

1 **MAST: A flexible statistical framework for assessing transcriptional changes and** 2 **characterizing heterogeneity in single-cell RNA-seq data.**

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16

17 **Abstract**

18 Single-cell transcriptomic profiling enables the unprecedented interrogation of gene
19 expression heterogeneity in rare cell populations that would otherwise be obscured in
20 bulk RNA sequencing experiments. The stochastic nature of transcription is revealed in
21 the bimodality of single-cell transcriptomic data, a feature shared across single-cell
22 expression platforms. There is, however, a paucity of computational tools that take
23 advantage of this unique characteristic. We present a new methodology to analyze
24 single-cell transcriptomic data that models this bimodality within a coherent generalized
25 linear modeling framework. We propose a two-part, generalized linear model that allows
26 one to characterize biological changes in the proportions of cells that are expressing
27 each gene, and in the positive mean expression level of that gene. We introduce the
28 *cellular detection rate*, the fraction of genes turned on in a cell, and show how it can be
29 used to simultaneously adjust for technical variation and so-called “extrinsic noise” at the
30 single-cell level without the use of control genes. Our model permits direct inference on
31 statistics formed by collections of genes, facilitating gene set enrichment analysis. The
32 residuals defined by such models can be manipulated to interrogate cellular
33 heterogeneity and gene-gene correlation across cells and conditions, providing insights
34 into the temporal evolution of networks of co-expressed genes at the single-cell level.
35 Using two single-cell RNA-seq datasets, including newly generated data from Mucosal
36 Associated Invariant T (MAIT) cells, we show how model residuals can be used to
37 identify significant changes across biologically relevant gene sets that are missed by
38 other methods and characterize cellular heterogeneity in response to stimulation.

39

40 **Introduction:**

41 Whole transcriptome expression profiling of single cells via RNA-seq (scRNA-seq) is the logical
42 apex to single cell gene expression experiments. In contrast to transcriptomic experiments on
43 mRNA derived from bulk samples, this technology provides powerful multi-parametric
44 measurements of gene co-expression at the single-cell level. However, the development of
45 equally potent analytic tools has trailed the rapid advances in the biochemistry and molecular

46 biology, and several challenges need to be addressed to fully leverage the information in single-
47 cell expression profiles.

48
49 First, single-cell expression has repeatedly been shown to exhibit a characteristic bimodal
50 expression pattern, wherein the expression of otherwise abundant genes is either strongly
51 positive, or undetected within individual cells. This is due in part to low starting quantities of
52 RNA such that many genes will be below the threshold of detection, but there is also a biological
53 component to this variation (termed extrinsic noise in the literature) that is conflated with the
54 technical variability¹⁻³. We and other groups⁴⁻⁶ have shown that the proportion of cells with
55 detectable expression reflects both technical and biological differences between samples.
56 Results from synthetic biology also support the notion that bimodality can arise from the
57 stochastic nature of gene expression^{2,3,7,8}.

58
59 Secondly, measuring single cell gene expression might seem to obviate the need to normalize
60 for starting RNA quantities. Recent work shows that cells scale transcript copy number with cell
61 volume (a factor that affects gene expression globally) to maintain a constant mRNA
62 concentration and thus constant biochemical reaction rates^{9,10}. In scRNA-seq, cells of varying
63 volume are diluted to an approximately fixed reaction volume leading to differences in detection
64 rates of various mRNA species that are driven by the initial cell volumes. Technical assay
65 variability (e.g. mRNA quality, pre-amplification efficiency) and extrinsic biological factors (e.g.
66 nuisance biological variability due to cell size) remain, and can significantly influence expression
67 level measurements. Consequently, this may render traditional normalization strategies using
68 the expression level of a few “housekeeping” genes, like GAPDH, infeasible¹⁰. Recently, Shalek
69 et al⁵ observed a strong relationship between average expression and detection efficiency, and
70 have proposed a computational approach to correct the estimated gene-specific probability of
71 detection. Our approach easily allows for estimation and control of the CDR simultaneously
72 while estimating treatment effects as opposed to previous approaches⁵ that relied on a set of
73 control genes and could not jointly model both factors.

74
75 Previously, Kharchenko et al⁶ developed a so-called three-component mixture model to test for
76 differential gene expression while accounting for bimodal expression. Their approach is limited
77 to two-class comparisons and cannot adjust for important biological covariates such as multiple
78 treatment groups and technical factors such as batch or time information, severely limiting its
79 utility in more complex experimental designs. On the other hand, several methods have been
80 proposed for modeling bulk RNA-seq data that permit complex modeling through linear¹¹ or
81 generalized linear models^{12,13} but these models have not yet been adapted to single-cell data as
82 they do not properly account for the observed bimodality in expression levels. This is particularly
83 important when adjusting for covariates that might affect the expression rates. As we will
84 demonstrate later, such model mis-specification can significantly affect sensitivity and specificity
85 when detecting differentially expressed genes and gene-sets.

86
87 Here, we propose a Hurdle model tailored to the analysis of scRNA-seq data, providing a
88 mechanism to address the challenges noted above. It is a two-part generalized linear model that
89 simultaneously models the rate of expression over background of various transcripts, and the

90 positive expression mean. Leveraging the established theory for generalized linear modeling
91 allows us to accommodate complex experimental designs while controlling for covariates
92 (including technical factors) in both the discrete and continuous parts of the model. We
93 introduce the *cellular detection rate (CDR)*: the fraction of genes that are turned on / detected in
94 each cell, which, as discussed above, acts as a proxy for both technical (e.g. dropout,
95 amplification efficiency, etc.) and biological factors (e.g. cell volume and other extrinsic factors
96 other than treatment of interest) that can influence gene expression. As a result it represents an
97 important source of variability in scRNA-seq data that needs to be considered (Figure 1). Our
98 approach of modeling the CDR as a covariate, offers an alternative to the weight correction of
99 Shalek et al⁵ that does not depend on the use of control genes and allows us to jointly estimate
100 nuisance and treatment effects. Our framework permits the analysis of complex experiments,
101 such as repeated single cell measurements under various treatments and/or longitudinal
102 sampling of single cells from multiple subjects with a variety of background characteristics (e.g.
103 gender, age, etc.) as it is easily extended to accommodate random effects. Differences between
104 treatment groups are summarized with pairs of regression coefficients whose sampling
105 distributions are available through bootstrap or asymptotic expressions, enabling us to perform
106 complementary differential gene expression and gene set enrichment analyses (GSEA). We use
107 an empirical Bayesian framework to regularize model parameters, which helps improve
108 inference for genes with sparse expression, much like what has been done for bulk gene
109 expression¹⁴. Our GSEA approach accounts for gene-gene correlations, which is important for
110 proper control of type I errors¹⁵. This GSEA framework is particularly useful for synthesizing
111 observed gene-level differences into statements about pathways or modules. Finally, our model
112 yields *single cell residuals* that can be manipulated to interrogate cellular heterogeneity and
113 gene-gene correlations across cells and conditions. We have named our approach MAST for
114 Model-based Analysis of Single-cell Transcriptomics.

115
116 We illustrate the method on two data sets. We first apply our approach to an experiment
117 comparing primary human non-stimulated and cytokine-activated Mucosal-Associated Invariant
118 T (MAIT) cells. MAST identifies novel expression signatures of activation, and the single-cell
119 residuals produced by the model highlights a population of MAIT cells showing partial activation
120 but no induction of effector function. We then illustrate the application of MAST to a previously-
121 published complex experiment studying temporal changes in murine bone marrow-derived
122 dendritic cells subjected to LPS stimulation. We both recapitulate the findings of the original
123 publication and describe additional coordinated gene expression changes at the single-cell level
124 across time in LPS stimulated mDC cells.

125 126 **Results**

127
128 **MAST can account for variation in the cellular detection rate.** As discussed previously and
129 as shown on Figure 1 by principal component analysis (PCA), the cellular detection rate (CDR,
130 see Methods for exact definition), is an important source of variability. It is highly correlated with
131 the second principal component (PC, Pearson's rho=0.76 grouped, 0.91 stimulated, 0.97 non-
132 stimulated) in the MAIT dataset and the first PC (rho=0.92 grouped, 0.97 non-stimulated, 0.92
133 LPS, 0.89 PAM, 0.92 PIC) in the mDC dataset. We observe larger CDR variability within

134 treatment groups than across groups, suggesting that it is likely to be a nuisance factor. This is
135 further supported by the fact that the CDR calculated within control (e.g. housekeeping) genes
136 is highly correlated with the CDR calculated over all genes (Supplementary Figure 1). Its role as
137 a principal source of variation persists across experiments (Figure 1).

138
139 We thus conjecture that CDR is a proxy for unobserved nuisance factors that should be
140 explicitly modeled. In particular, it is not unreasonable to suggest that the CDR captures
141 variation in global transcription rate due to variations in cell size (among other factors)¹⁰, as well
142 as technical variation such as dropout, with dropout rates possibly correlated with cell-size.
143 Fortunately, MAST easily accommodates covariates, such as the CDR, and more importantly
144 allows joint, additive modeling of them with other biological variables of interest, with the effect
145 of each covariate decomposed into its discrete and continuous parts. This two-part modeling is
146 key to account for the CDR that directly reflects the gene-level transcription rates. Applying an
147 analysis of deviance with MAST (see Methods), we quantified the amount of variability that
148 could be attributed to CDR. The CDR accounts for 5.2% of the deviance in the MAIT data set
149 and 4.8% in the mDC data set for the average gene, and often times much more than that: it
150 comprises more than 9% of the deviance in over 10% of genes in both data sets, particularly for
151 the discrete component of the model (Supplementary Figure 2). It should also be noted that the
152 CDR deviance estimates for many of the genes are comparable (if not greater) to the treatment
153 deviance estimates showing that it.

154
155 That CDR predicts expression levels contradicts the model of independent expression between
156 genes, since the level of expression (averaged across many genes) would not affect the level in
157 any given gene were expression independent. This pervasiveness suggests latent factors are
158 creating coordinated changes in expression across genes. In light of the work of Padovan-
159 Merhar et al¹⁰, we conjecture the latent factor relates to differences in cell volumes, since cells
160 of different volumes compensate to conserve mRNA species molarity, which implies higher copy
161 numbers of all transcripts in larger cells. Higher copy numbers result in higher scRNA-seq
162 detection rates globally across transcripts.

163
164 Finally, we have investigated the relationship between our approach and the weight correction
165 of Shalek et al⁵ (Supplementary Figure 3). We observe a strong linear relationship between the
166 CDR and the weights of Shalek et al⁵. Thus, use of the CDR as a covariate can be seen as a
167 statistically rigorous way to correct for the dropout biases of Shalek et al⁵, without the need to
168 use control genes, and more importantly with the ability to control for these while estimating
169 treatment effects.

170 171 **Single-cell sequencing identifies a transcriptional profile of MAIT cell activation**

172 We applied MAST to our MAIT dataset to identify genes up- or down-regulated by cytokine
173 stimulation while accounting for variation in the CDR (see Methods). We detected 291
174 differentially expressed genes, as opposed to 1413 when excluding CDR. To determine whether
175 this was due to a change in ranking or a simply a shift in significance, we compared the overlap
176 between the top n genes in both models (varying n from 100 to 1413), and found that, on
177 average, 35% (range 32% - 38%) of genes are excluded when CDR is modeled, suggesting that

178 inclusion of this variable allows global changes in expression, manifest in the CDR, to be
179 decomposed from local changes in expression. This is supported by gene ontology enrichment
180 analysis (Supplementary Figure 4) of these CDR-specific genes (n=539), where we see no
181 enrichment for modules associated with treatment of interest.

182

183 In order to assess the type-I error rate of our approach, we also applied MAST to identify
184 differentially expressed genes across random splits of the non-stimulated MAIT cells. As
185 expected, MAST did not detect any significant differences (Supplementary Figure 5A), whereas
186 DESeq and edgeR, designed for bulk RNA-seq, detected large number of differentially
187 expressed genes even at very low FDR thresholds. We examined the GO enrichment of genes
188 detected by limma or edgeR or DESeq but not MAST and found that these sets lacked
189 significant enrichment for modules related to the treatment of interest (Supplementary Figures
190 5B and 6-8). MAST's testing framework evidently has better specificity than these approaches.

191

192 Figure 2A shows the single-cell expression (\log_2 -TPM) of the top 100 genes identified as
193 differentially expressed between cytokine (IL18, IL15, IL12) stimulated (purple) and non-
194 stimulated (pink) MAIT cells using MAST. Following stimulation with IL12/15/18, we observe
195 increased expression in genes with effector function including Interferon- γ (IFN- γ), granzyme-
196 B (GZMB) as has been reported in NK, NKT and memory T cells, and a concomitant
197 downregulation of the AP-1 transcription factor network. CD69 is an early and only transient
198 marker of activation that can be induced by stimulation of the T cell receptor or by cytokine
199 signals. Its downregulation at the mRNA level after 24h is likely preceding subsequent protein-
200 level downregulation¹⁶⁻¹⁸.

201

202 We used these lists of up- and down-regulated genes to define a MAIT activation score that
203 differentiates between stimulated and non-stimulated MAITs as shown in Figure 2B. This score
204 (see Methods), for each cell, is defined as the expected expression level across genes in a
205 module (based on the model fit) corrected for nuisance factors (such as CDR, see Methods).
206 The score enables us to cleanly differentiate stimulated and non-stimulated cells, and
207 demonstrates that the stimulated MAIT population is much more heterogeneous in its
208 expression phenotype. In particular, a few stimulated MAIT cells (SC08, SC54, SC48, SC15,
209 SC46, and SC61 in Figure 2A) exhibit low expression of IFN- γ response genes, suggesting
210 these cells did not fully activate despite stimulation. Post-sort experiments via FCM show that
211 the sorted populations were over 99% pure MAITs (Supplementary Figure 9A), and exhibited a
212 change in cell size upon stimulation (Supplementary Figure 9B), and that up to 26% of
213 stimulated MAITs didn't express IFN- γ or GZMB following cytokine stimulation (Supplementary
214 Figure 9C). The non-responding cells in the RNA-seq experiment likely correspond to these
215 non-responding cells from the flow cytometry experiment, and the observed frequencies of
216 these cells in the RNA-seq and flow populations are consistent with each other (Pr(observing 6
217 or fewer non-responding cells) = 0.16 under binomial sampling). We discuss this heterogeneity
218 in a further section. Importantly, the lists of up- and down-regulated genes can be used to define
219 gene sets for gene set enrichment analysis in order to identify transcriptional changes related to
220 MAIT activation in bulk experiments.

221

222 **Gene set enrichment analysis highlights pathways implicated in MAIT cell activation.**

223 We used MAST to perform gene set enrichment analysis (GSEA, see methods) in the MAIT
224 data using the blood transcriptional modules of Li et al¹⁹. The cell-level scores for the top 9
225 enriched modules (Figure 3A) continue to show significant heterogeneity in the stimulated cells,
226 particularly for modules related to T-cell signaling, protein folding, proteasome function, and the
227 AP-1 transcription factor network. Enrichment in stimulated cells (green) and non-stimulated
228 cells (pink) is displayed for each module for the discrete and continuous components of the
229 model (Figure 3B, see Methods), as well as a Z-score combining the discrete and continuous
230 parts. The enrichment in the T-cell signaling module is driven by the increased expression of
231 IFN- γ , GZMB, IL2RA, IL2RB, and TNFRSF9, 5 of the 6 genes in the module. Stimulated cells
232 also exhibit increased energy usage, translation and protein synthesis, while down-regulating
233 genes involved in cell cycle growth and arrest (and other cell cycle related modules). The down-
234 regulation of cell cycle growth inhibition genes indicates that IL-12/15/18 signals are sufficient to
235 prepare MAIT cells for cell proliferation. Interestingly, we observe down-regulation of mRNA
236 transcripts from genes in the AP-1 transcription factor network. This has been previously
237 described in dendritic cells in response to LPS stimulation²⁰ and, indeed, we observe this effect
238 in the mDC data set analyzed here (Supplementary Figure 10).

239
240 Our GSEA approach is more powerful than existing methods for bulk RNA-seq data
241 (Supplementary Figure 11), and we discover significantly enriched modules with clear patterns
242 of stimulation-induced changes that other methods omit (Supplementary Figure 12). Two such
243 modules include the “T-cell surface signature” and “chaperonin mediated protein folding, whose
244 component genes show elevated expression in response to stimulation (Supplementary Figure
245 12A-D). These additional discoveries are not solely due to greater permissiveness in MAST.
246 We applied MAST to identify differentially expressed gene sets across random partitions of the
247 non-stimulated cells, to examine its false discovery rate. As expected, MAST did not detect any
248 significant differences, which suggests that it has good type I error control.

249
250 **Residual analysis identifies networks of co-expressed genes implicated in MAIT cell**
251 **activation.** Much of the heterogeneity between the non-responding and responding stimulated
252 cells remains even after removal of marginal (gene level) stimulation effects. Since, MAST
253 models the expected expression value for each cell, we can compute residuals adjusted for
254 known sources of variability (See Methods). The residuals can be compared across genes to
255 characterize cellular heterogeneity and correlation. We observe co-expression in the residuals
256 from stimulated cells that is not evident in the non-stimulated group (Figure 4A,B). Since the
257 residuals have removed any marginal changes due to stimulation in each gene, the average
258 residual in the two groups is comparable. The co-expression observed, meanwhile, is due to
259 individual cells expressing these genes *dependently*, where pairs of genes appear together
260 more often than expected under a model of independent expression.

261
262 Two clusters of co-expressed genes stand out in the residuals of the stimulated cells (Figure 4
263 B). These clusters show coordinated, early up-regulation of GZMB and IFN- γ in response to
264 stimulation in MAIT cells and a concomitant decrease in CD69 expression, an early and

265 transient activation marker. PCA of the model residuals highlights the non-responsive stimulated
266 MAIT cells (Figure 4C).

267

268 Accounting for the CDR reduces the background correlation observed between genes
269 (Supplementary Figure 13) where nearly 25% of pairwise correlations decrease after CDR
270 correction. When the CDR is included in the model, the number of differentially expressed
271 genes with significant correlations across cells (FDR adjusted p-value < 1%) decreases from 73
272 to 61 in the stimulated cells, and from 808 to 15 in non-stimulated cells. This shows that
273 adjusting for CDR is also important for co-expression analyses as it reduces background co-
274 expression attributable to cell volume, which otherwise results in dense, un-interpretable gene
275 networks.

276

277

278 **MAST on complex experimental designs: temporal expression patterns of mouse** 279 **dendritic cell maturation**

280 Shalek et al⁵ analyzed murine bone-marrow derived dendritic cells simulated using three
281 pathogenic components over the course of six hours and estimated the proportion of cells that
282 expressed a gene and the expression level of expressing cells. We compared results from
283 applying our model to those obtained by Shalek et al⁵ when analyzing their lipopolysaccharide
284 (LPS) stimulated cells. As with the MAIT analysis, we used MAST adjusting for the CDR. MAST
285 identified a total of 1359 differentially expressed genes (1996 omitting the CDR), and the CDR
286 accounted for 5.2% of the model deviance in the average gene.

287 The most significantly elevated genes at 6h include CCL5, CD40, IL12B, and Interferon-
288 inducible (IFIT) gene family members, while down-regulation was observed for EGR1 and
289 EGR2, transcription factors that are known to negatively regulate dendritic cell
290 immunogenicity²¹.

291

292 **GSEA of mouse bone marrow-derived dendritic cells**

293 We performed GSEA with the Mouse GO modules and three modules Shalek et al⁵ identified.
294 The blood transcriptional modules of Li et al¹⁹ are shown in Supplementary Figure 10. Figure 5
295 shows module scores for significant GSEA modules for the LPS stimulated cells where the
296 heatmap represents Z values (see methods for details). Besides finding signatures consistent
297 with the modules from Shalek et. al. (Figure 5A), we identify modules that show similar
298 annotation and overlap significantly with the *core antiviral* and *sustained inflammatory*
299 signatures, including several modules linked to type 1 interferon response and antiviral
300 signatures (Figure 5B). The “cellular response to interferon- beta” signature (n = 22) overlaps
301 with the original core antiviral signature (n = 99) by 13 genes (hypergeometric p = 1.24x10⁻²³).
302 The *response* and *defense response to virus* signatures overlap with the core antiviral signature
303 by 17 of 43 and 22 of 74 genes (hypergeometric p=3.64x10⁻²⁶ and 4.08x10⁻²⁹, respectively),
304 suggesting the core antiviral signature captures elements of these known signatures. The
305 *chemokine* (n=16) and *cytokine activity* (n=51) modules overlap with the sustained inflammatory
306 (n = 95) module by 5 and 12 genes, respectively (hypergeometric p=5.10x10⁻⁹ and 9.53x10⁻¹⁶).
307 Our modeling approach identifies the two “early marcher” cells in the core antiviral module
308 (marked with triangles on Figure 5A) corresponding to the same cells highlighted in Figure 4b of

309 Shalek et al⁵. Other modules exhibiting significant time-dependent trends include a module of
310 genes involved in the AP-1 transcription factor network that is down-regulated (Supplementary
311 Figure 10), a finding which has been previously shown in human monocytes following LPS
312 stimulation²⁰. As with the MAITs, GSEA permutation analysis to evaluate type I error rates did
313 not identify any significant modules (data not shown). These results further confirm the original
314 findings and demonstrate the increased sensitivity of our approach. GSEA heatmaps for the
315 other stimulations can be found in Supplementary Figure 14.

316

317 **Residual analysis of mouse bone marrow-derived dendritic cells identifies sets of co-** 318 **expressed genes.**

319 We also explored stimulation-driven correlation patterns. Principal component analysis (Figure
320 6A) of the model residuals demonstrates a clear time trend associated with PC1, as cells
321 increase co-expression of interferon-activated genes. After removing the marginal stimulation
322 and adjusting for the CDR, we observe correlation between chemokines CCL5, TNF receptor
323 CD40, and interferon-inducible (IFIT) genes (Figure 6B). A principal finding of the original
324 publication was the identification of a subset of cells that exhibited an early temporal response
325 to LPS stimulation. Recapitulating the original results here, when we examine the PCA of the
326 residuals using the genes in the core antiviral module, we can identify the “early marcher” cells
327 at the 1h time-point (Supplementary Figure 15). The co-expression plot for other stimulations
328 can be found in the supplementary material (Supplementary Figures 16 and 17).

329

330 **Discussion**

331 We have presented MAST, a flexible statistical framework for the analysis of scRNA-seq data.
332 MAST is suitable for supervised analyses about differential expression of genes and gene-
333 modules, as well as unsupervised analyses of model residuals, to generate hypotheses
334 regarding co-expression of genes. MAST accounts for the bimodality of single-cell data by
335 jointly modeling rates of expression (discrete) and positive mean expression (continuous)
336 values. Information from the discrete and continuous parts is combined to perform inference
337 about changes in expression levels using gene or gene-set based statistics. Because our
338 approach uses a generalized linear framework, it can be used to jointly estimate nuisance
339 variation from biological and technical sources, as well as biological effects of interest. In
340 particular, we have shown that it is important to control for the proportion of genes detected in
341 each cell, which we refer to as the cellular detection rate (CDR), as this factor can single-
342 handedly explain 13% of the variability in the 90% percentile gene. Adjusting for CDR at least
343 partially controls for differences in abundance due to cell size (and other extrinsic biological and
344 technical effects), while omitting it would lead to overestimated effects of the treatment on the
345 system. Using several scRNA-seq datasets, we showed that our approach provides a
346 statistically rigorous improvement to methods proposed by other groups in this context⁵.

347

348 Because our approach is regression-based, it can be used to compute residuals to explore
349 cellular heterogeneity and gene-gene correlations after selected technical and/or biological
350 effects have been removed. In particular, using this approach, we identify MAIT cells that do not
351 have a typical activated expression profile in response to stimulation (Figures 2 and 3). The
352 proportion of these cells detected in the scRNASeq experiment is consistent with what was

353 detected in the flow cytometry experiment. These cells do not produce IFN- γ or GZMB upon to
354 cytokine stimulation and exhibit expression profiles intermediate to non-stimulated and
355 stimulated cells (Supplementary Figure 18C). The cells exhibit lower levels of IFN- γ and GZMB
356 than activated cells (Supplementary Figure 18A), but also exhibit decreased expression of AP-1
357 component genes Fos and FosB, consistent with other stimulated cells (Supplementary Figure
358 18B).

359
360 As discussed by Padovan-Merhar et al¹⁰, care must be taken when interpreting experiments
361 where the system shows global changes in CDR across treatment groups, as this could result in
362 confounding treatment effect with differences in cell volume, which are not necessarily of
363 biological interest. Our approach addresses this issue as MAST allows joint modeling of CDR
364 and treatment effects, so the interpretation of the treatment effect is that the cell volume/CDR
365 has been held constant. It is also possible to only use CDR as a precision variable by centering
366 the CDR within each treatment groups, which makes the CDR measurement orthogonal to
367 treatment. This would implicitly assume that the observed changes are treatment induced,
368 while still modeling the heterogeneity in cell volume within each treatment group. An alternative
369 approach would be to estimate the CDR coefficient using a set of control genes assumed to be
370 treatment invariant, such as housekeeping or ERCC spike-ins^{22,23} and including it as an offset to
371 the linear predictors in the regression. An analogous approach is undertaken by Buettner et.
372 al.²², however it does not account for bimodality and does not jointly model technical and
373 biological effects.

374
375 MAST is available as an R package (<http://www.github.com/RGLab/MAST>, doi:
376 [10.5281/zenodo.18539](https://doi.org/10.5281/zenodo.18539)). All data and results presented in this paper – including code to
377 reproduce the results – are available at:
378 (<http://github.com/RGLab/MASTdata/archive/v1.0.0.tar.gz>, doi: 10.5281/zenodo.18540). It
379 should also be noted that while most of the methodology presented here was developed for
380 scRNA-seq, it should be applicable to other single-cell gene expression platforms.

381 382 **Figure Captions**

383 **Figure 1.** The fraction of genes expressed, or cellular detection rate (CDR), explains the
384 principal components of variation in MAIT and DC data sets.

385
386 **Figure 2.** Single-cell expression (\log_2 -TPM) of the top 100 genes identified as differentially
387 expressed between cytokine (IL18, IL15, IL12) stimulated (purple) and non-stimulated (pink)
388 MAIT cells using MAST (A). Partial residuals for up- and down- regulated genes are
389 accumulated to yield an activation score (B), and this score suggests that the stimulated cells
390 have a more heterogeneous response to stimulation than do the non-stimulated cells.

391
392 **Figure 3.** Module scores for individual cells for the top 9 enriched modules (A) and decomposed
393 Z-scores (B) for single-cell gene set enrichment analysis in MAIT data set, using the blood
394 transcription modules (BTM) database. The distribution of module scores suggests
395 heterogeneity among individual cells with respect to different biological processes. Enrichment
396 of modules in stimulated and non-stimulated cells is due to a combination of differences in the

397 discrete (proportion) and continuous (mean conditional expression) of genes in modules. The
398 combined Z-score reflects the enrichment due to differences in the continuous and discrete
399 components.

400

401 **Figure 4.** Gene-gene correlation (Pearson's rho) of model residuals in non-stimulated (A) and
402 stimulated (B) cells, and principal components analysis biplot of model residuals (C) on both
403 populations using the top 50 marginally differentially expressed genes. As marginal changes in
404 the genes attributable to stimulation and CDR have been removed, clustering of subpopulations
405 in (C) indicates co-expression of the indicated genes on a cellular basis.

406

407 **Figure 5.** Module scores (A) and decomposed Z-scores (B) for single-cell gene set enrichment
408 analysis for LPS stimulated cells, mDC data set, using the mouse GO biological process
409 database. The change in single-cell module scores over time for the nine most significantly
410 enriched modules in response to LPS stimulation are shown in A. The *core antiviral, peaked*
411 *inflammatory* and *sustained inflammatory* modules are among the top enriched modules,
412 consistent with the original publication. Additionally we identify GO modules *cellular response to*
413 *interferon-beta* and *response to virus*, which behave analogously to the core antiviral and
414 sustained inflammatory modules. No GO analog for the *peaked inflammatory* module was
415 detected. The majority of modules detected exhibit enrichment relative to the 1h time point (thus
416 increasing with time). The “early marcher” cells identified in the original publication are
417 highlighted here with triangles. We show the top 50 most significant modules (B). The combined
418 Z-score summarizes the changes in the discrete and continuous components of expression.

419

420 **Figure 6.** Principal components analysis biplot of model residuals (A) and Gene-gene
421 correlation (Pearson's R) of model residuals (B) by time point for LPS cells, mDC experiment
422 using 20 genes with largest log-fold changes, given significant (FDR $q < .01$) marginal changes
423 in expression. PC1 is correlated with change over time. The two “early marcher” cells are
424 highlighted by an asterisk at the 1h time-point. Correlation structure in the residuals is
425 increasingly evident over time and can be clearly observed at the 6h time-point compared to the
426 earlier time-points.

427

428 **METHODS**

429

430 **Data Sets**

431 Data for the MAIT study were derived from a single donor who provided written informed
432 consent for immune response exploratory analyses. The study was approved by the
433 relevant institutional review boards.

434

435 **MAIT cell isolation and stimulation**

436 Cryopreserved PBMC were thawed and stained with Aqua Live/Dead Fixable Dead Cell Stain
437 and the following antibodies: CD3, CD8, CD4, CD161, V α 7.2, CD56 and CD16. CD8⁺ MAIT
438 cells were sorted as live CD3⁺CD8⁺ CD4⁺CD161^{hi}V α 7.2⁺ cells and purity was confirmed by post-
439 sort FACS analysis. Sorted MAIT cells were divided into aliquots and immediately processed on

440 a C1 Fluidigm machine or treated with a combination of IL-12 (eBioscience), IL-15
441 (eBioscience), and IL-18 (MBL) at 100ng/mL for 24 hours followed by C1 processing.

442

443

444 **C1 processing, Sequencing, and Alignment**

445 After flow sorting, single cells were captured on the Fluidigm™ C1 Single-Cell Auto Prep
446 System (C1), lysed on chip and subjected to reverse transcription and cDNA amplification using
447 the SMARTer® Ultra™ Low Input RNA Kit for C1 System (Clontech). Sequencing libraries were
448 prepared using the Nextera XT DNA Library Preparation Kit (Illumina) according to C1 protocols
449 (Fluidigm). Barcoded libraries were pooled and quantified using a Qubit® Fluorometer (Life
450 Technologies). Single-read sequencing of the pooled libraries was carried out either on a
451 HiScanSQ or a HiSeq2500 sequencer (Illumina) with 100-base reads, using TruSeq v3 Cluster
452 and SBS kits (Illumina) with a target depth of >2.5M reads. Sequences were aligned to the
453 UCSC Human genome assembly version 19 and gene expression levels quantified using
454 RSEM²⁵ and TPM values were loaded into R²⁶ for analyses. See supplement for more details
455 on data processing procedures.

456

457

458 **Time-series stimulation of mouse bone-marrow derived dendritic cells (mDC)**

459 Processed RNA-seq data (transcripts-per-million, TPM) were downloaded from GEO under
460 accession number GSE41265. Alignment, pre-processing and filtering steps have been
461 previously described⁵. Low quality cells were filtered as described in Shalek et al⁵.

462

463 **Single Cell RNA Seq Hurdle model**

464 We model the $\log_2(\text{TPM}+1)$ expression matrix as a two part generalized regression model. The
465 cell expression rate given a design is modeled using logistic regression and the expression level
466 is modeled as conditionally Gaussian given that they are expressed.

467

468 Given normalized, possibly thresholded (see supplementary material), scRNA-seq expression
469 $Y = [y_{ig}]$, the rate of expression and the level of expression for the expressed cells are modeled
470 conditionally independent for each gene g . Define the indicator $Z = [z_{ig}]$ indicating whether
471 gene g is expressed in cell i , i.e. $z_{ig} = 0$ if $y_{ig} = 0$ and $z_{ig} = 1$ if $y_{ig} > 0$. We fit logistic
472 regression models for the discrete variable Z and Gaussian linear model for the continuous
473 variable ($Y | Z = 1$) independently, as follows,

474

$$\begin{aligned} \text{logit} \left(P(Z_{ig} = 1) \right) &= X_i \beta_g^D \\ \Pr(Y_{ig} = y | Z_{ig} = 1) &= N(X_i \beta_g^C, \sigma_g^2) \end{aligned}$$

475

476 The regression coefficients of the discrete component are regularized using a Bayesian
477 approach as implemented in the *bayesglm* function of the *arm* R package, which uses weakly
478 informative priors²⁷ to provide sensible estimates under linear separation (See supplementary
479 material for details). We also perform regularization of the continuous model variance

480 parameter, as described below, which helps increases robustness of gene-level differential
481 expression analysis when a gene is only expressed in a few cells.

482
483 We define the *cellular detection rate* (CDR) as the proportion of genes detected in each cell.
484 The CDR for cell i is:

$$\text{CDR}_i = 1/N \sum_{g=1}^N z_{ig}$$

485 An advantage of our approach is that it is straightforward to account for CDR variability by
486 adding the variable as a covariate in the discrete and continuous models (column of the design
487 matrix, X , defined above). In the context of our hurdle model, inclusion the CDR covariate can
488 be thought of as the discrete analog of global normalization, and as we show in the examples,
489 this normalization yields more interpretable results and helps decrease background correlation
490 between genes, which is desirable for detecting genuine gene co-expression.

491
492 **Shrinkage of the continuous variance**

493 As the number of expressed cells varies from gene to gene, so does the amount of information
494 available to estimate the residual variance of the gene. On the other hand, many genes can be
495 expected to have similar variances. To accommodate this feature of the assay, we shrink the
496 gene-specific variances estimates to a global estimate of the variance using an empirical Bayes
497 method. Let τ_g^2 be the precision (1/variance) for $Y_g|Z_g = 1$ in gene g . We suppose
498 $\tau_g^2 \sim \text{Gamma}(\alpha, \beta)$, find the joint likelihood (across genes) and integrate out the gene-specific
499 inverse variances. Then maximum likelihood is used to estimate α and β . Due to conjugacy,
500 these parameters are interpretable providing 2α pseudo-observations with precision β/α . This
501 leads to a simple procedure where the shrunken gene-specific precision is a convex
502 combination of its MLE and the common precision. This approach accounts for the fact that the
503 number of cells expressing a gene varies from gene to gene. Genes with fewer expressed cells
504 end up with proportionally stronger shrinkage, as the ratio of pseudo observations to actual
505 observations is greater. Further details are available in the supplement.

506
507 **Testing for differential expression**

508 Because Z_g and Y_g are defined conditionally independent for each gene, tests with asymptotic
509 χ^2 null distributions, such as the likelihood ratio or Wald tests can be summed and remain
510 asymptotically χ^2 , with the degrees of freedom of the component tests added. For the
511 continuous part, we use the shrunken variance estimates derived through our empirical Bayes
512 approach described above. The test results across genes can be combined and adjusted for
513 multiplicity using the false discovery rate (FDR) adjustment²⁸. In this paper, we declare a gene
514 differentially expressed if the FDR adjusted p-value is less than 0.01 and the estimated fold-
515 change is greater than 1.5 (on \log_2 scale).

516
517 **Gene Set Enrichment Analysis (GSEA)**

518 Our competitive GSEA compares the average model coefficient in the *test* set (gene set of
519 interest) to the average model coefficient in the *null* set (everything else) with a Z-test. Suppose
520 the genes are sorted so that the first G_0 genes are in the null set, and the last $G - G_0$ genes are

521 in the test set. Then, for example, to test the continuous coefficients in the gene set, the sample
522 means of the coefficients in the test and null sets are calculated, that is, calculate $\hat{\theta} =$
523 $1/(G - G_0) \sum_{g=G_0+1}^G \hat{\beta}_g$ and $\hat{\theta}_0 = 1/G_0 \sum_{g=1}^{G_0} \hat{\beta}_g$. The sampling variance of $\hat{\theta}_0$, in principle, is
524 equal to $1/G_0 (\sum_{g=1}^{G_0} Var(\hat{\beta}_g) + 2 \sum_{1 \leq g < h < G_0} Cov(\hat{\beta}_g, \hat{\beta}_h))$, and similarly for $\hat{\theta}$.

525 Given this sampling variance, a Z test can be formed by comparing $Z = \frac{\hat{\theta} - \hat{\theta}_0}{\sqrt{Var(\hat{\theta}) + Var(\hat{\theta}_0)}}$.

526

527 We estimate $Var(\hat{\beta}_g)$ and $Cov(\hat{\beta}_g, \hat{\beta}_h)$ via bootstrap, to avoid relying on asymptotic
528 approximations. In practice, we find only a few (<100) bootstrap replicates are necessary to
529 provide stable variance-covariance estimates, however even this modest requirement can be
530 relaxed for exploratory analysis by assuming independence across genes and using model-
531 based (asymptotic) estimates.

532

533 Z scores are formed and calculated equivalently for the logistic regression coefficients. GSEA
534 tests are done separately on the two components of the hurdle model and the results from the
535 two components are combined using the Stouffer's method²⁹, which favors consensus in the two
536 components³⁰ (see supplement for details). The approach is similar to that used by CAMERA¹⁵
537 for bulk experiments in its accounting for inter-gene correlation that is known to inflate the false
538 significance (type-I error) in permutation-based GSEA protocols¹⁵, although it differs in that it
539 uses the sampling variance of each model coefficient to find the variance of the average
540 coefficient, whereas CAMERA uses the empirical variance of the model coefficients. In our
541 analyses we used the Emory blood transcriptional modules¹⁹ as well as mouse gene ontology
542 annotations available from the Mouse Genome Informatics web site³².

543

544 **GO Enrichment Analysis**

545 Testing for enriched Gene Ontology terms based on list of genes was performed with the
546 GOrilla online tool using the approach of comparing an unranked target list against a
547 background list³³.

548

549 **Residual Analysis**

550 The hurdle model, in general, provides two residuals: one for the discrete component and one
551 for the continuous component. Standardized deviance residuals are calculated for the discrete
552 and continuous component separately, and then we combine the residuals by averaging them.
553 If a cell is unexpressed, then its residual is missing and it is omitted from the average. See the
554 supplement for details.

555

556 **Module Scores**

557 In order to assess the degree to which each cell exhibits enrichment for each gene module, we
558 use quantities available through our model to define module "scores", which are defined as the
559 observed expression corrected for CDR effect, analogous to those defined by Shalek et al⁵. The
560 score s_{ij} for cell i and gene j is defined as the observed expression corrected for the CDR
561 effect: $s_{ij} = y_{ij} - \tilde{y}_{ij}$ where \tilde{y}_{ij} is the predicted effect from the fitted model that excludes the
562 treatment effects of interest. This can be interpreted as correcting the observed expression of

563 gene j in cell i by subtracting the conditional expectation of nuisance effects. In our two part
564 model, $\tilde{y}_{ij} = \hat{z}_{ij} \hat{y}_{ij}$ where \hat{z}_{ij} and \hat{y}_{ij} are the predicted values from the discrete and continuous
565 components of our hurdle model.

566 A gene module score for cell l is the average of the scores for the genes contained in the
567 module, i.e. $\sum_{\{j \in \text{module}\}} s_{ij} / |\text{module}|$

568

569 **Author Contributions:**

570 GF, AM, MY and RG developed the statistical methods, and wrote the manuscript. AM, MY,
571 and GF wrote the R package and performed data analysis. CKS, HWM, and MP designed and
572 performed the MAIT cell experiments and contributed to data interpretation and provided
573 manuscript feedback. VG and PL coordinated the collection of the single cell sequencing data
574 and contributed manuscript preparation and feedback and data interpretation. AKS contributed
575 the mDC data and contributed manuscript feedback and to data interpretation. JD contributed to
576 data analysis of the single-cell expression data. MJM contributed samples and to study design.

577

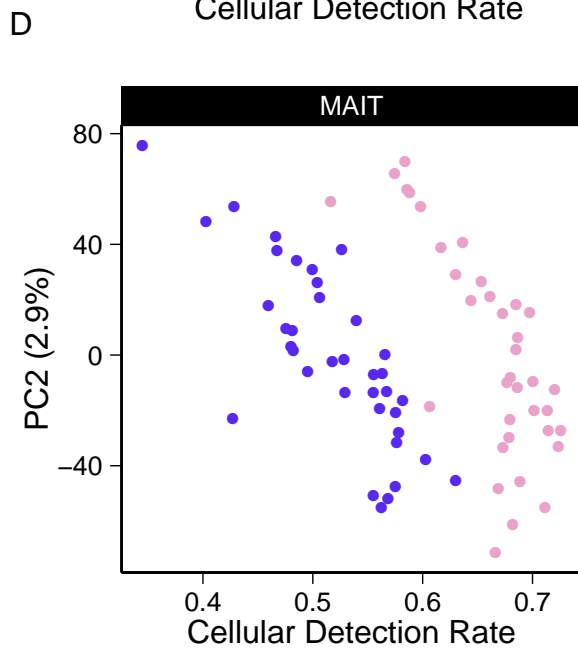
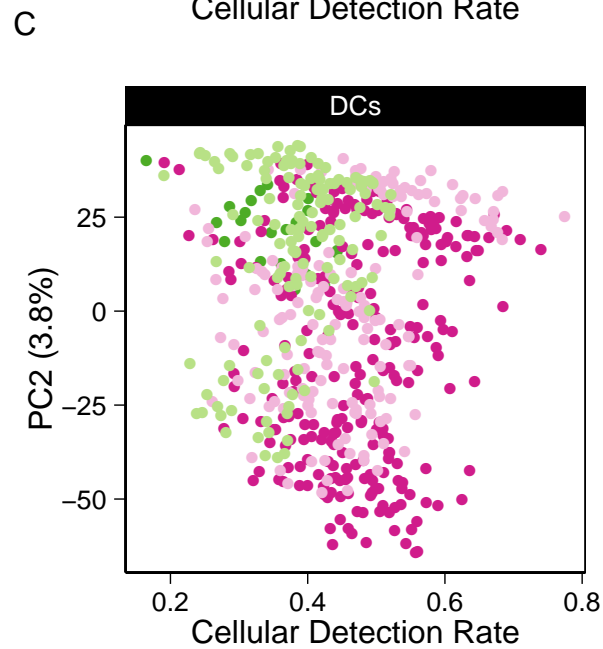
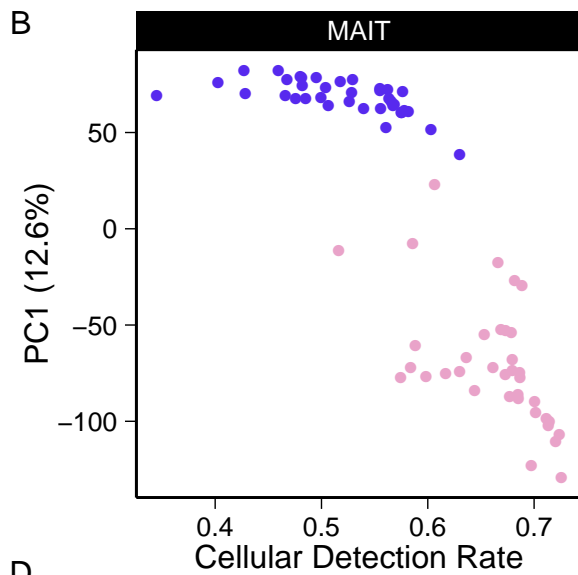
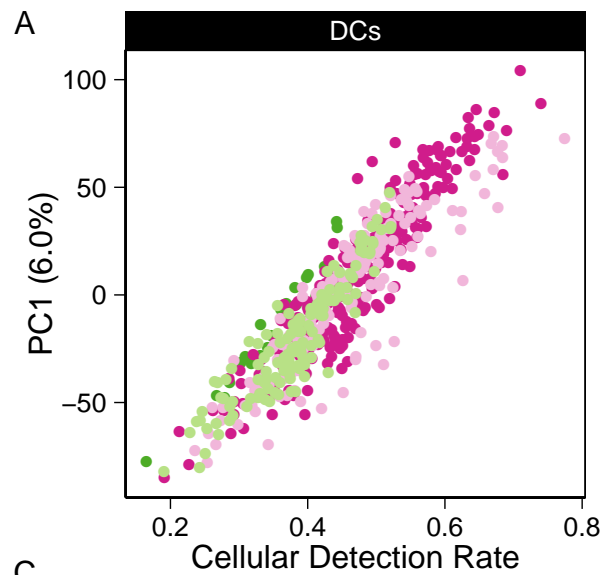
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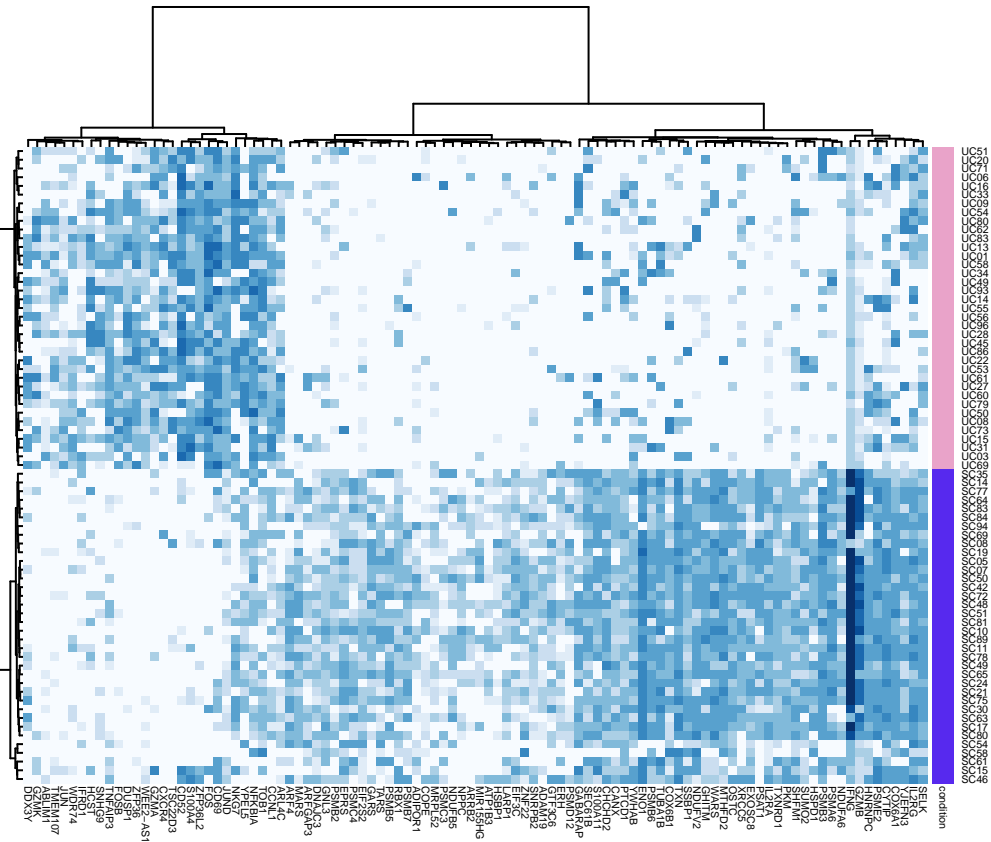
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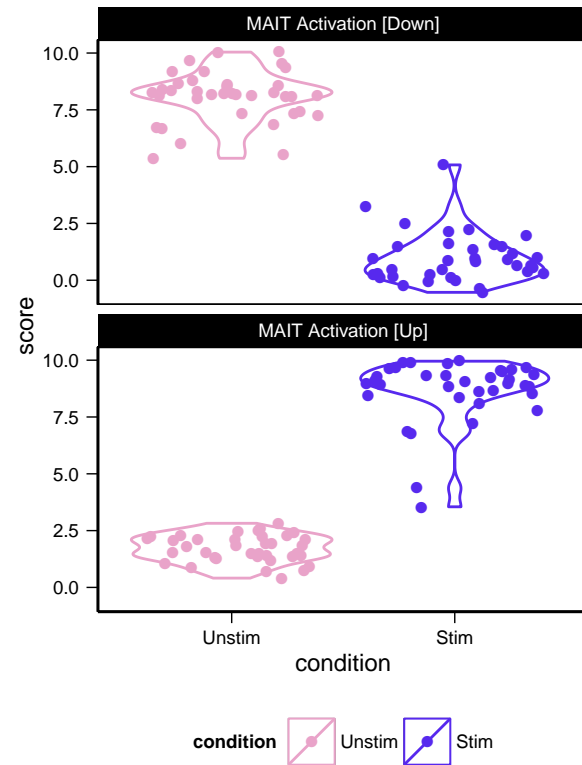
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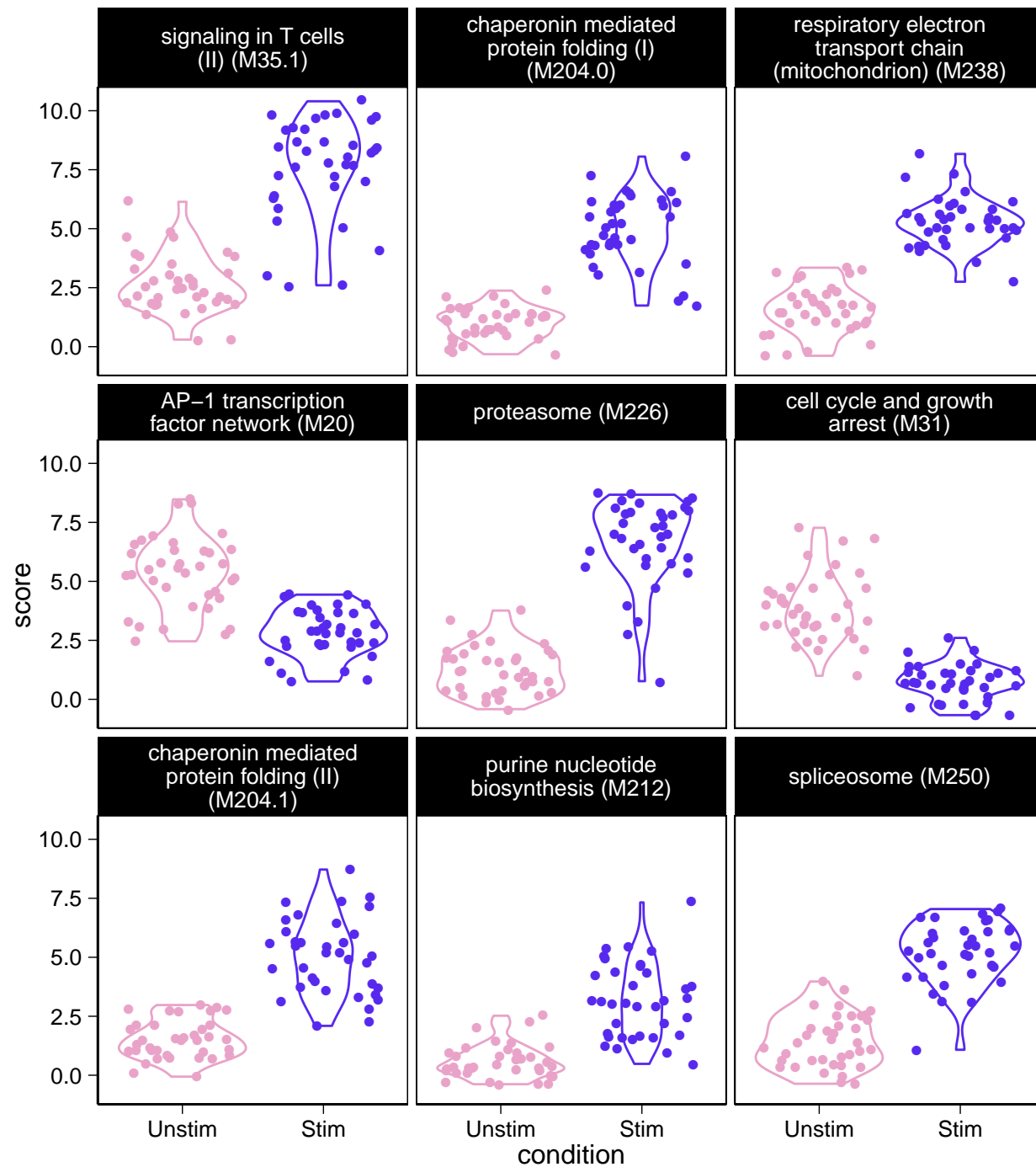
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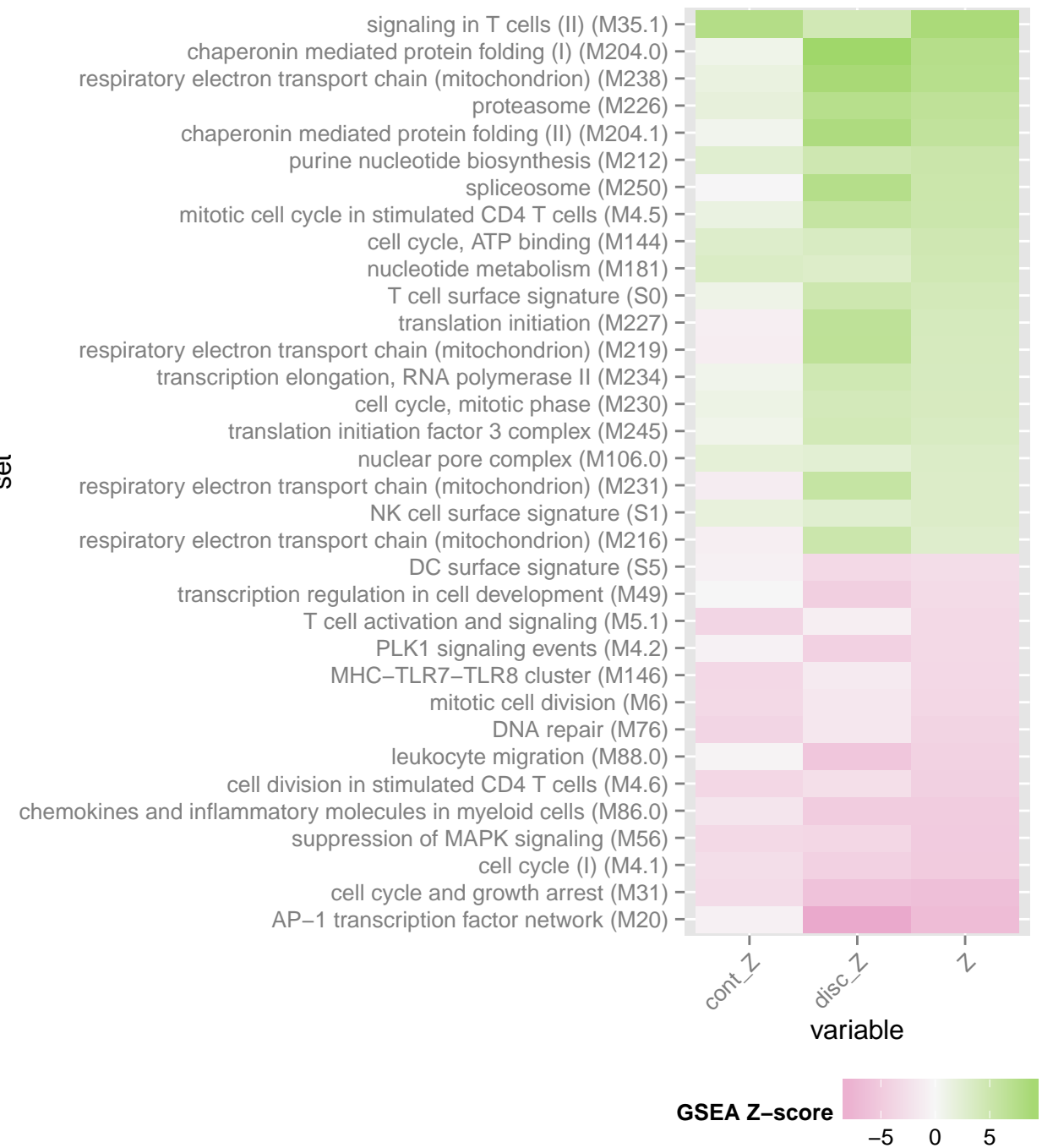
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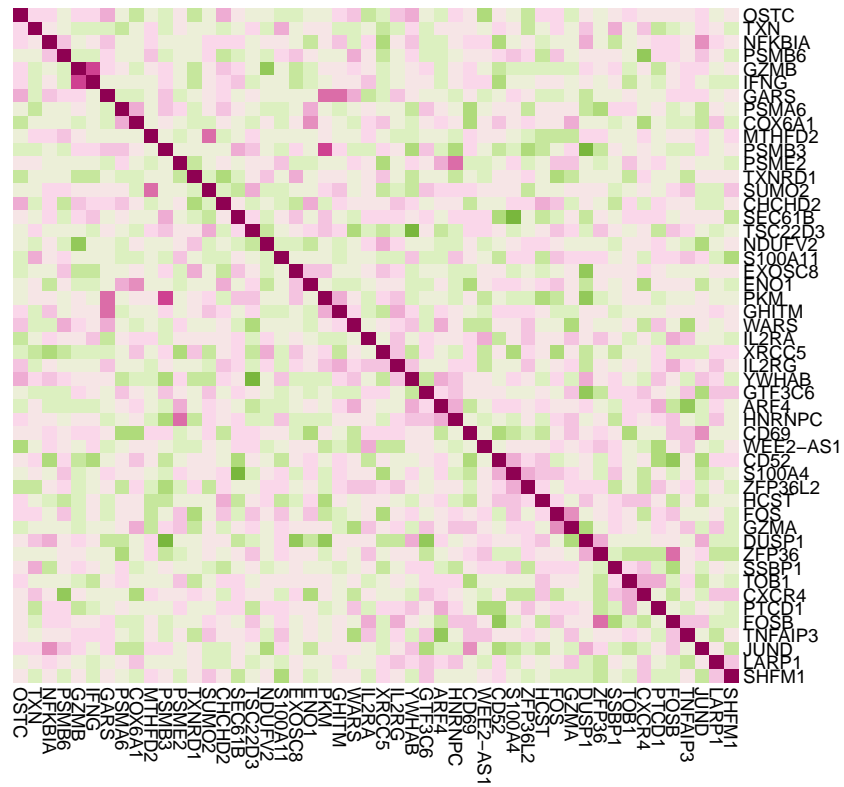
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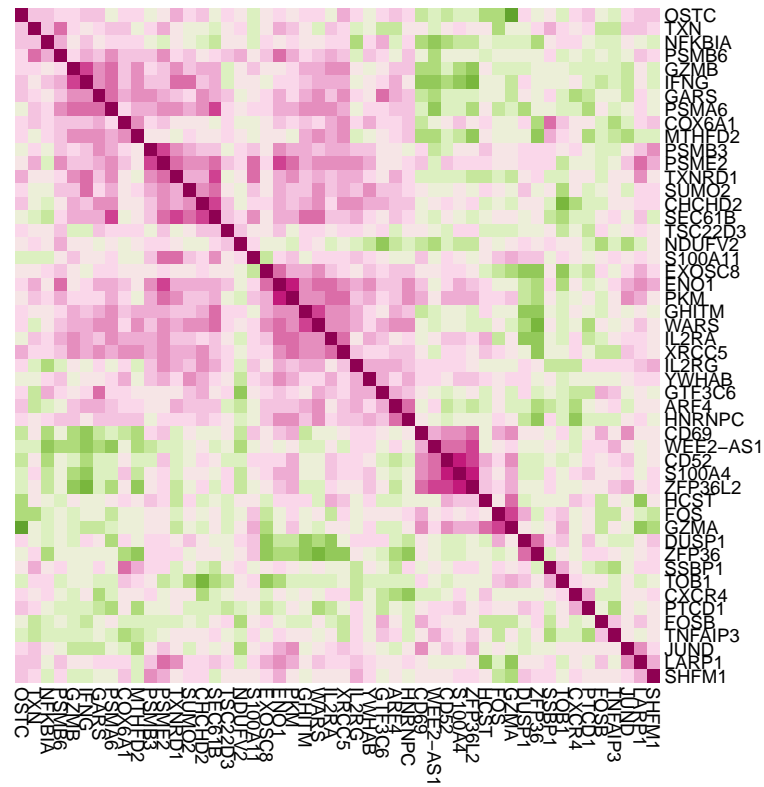
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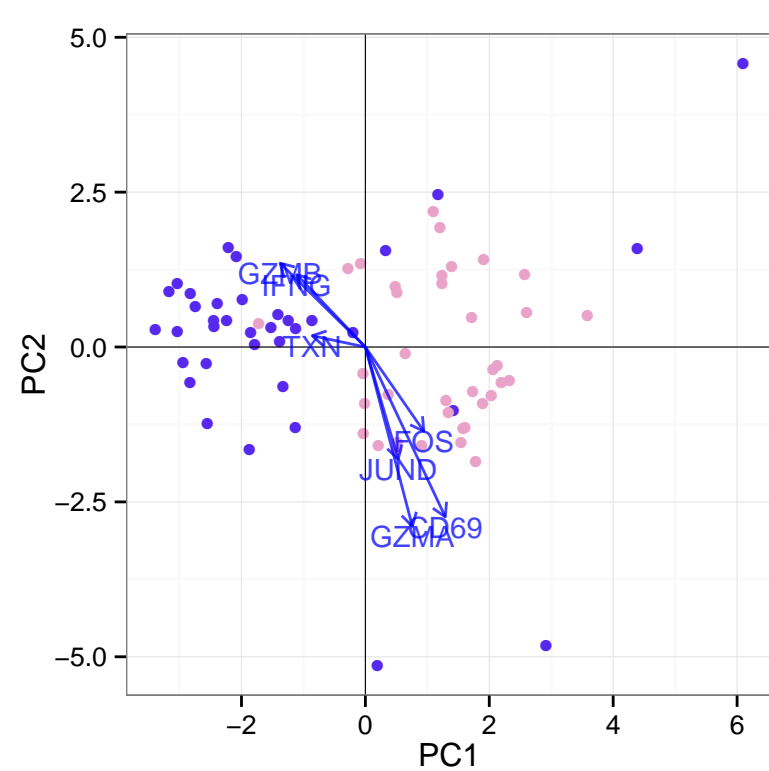
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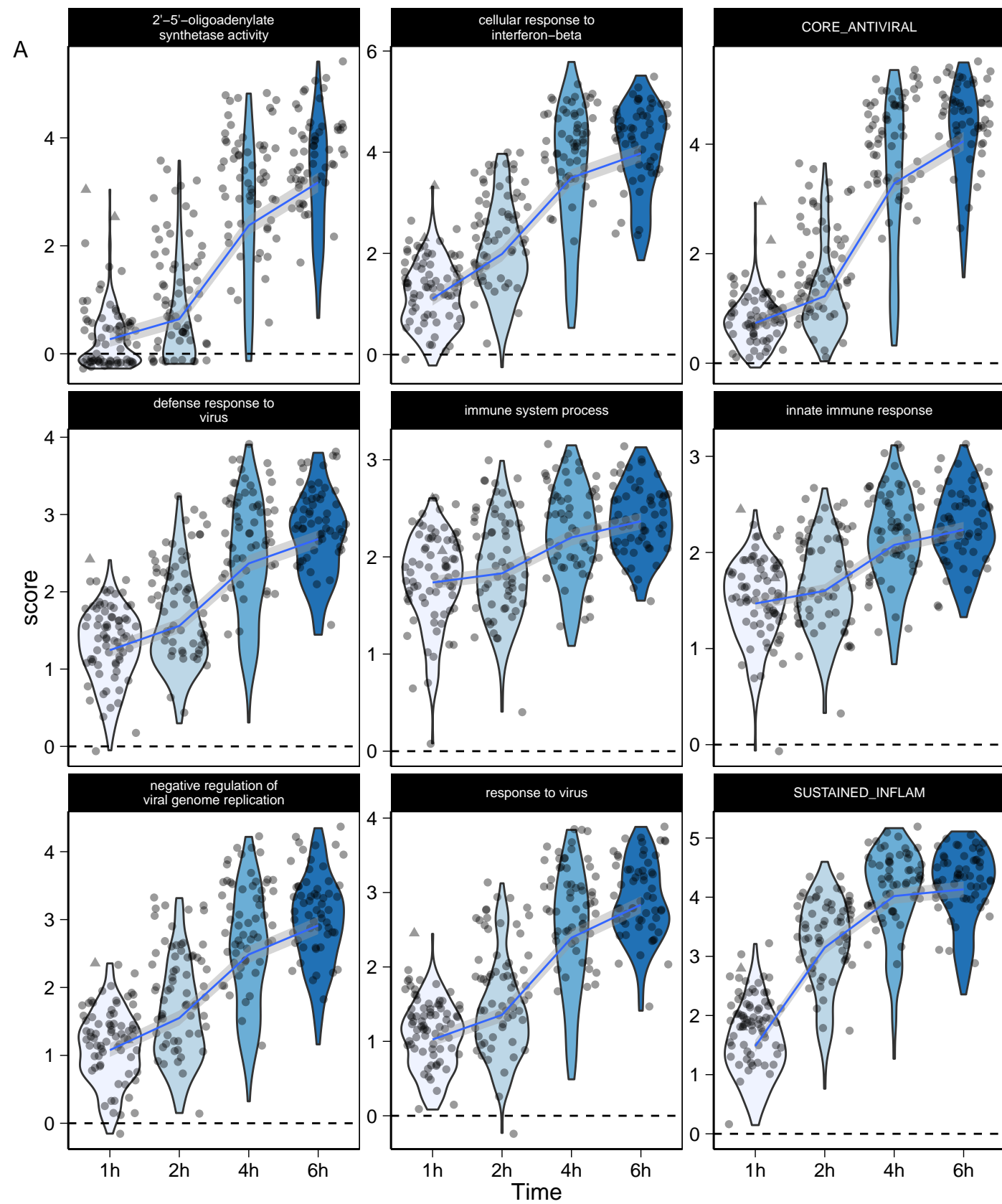


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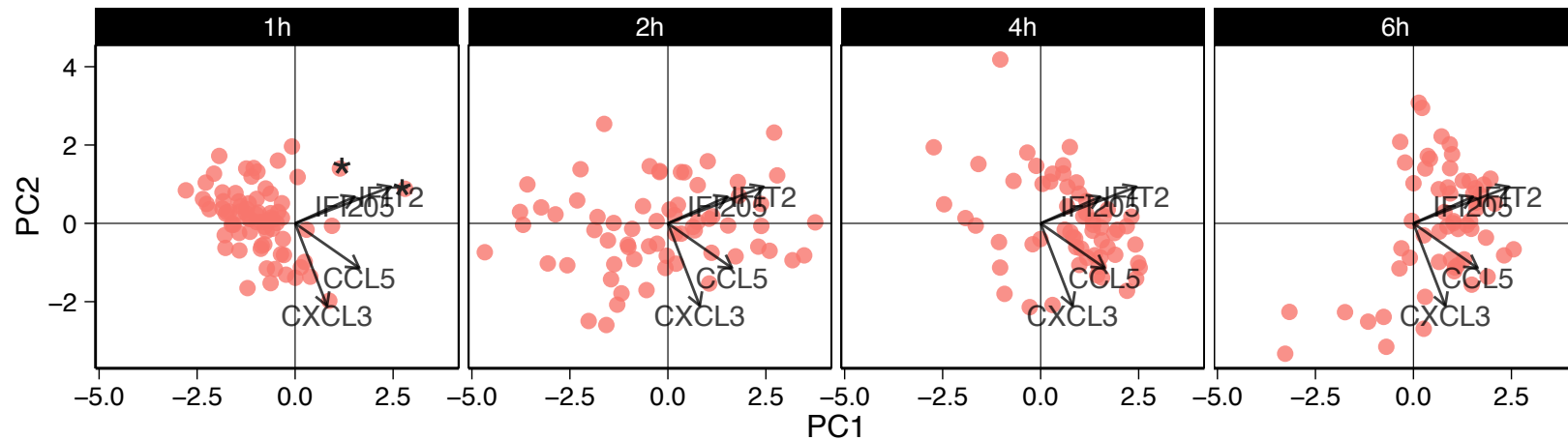


C





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