within a host population, and as a consequence, a transient decrease in
603 reproductive rate. Here, the transients only last a few generations and this pertur-
604 bation of the population's microbiome is the first step in a destabilizing cascade.
605 We show here that the population can then be pushed toward a second step of
606 destabilization, triggered by a mate-finding Allee effect that can be reinforced by
607 the application of mating disruption, which potentially drives the population to
608 extinction.

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613 **Authors' contributions statement**

614 JCB and XF conceived the study. JCB, XF, and RV discussed the model. JCB and
615 RV developed, ran, and analyzed the model. JCB and XF wrote the manuscript.

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782 **Tables**

Parameter	Description	Value	Source
N_0	initial population size	varies	
m, m_e	<i>per capita</i> fecundity	25	
σ^*	standard deviation of fecundity	5	
S_0^*	survivorship in absence of competition	varies	
α	related to carrying capacity	0.00002	[1]
γ	related to intraspecific competition	1	[1]
μ_X^*, μ_Y	maternal transmission failure	0.03	[1]
F_X^*, F_Y	relative fecundity of infected individuals	0.95	[1]
H_X, H_Y	proportion of offspring surviving CI	0.05	[2]
θ	strength of Allee effect	varies	
P	number of pheromone sources	varies	

Table 1: List of model parameters. Unless otherwise stated, all figures use these parameter values. Parameter values with a “*” have associated sensitivity analyses (as discussed in the main text) in the Supplementary Information. [1] refers to the reference Dobson *et al.* 2002, and [2] refers to Charlat *et al.* (2005).

$\begin{matrix} \diagdown \\ \text{♀} \end{matrix} \quad \begin{matrix} \text{♂} \\ \diagup \end{matrix}$	W	X	Y
W	$W: c_t q_t$ $X: 0$ $Y: 0$	$W: H_X c_t i_t$ $X: 0$ $Y: 0$	$W: H_Y c_t j_t$ $X: 0$ $Y: 0$
X	$W: \mu_X F_X a_t q_t$ $X: (1 - \mu_X) F_X a_t q_t$ $Y: 0$	$W: \mu_X F_X H_X a_t i_t$ $X: (1 - \mu_X) F_X a_t i_t$ $Y: 0$	$W: \mu_X F_X H_Y a_t j_t$ $X: (1 - \mu_X) F_X H_Y a_t j_t$ $Y: 0$
Y	$W: \mu_Y F_Y b_t q_t$ $X: 0$ $Y: (1 - \mu_Y) F_Y b_t q_t$	$W: \mu_Y F_Y H_X b_t i_t$ $X: 0$ $Y: (1 - \mu_Y) F_Y H_X b_t i_t$	$W: \mu_Y F_Y H_Y b_t j_t$ $X: 0$ $Y: (1 - \mu_Y) F_Y b_t j_t$

Table 2: Summary of *Wolbachia* transmission and its effects on reproduction in its host population. The first column states the maternal *Wolbachia* cytotype and the first row states the paternal *Wolbachia* cytotype. Each box in the table corresponding to a particular pairing between a female and male provides that proportion of the offspring from that pair that are uninfected (W), infected with cytotype X , and infected with cytotype Y .

783 Figure captions

784 **Figure 1.** (A) Verification that our model accurately predicts the invasion thresh-
785 old as analytically determined in Hoffman *et al.* 1990. Here, we ignore Allee effects,
786 stochasticity, and assume $P = 0$. The thick black curve is the frequency of infected
787 individuals at time $t + 1$ given the frequency at t ; equilibria occur when this curve
788 and the gray line (which corresponds to the case that the frequency at generations
789 t and $t + 1$ are equal) intersect. Note that when the black curve lies above the gray
790 line, then the infection frequency is increasing; similarly, the infection frequency is
791 decreasing when the black curve falls below the gray line. The dashed lines indi-
792 cate the analytically predicted equilibrium. The smaller intersection is an unstable
793 equilibrium that defines the invasion threshold: i.e. individuals introduced at a
794 frequency higher than this threshold will successfully invade the population and
795 approach the higher stable equilibrium. (B) Demonstration that the Allee effect
796 does not change the invasion threshold (dashed line is our model prediction, solid
797 line is the analytically predicted non-zero, stable equilibrium). In this figure, we
798 use more extreme values for parameters related to CI to more clearly demonstrate
799 the location of the invasion threshold (specifically, $\mu_X = 0.2$, $H_X = 0.1$).

800

801 **Figure 2.** Reproductive rate as a function of population size when $\theta = 100$ (gray)
802 and $\theta = 1500$ (red). Values above one correspond to population growth, and val-
803 ues below one correspond to decline. The populations corresponding to the solid
804 lines have no *Wolbachia*-infected individuals, populations with dashed lines have
805 10% of the population infected, and dash-dotted lines have 50% of the population
806 infected at generation t .

807

808 **Figure 3.** Deterministic results. (A) single introduction; (B) two introductions.
809 Plot displays the minimum population size relative to K over 50 generations as-
810 suming that $N_0 = K$. The solid line has $S_0 = 0.08$ (maximum reproductive
811 rate of 2 in the absence of AE, as in Dobson *et al.* (2002)), the dashed line has
812 $S_0 = 0.15$ (maximum reproductive rate of 3.75 in the absence of AE), dash-dotted
813 line has $S_0 = 0.2$ (maximum reproductive rate of 5 in the absence of AE), and
814 dotted line has $S_0 = 0.25$ (maximum reproductive rate of 6.25 in absence of AE).
815 (C) Results when θ relative to K is fixed 0.1 (as displayed in A and B) with
816 $S_0 = 0.15$, and instead varies the number of pheromone sources P relative to K .
817 The black line corresponds to the case with no *Wolbachia*-infected individuals,
818 dashed corresponds to a single release, and dash-dotted corresponds to a release of
819 two cytotypes. In all plots, each release is created so the initial infection frequency
820 of that cytotype is 10%. The first release is at generation one, and the second is
821 at generation six.

822

823 **Figure 4.** Using $S_0 = 0.15$, the colors of each plot represent the extinction prob-
824 ability for a given parameter combination based on 500 realizations of the model.
825 In each plot, the initial population size and the strength of the Allee effect θ rela-
826 tive to K are varied. We note that the carrying capacity of the population in the
827 absence of Allee effects is 5500 with these parameters; therefore, the introduction
828 sizes – which adjust the population size so that there is a 10% infection frequency
829 – do not exceed ~ 612 insects. Top row: demographic stochasticity only. Bottom
830 row: both demographic and environmental stochasticity. First column: no intro-
831 duction. Second column: introduction such that infection frequency is at 10%.

832 Third column: two subsequent introductions, both of which ensure the infection
833 frequency is 10% for each cytotype at time of introduction (see Supplementary In-
834 formation for generation of second introduction). Gray line is the Allee threshold
835 (i.e. initial populations below the gray line go to extinction in the deterministic
836 model).

837

838 **Figure 5.** Using $S_0 = 0.15$ and fixing $\theta/K = 0.1$, the colors of each plot represent
839 the extinction probability for a given parameter combination based on 500 real-
840 izations of the model. In each plot, the initial population size and the number of
841 pheromone sources (P) relative to K are varied. Top row: demographic stochas-
842 ticity only. Bottom row: both demographic and environmental stochasticity. First
843 column: no introduction. Second column: introduction such that infection fre-
844 quency is at 10%. Third column: two subsequent introductions.

845

Figure 1:

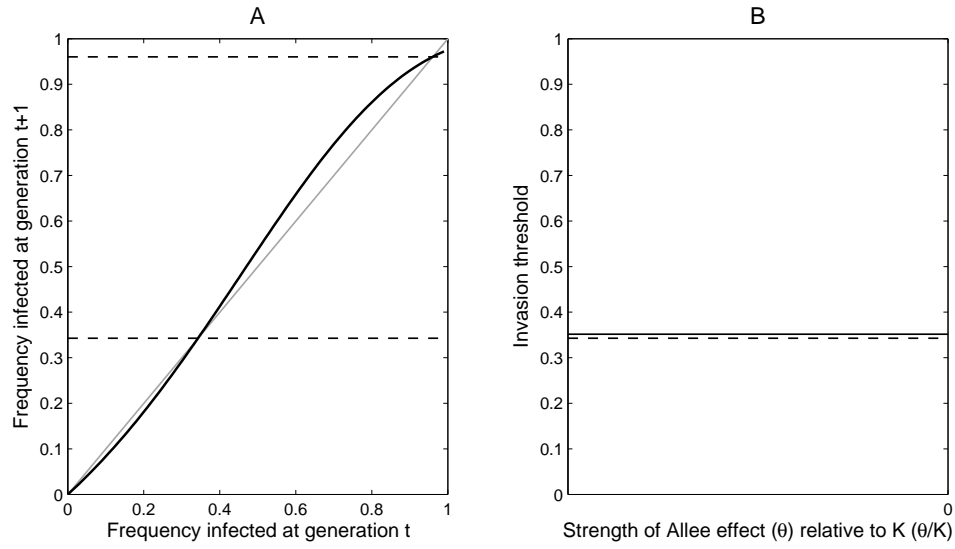


Figure 2:

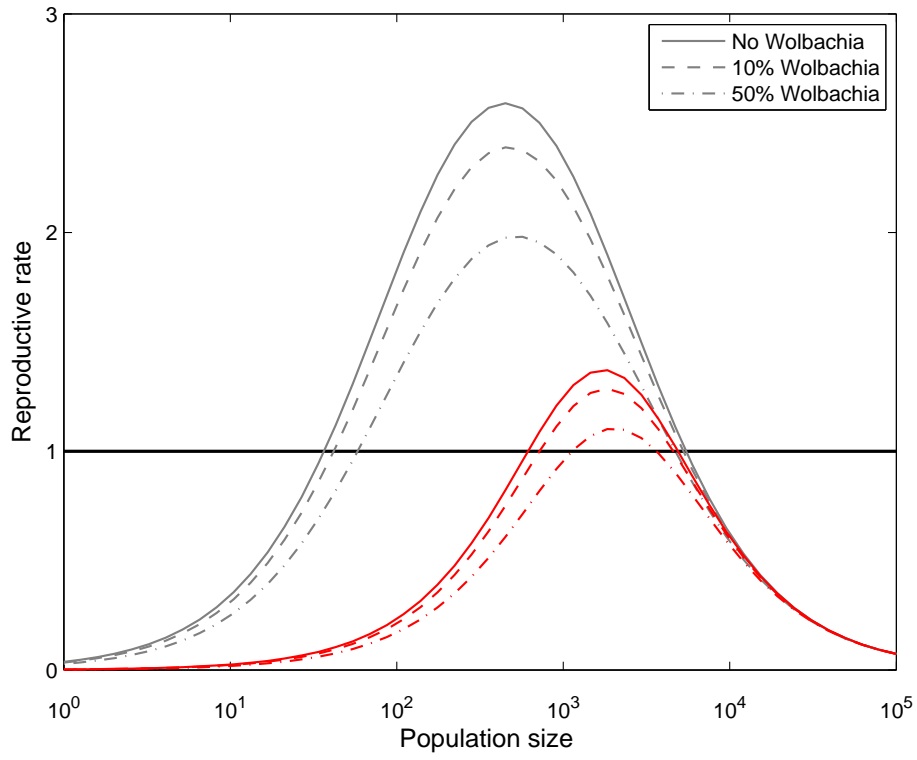


Figure 3:

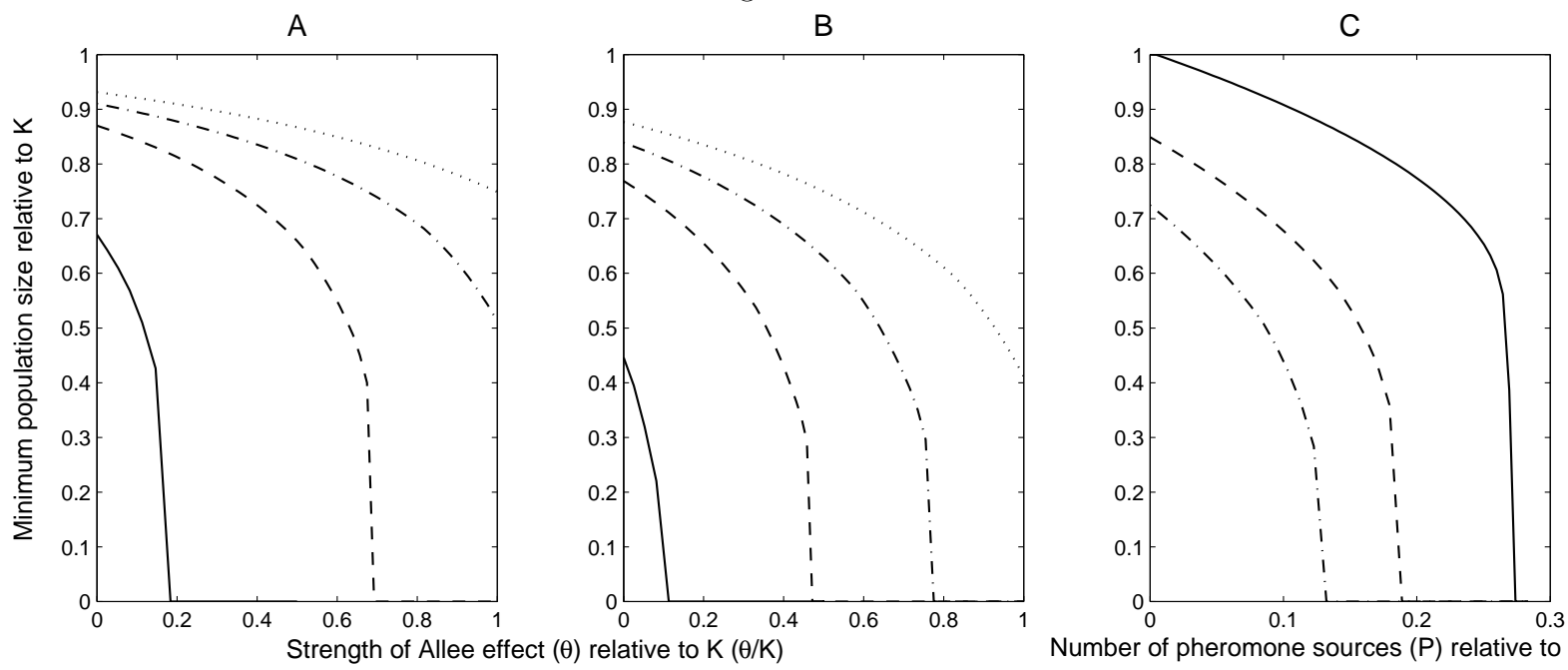


Figure 4:

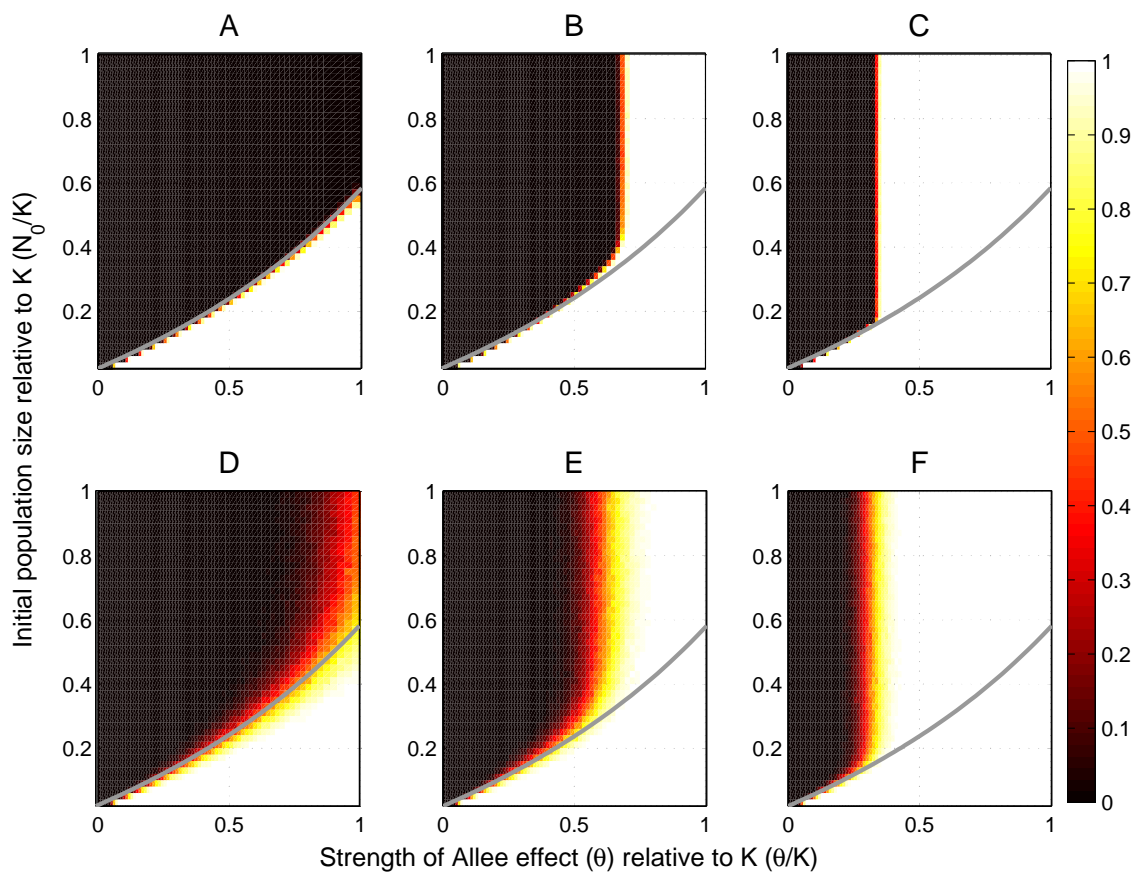


Figure 5:

