Catch me if you can: Adaptation from standing genetic variation to a moving phenotypic optimum

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ABSTRACT

Adaptation lies at the heart of Darwinian evolution. Accordingly, numerous studies have tried 15 to provide a formal framework for the description of the adaptive process. Out of these, two 16 complementary modelling approaches have emerged: While so-called adaptive-walk models 17 consider adaptation from the successive fixation of *de-novo* mutations only, quantitative ge-18 netic models assume that adaptation proceeds exclusively from pre-existing standing genetic 19 variation. The latter approach, however, has focused on short-term evolution of population 20 means and variances rather than on the statistical properties of adaptive substitutions. Our 21 aim is to combine these two approaches by describing the ecological and genetic factors that 22 determine the genetic basis of adaptation from standing genetic variation in terms of the 23 effect-size distribution of individual alleles. Specifically, we consider the evolution of a quan-24 titative trait to a gradually changing environment. By means of analytical approximations, 25 we derive the distribution of adaptive substitutions from standing genetic variation, that is, 26 the distribution of the phenotypic effects of those alleles from the standing variation that be-27 come fixed during adaptation. Our results are checked against individual-based simulations. 28 We find that, compared to adaptation from *de-novo* mutations, (i) adaptation from standing 29 variation proceeds by the fixation of more alleles of small effect; (ii) populations that adapt 30 from standing genetic variation can traverse larger distances in phenotype space and, thus, 31 have a higher potential for adaptation if the rate of environmental change is fast rather than 32 slow. 33

INTRODUCTION

One of the biggest surprises that has emerged from evolutionary research in the past few 34 decades is that, in contrast to what has been claimed by the neutral theory (KIMURA 1983), 35 adaptive evolution at the molecular level is wide-spread. In fact, some empirical studies 36 concluded that up to 45% of all amino acid changes between *Drosophila simulans* and *D*. 37 yakuba are adaptive (SMITH and EYRE-WALKER 2002; ORR 2005b). Along the same line, 38 WICHMAN et al. (1999) evolved the single-stranded DNA bacteriophage $\Phi X174$ to high tem-39 perature and a novel host and found that 80 - 90% of the observed nucleotide substitutions 40 had an adaptive effect. These and other results have led to an increased interest in providing 41 a formal framework for the adaptive process that goes beyond traditional population- and 42 quantitative-genetic approaches and considers the statistical properties of suites of substitu-43 tions in terms of "individual mutations that have individual effects" (ORR 2005a). In general, 44 selection following a change in the environmental conditions may act either on *de-novo* muta-45 tions or on alleles already present in the population, also known as standing genetic variation. 46 Consequently, from the numerous studies that have attempted to address this subject, two 47 complementary modelling approaches have emerged. 48

So-called adaptive-walk models (GILLESPIE 1984; KAUFFMAN and LEVIN 1987; ORR 2002, 49 2005b) typically assume that selection is strong compared to mutation, such that the popula-50 tion can be considered monomorphic all the time and all observed evolutionary change is the 51 result of *de-novo* mutations. These models have produced several robust predictions (ORR 52 1998, 2000; MARTIN and LENORMAND 2006a,b), which are supported by growing empirical 53 evidence (COOPER et al. 2007; ROCKMAN 2012; HIETPAS et al. 2013; but see Bell 2009), 54 and has provided a statistical framework for the fundamental event during adaptation, that 55 is, the substitution of a resident allele by a beneficial mutation. Specifically, the majority 56 of models (e.g., GILLESPIE 1984; ORR 1998; MARTIN and LENORMAND 2006a) consider the 57 effect-size distribution of adaptive substitutions following a sudden change in the environ-58

⁵⁹ ment. Recently, KOPP and HERMISSON (2009b) and MATUSZEWSKI *et al.* (2014) extended ⁶⁰ this framework to gradual environmental change.

In contrast, most quantitative-genetic models consider an essentially inexhaustible pool of 61 pre-existing standing genetic variants as the sole source for adaptation (LANDE 1976). Evolv-62 ing traits are assumed to have a polygenic basis, where many loci contribute small individual 63 effects, such that the distribution of trait values approximately follows a Gaussian distribution 64 (BULMER 1980; BARTON and TURELLI 1991; KIRKPATRICK et al. 2002). Since the origins 65 of quantitative genetics lie in the design of plant and animal breeding schemes (WRICKE and 66 WEBER 1986; TOBIN et al. 2006; HALLAUER et al. 2010), the traditional focus of these mod-67 els was on predicting short-term changes in the population mean phenotype (often assuming 68 constant genetic variances and covariances), and not on the fate and effect of individual 69 alleles. The same is true for the relatively small number of models that have studied the 70 contribution of new mutations in the response to artificial selection (e.g. HILL and RASBASH 71 1986a) and the shape and stability of the **G**-matrix (i.e., the additive variance-covariance 72 matrix of genotypes; JONES et al. 2004, 2012). 73

It is only in the past decade that population geneticists have thoroughly addressed adaptation 74 from standing genetic variation at the level of individual substitutions (ORR and BETAN-75 COURT 2001; HERMISSON and PENNINGS 2005; CHEVIN and HOSPITAL 2008). HERMISSON 76 and PENNINGS (2005) calculated the probability of adaptation from standing genetic vari-77 ation following a sudden change in the selection regime. They found that, for small-effect 78 alleles, the fixation probability is considerably increased relative to that from new mutations. 79 Furthermore, CHEVIN and HOSPITAL (2008) showed that the selective dynamics at a focal 80 locus are substantially affected by genetic background variation. These results where experi-81 mentally confirmed by LANG et al. (2011), who followed beneficial mutations in hundreds of 82 evolving yeast populations and showed that the selective advantage of a mutation plays only a 83 limited role in determining its ultimate fate. Instead, fixation or loss is largely determined by 84 variation in the genetic background – which need not to be preexisting, but could quickly be 85

generated by a large number of new mutations. Still, predictions beyond these single-locus 86 results have been verbal at best, stating that "compared with new mutations, adaptation 87 from standing genetic variation is likely to lead to faster evolution [and] the fixation of more 88 alleles of small effect [...]" (BARRETT and SCHLUTER 2008). Thus, despite recent progress, 89 one of the central questions still remains unanswered: From the multitude of standing genetic 90 variants segregating in a population, which are the ones that ultimately become fixed and 91 contribute to adaptation, and how does their distribution differ from that of (fixed) de-novo 92 mutations? 93

The aim of the present article is to contribute to overcoming what has been referred to as "the 94 most obvious limitation" (ORR 2005b) of adaptive-walk models and to study the ecological 95 and genetic factors that determine the genetic basis of adaptation from standing genetic vari-96 ation. Specifically, we consider the evolution of a quantitative trait in a gradually changing 97 environment. We develop an analytical framework that accurately describes the distribu-98 tion of adaptive substitutions from standing genetic variation and discuss its dependence on 99 the effective population size, the strength of selection and the rate of environmental change. 100 In line with BARRETT and SCHLUTER (2008), we find that, compared to adaptation from 101 *de-novo* mutations, adaptation from standing genetic variation proceeds, on average, by the 102 fixation of more alleles of small effect. Furthermore, when standing genetic variation is the 103 sole source for adaptation, faster environmental change can enable the population to remain 104 better adapted and to traverse larger distances in phenotype space. 105

MODEL AND METHODS

¹⁰⁶ Phenotype, Selection and Mutation

We consider the evolution of a diploid population of N individuals with discrete and nonoverlapping generations characterized by a single phenotypic trait z, which is under Gaussian stabilizing selection with regard to a time-dependent optimum $z_{opt}(t)$:

$$w(z,t) = \exp\left[-\frac{(z-z_{\rm opt}(t))^2}{2\sigma_s^2}\right],\tag{1}$$

where σ_s^2 describes the width of the fitness landscape. Throughout this paper we choose the linearly moving optimum,

$$z_{\rm opt}(t) = vt, \tag{2}$$

where v is the rate of environmental change.

¹¹³ Mutations enter the population at rate $\frac{\Theta}{2}$ (with $\Theta = 4Nu$ where u is the per-haplotype muta-¹¹⁴ tion rate), and we assume that their phenotypic effect size α follows a Gaussian distribution ¹¹⁵ with mean 0 and variance σ_m^2 (which we will refer to as the distribution of new mutations), ¹¹⁶ that is

$$p(\alpha) = \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{\alpha^2}{2\sigma_m^2}\right).$$
(3)

Throughout this paper we equate genotypic with phenotypic values and, thus, neglect any environmental variance. Note that this model is, so far, identical to the moving-optimum model proposed by KOPP and HERMISSON (2009b) (see also BÜRGER 2000).

¹²⁰ Genetic assumptions and simulation model

¹²¹ To study the distribution of adaptive substitutions from standing genetic variation, we con-

ducted individual-based simulations (IBS; available upon request; see BÜRGER 2000; KOPP and HERMISSON 2009b) that explicitly model the simultaneous evolution at multiple loci, while making additional assumptions about the genetic architecture of the selected trait, the life cycle of individuals and the regulation of population size. This will serve as our main model.

Individuals are characterized by a linear (continuous) genome of diploid loci, Genome 127 which determine the phenotype z additively (i.e., there is no phenotypic epistasis; note, 128 however, that there is epistasis for fitness). Mutations occur at constant rate $\frac{\Theta}{2N} = u$ per 129 haplotype. In contrast to the majority of individual-based models (e.g., JONES et al. 2004; 130 KOPP and HERMISSON 2009b; MATUSZEWSKI et al. 2014), we do not fix the number of 131 loci *a-priori*, but instead assume that each mutation creates a unique polymorphic locus, 132 whose position is drawn randomly from a uniform distribution over the entire genome (where 133 genome length is determined by the recombination parameter r described below). Thus, each 134 locus consists only of a wild-type allele with phenotypic effect 0 and a mutant allele with 135 phenotypic effect α , which is drawn from equation (3). Thus, we effectively design a bi-allelic 136 infinite-sites model with a continuum of alleles. 137

To monitor adaptive substitutions, we introduce a population-consensus genome \mathcal{G} that keeps track of all loci, that is, of all mutant alleles that are segregating in the population. If a mutant allele becomes fixed in the population it is declared the new wild-type allele and its phenotypic effect is reset to 0. The phenotypic effects of all fixed mutations are taken into account by a variable z_{fix} , which can be interpreted as a phenotypic baseline effect. Thus, the phenotype z of an individual *i* is given by

$$z_i = z_{\text{fix}} + \sum_{h \in \{1,2\}} \sum_{l \in \mathcal{G}} \mathbb{1}(i, l, h) \alpha_l.$$

144 where

 $\mathbb{1}(i,l,h) = \begin{cases} 1 & \text{if individual } i \text{ carries mutant allele } \alpha \text{ at locus } l \text{ on haplotype } h \\ 0 & \text{otherwise.} \end{cases}$

¹⁴⁵ Life cycle Each generation, the following steps are performed:

- 146 1. Viability selection: Individuals are removed with probability 1 w(z) (see eq. 1).
- ¹⁴⁷ 2. Population regulation: If, after selection, the population size N exceeds the carrying ¹⁴⁸ capacity K, N - K randomly chosen individuals are removed.

3. Reproduction: The surviving individuals are randomly assigned to mating pairs, and each mating pair produces exactly 2B = 4 offspring. Note that under this scheme, the effective population size N_e equals 4/3 times the census size (BüRGER 2000, p. 274). To account for this difference, Θ in the analytical approximations needs to calculated on the basis of this effective size, i.e., $\Theta = 4N_e u$. The offspring genotypes are derived from the parent genotypes by taking into account segregation, recombination and mutation.

Recombination For each reproducing individual, the number of crossing-over events dur-155 ing gamete formation (i.e., the number of recombination breakpoints) is drawn from a Poisson 156 distribution with (genome-wide recombination) parameter r (i.e., the total genome length is 157 $r \cdot 100$ cM, see Supporting Information 1). The genomic position of each recombination break-158 point is then drawn from a uniform distribution over the entire genome, and the offspring 159 haplotype is created by alternating between the maternal and paternal haplotype depending 160 on the recombination breakpoints. Free recombination (where all loci are assumed to be 161 unlinked) corresponds to $r \to \infty$. In this case, for each locus a Bernoulli-distributed random 162 number is drawn to determine whether the offspring haplotype will receive the maternal or 163 the paternal allele at that locus. 164

Simulation initialization and termination Starting from a population of K wild-type 165 individuals with phenotype z = 0 (i.e., the population was perfectly adapted at t = 0), 166 we allowed for the establishment of genetic variation, σ_g^2 , by letting the population evolve 167 for 10,000 generations under stabilizing selection with a constant optimum. Increasing the 168 number of generations had no effect on the average σ_q^2 . Following this equilibration time, the 169 optimum started moving under ongoing mutational input, and the simulation was stopped 170 once all alleles from the standing genetic variation had either been fixed or lost (i.e., when 171 $\sigma_{sgv}^2 = 0$). Simulations were replicated until a total number of 5000 adaptive substitutions 172 from standing genetic variation was recorded. 173

Analytical approximations: Evolution of a focal locus in the presence of genetic background variation

In order to obtain an analytically tractable model, we need to approximate the multi-locus 176 dynamics. Clearly, simple interpolation of single locus theory will fail, because when alleles 177 at different loci influencing the same trait segregate in the standing genetic variation, the 178 selective dynamics of any individual allele are critically affected by the collective evolutionary 179 response at other loci. In particular, any allele that brings the mean phenotype closer to the 180 optimum simultaneously decreases the selective advantage of other such alleles (epistasis for 181 fitness). Thus, if simultaneous evolution at many loci allows the population to closely follow 182 the optimum, large-effect alleles at any given locus are likely to remain deleterious (as their 183 carriers would overshoot the optimum). To account for these effects, we adopt a quantitative-184 genetics approach originally developed by LANDE (1983) and introduce a genetic background 185 $z_{\rm B}$ that evolves according to Lande's equation 186

$$\Delta \bar{z} = \sigma_q^2 \beta, \tag{5a}$$

187 where

$$\beta = \frac{\partial \log(\bar{w})}{\partial \bar{z}} \tag{5b}$$

denotes the selection gradient, which measures the change in log mean fitness per unit change of the mean phenotype and σ_g^2 gives the genetic variance (LANDE 1976). Furthermore, assuming that the distribution of phenotypic values from the genetic background is Gaussian and the genetic variance remains constant, the mean background phenotype evolves according to

$$\bar{z}_{\rm B}(t) \approx vt - \frac{v}{\gamma} (1 - (1 - \gamma)^t)$$
(6a)

193 with

$$\gamma = \frac{\sigma_g^2}{\sigma_g^2 + \sigma_s^2} \tag{6b}$$

194 (BÜRGER and LYNCH 1995).

Given the dynamics of the genetic background, we choose one focal locus and derive the 195 time-dependent selection coefficient $s(\alpha, t)$ for an allele with phenotypic effect α (for details 196 see below). We then use theory for adaptation from standing genetic variation (HERMISSON 197 and PENNINGS 2005) and for fixation under time-inhomogeneous selection (UECKER and 198 HERMISSON 2011) to estimate the fixation probability for this allele (see also Appendix 1). 199 As long as there is no linkage (i.e., there is free recombination between all loci), each locus 200 can be viewed as the focal locus (with a specific phenotypic effect α), allowing us to get an 201 estimate for the overall distribution of adaptive substitutions from standing genetic varia-202 tion. Thus, in these approximations, our multi-locus model is effectively treated within a 203 single-locus framework. Note that a similar focal-locus approach has recently been used to 204 analyze the effect of genetic background variation on the trajectory of an allele sweeping to 205 fixation (CHEVIN and HOSPITAL 2008), and to study the probability of adaptation to novel 206

environments (GOMULKIEWICZ *et al.* 2010), with both studies stressing the fact that genetic
background variation cannot be neglected and critically affects the adaptive outcome.

²⁰⁹ Wright-Fisher simulations: A focal locus with recurrent mutations

To simulate evolution at a focal locus, we followed HERMISSON and PENNINGS (2005) and implemented a multinomial Wright-Fisher (WF) sampling approach (available upon request). These simulations serve as an additional analysis tool that has been adjusted to the approximation method and allows the adaptive process to be simulated fast and efficiently. In addition, they go beyond the individual-based model in one aspect, as they do not make the infinite-sites assumption but allow for recurrent mutation at the focal locus.

Genome At the focal locus, mutations with a fixed allelic effect α appear recurrently 216 at rate θ and convert ancestral alleles into derived mutant alleles. Accordingly, despite a 217 genetic background with normally distributed genotypic values, there are at most two types 218 of (focal) alleles in the population, where each type "feels" only the mean background $\bar{z}_{\rm B}$, 219 which evolves according to Lande's equation (eq. 5, see above). The genetic background 220 variation σ_q^2 is assumed to be constant and serves as a free parameter that is independent of 221 θ , N_e and σ_s^2 . Note that the evolutionary response at the focal locus is influenced by that of 222 the genetic background, and vice versa, meaning that the two are interdependent. 223

Procedure We follow the evolution of $2N_e$ alleles at the focal locus. Each generation is generated by multinomial sampling, where the probability of choosing an allele of a given type (ancestral or derived) is weighted by its respective (marginal) fitness. Furthermore, the mean phenotype of the genetic background $\bar{z}_{\rm B}$ evolves deterministically according to equation (5) with constant σ_g^2 . To let the population reach mutation-selection-drift equilibrium, each simulation is started $4N_e$ generations before the environment starts changing. Initially, the population consists of only ancestral alleles "0"; the derived allele "1" is created by mutation. If the derived allele reaches fixation before the environmental change (by drift), it is itself declared "ancestral"; i.e., the population is set back to the initial state. After $4N_e$ generations, the optimum starts moving, such that the selection coefficient of the derived allele, which is initially deleterious (i.e., $s(\alpha, t) \leq 0$), increases and may eventually become beneficial (i.e., $s(\alpha, t) > 0$), depending on the response at the genetic background. Simulations continue until the derived allele is either fixed or lost. Fixation probabilities are estimated from 100,000 simulation runs.

Both simulation programs are written in C++ and make use of the Gnu Scientific Library (GALASSI *et al.* 2009). Mathematica (Wolfram Research, Inc., Champaign, USA) was used for the numerical evaluation of integrals and to create plots and graphics, making use of the LevelScheme package (CAPRIO 2005).

A summary of our notation is given in Table 1.

Table 1 – A	summary	of notation	and	definitions.
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α	phenotypic effect of mutation
p(lpha)	(Gaussian) distribution of new mutations
z	phenotype
\bar{z}_{B}	mean genetic background phenotype
v	rate of environmental change
$w(z, z_{opt}(t))$	(Gaussian) fitness function
σ_s^2	width of Gaussian fitness function
σ_m^2	variance of new mutations
σ_g^2	(background) genetic variance
s(lpha,t)	time-dependent selection coefficient for allele with phenotypic effect α
x	frequency of mutant allele
N_e	effective population size
θ	per locus mutation rate
Θ	population-wide mutation rate (per trait)
Π_{fix}	fixation probability
ho(x, lpha)	Distribution of mutant allele frequency at a single locus with phenotypic effect α
$P_{\rm SGV}$	Probability to adapt from standing genetic variation
$p_{ m SGV}$	Distribution of adaptive substitutions from standing genetic variation
$\delta_{ m eq}$	equilibrium lag

RESULTS

In the following, we calculate, first, the probability that a focal allele from the standing genetic variation becomes fixed when the population adapts to a moving phenotypic optimum, and second, the effect-size distribution of such alleles. Note that the first result will be derived under the assumption of recurrent mutation (see "Wright-Fisher simulations"), and serves as an intermediate step for the second result, which is based on an infinite-sites model (see "Genetic assumptions and simulation model").

²⁴⁹ The probability for adaptation from standing genetic variation

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The probability that a focal mutant allele from the standing genetic variation contributes to adaptation depends on the dynamics of its the selection coefficient in the presence of genetic background variation. For an allele with effect α and a genetic background with mean $\bar{z}_{\rm B}$ and variance σ_g^2 , the selection coefficient can be calculated as

$$s(\alpha, t) = \frac{w(\alpha + \bar{z}_{\mathrm{B}}(t), t)}{w(\bar{z}_{\mathrm{B}}(t), t)} - 1$$
$$\approx -\frac{\alpha^2}{2\left(\sigma_s^2 + \sigma_g^2\right)} + \frac{\alpha}{\sigma_s^2 + \sigma_g^2}(vt - \bar{z}_{\mathrm{B}}(t)).$$
(7)

Note that the genetic background variance has the effect of broadening the fitness landscape experienced by the focal allele (the term $\sigma_s^2 + \sigma_g^2$).

Plugging equation (6a) into equation (7) then yields the selection coefficient,

$$s(\alpha, t) \approx -\frac{\alpha^2}{2\left(\sigma_s^2 + \sigma_g^2\right)} + \frac{\alpha v}{\gamma\left(\sigma_s^2 + \sigma_g^2\right)} (1 - (1 - \gamma)^t).$$
(8)

Assuming that the population is perfectly adapted at t = 0 ($\bar{z}_B = 0$), the initial (deleterious) selection coefficient is given by

$$s(\alpha, 0) = -\frac{\alpha^2}{2\left(\sigma_s^2 + \sigma_g^2\right)}$$

Unlike in the model without genetic background variation (KOPP and HERMISSON 2009b), $s(\alpha, t)$ does not increase linearly, but instead depends on the evolution of the phenotypic lag δ between the optimum and the mean background phenotype. In particular, the population will reach a dynamic equilibrium with $\Delta \bar{z}_{\rm B} = v$, where it follows the optimum with a constant lag

$$\delta_{\rm eq} = \frac{v}{\gamma} \tag{9}$$

(BÜRGER and LYNCH 1995). Consequently, the selection coefficient for α approaches

$$\lim_{t \to \infty} s(\alpha, t) = -\frac{\alpha^2}{2\left(\sigma_s^2 + \sigma_g^2\right)} + \frac{\alpha v}{\gamma\left(\sigma_s^2 + \sigma_g^2\right)}.$$
(10)

Note that the right-hand side can be written as $s(\alpha, 0) + \alpha \beta_{eq}$, where β_{eq} is the equilibrium selection gradient (KOPP and MATUSZEWSKI 2014). In this case, the largest obtainable selection coefficient is for $\alpha = \delta_{eq}$ and evaluates to

$$s_{\max} = s(\delta_{eq}, \infty) = \frac{v^2}{2\gamma^2 \left(\sigma_s^2 + \sigma_g^2\right)}.$$
(11)

The range of allelic effects α that can reach a positive selection coefficient is bounded by $\alpha_{\min} = 0$ and $\alpha_{\max} = 2\delta_{eq}$. Note that in previous adaptive-walk models (e.q., KOPP and HERMISSON 2009b; MATUSZEWSKI *et al.* 2014) there was no strict α_{\max} , since the population followed the optimum by stochastic jumps, whereas in the present model, the genetic background evolves deterministically and establishes a constant equilibrium lag.

Assuming that α was deleterious prior to the environmental change, its allele frequency spectrum $\rho(x, \alpha)$ is given by equation (A5). When genetic background variation is absent

the fixation probability $\Pi_{\text{fix}}(\alpha)$ (eq. A7) can be calculated explicitly using

$$\varphi_{\sigma_g^2=0}(\alpha) = 1 + \frac{1}{2} \sqrt{\frac{\pi}{2\frac{\alpha v}{\sigma_s^2}}} \exp\left(\frac{s(\alpha,0)^2}{2\frac{\alpha v}{\sigma_s^2}}\right) \operatorname{erfc}\left(\frac{s(\alpha,0)}{\sqrt{2\frac{\alpha v}{\sigma_s^2}}}\right).$$
(12)

For the general case, however, $\Pi_{\text{fix}}(\alpha)$ can only be calculated numerically using equation (8) in equation (A7b), yielding

$$2\varphi(\alpha) = 1 + \int_0^\infty (1 + s(\alpha, t)) \exp\left[-\left(\left(-\frac{\alpha^2}{2(\sigma_s^2 + \sigma_g^2)}\right) + \left((1 - (1 - \gamma)^t)\frac{1}{\log[(1 - \gamma)^t]} + 1\right)\frac{\alpha v}{\gamma(\sigma_s^2 + \sigma_g^2)}\right)t\right] \mathrm{d}t.$$
 (13)

The fixation probability for an allele from the standing genetic variation with allelic effect α and a recurrent (per locus) mutation rate θ can then be calculated as

$$P_{\text{SGV}}(\alpha) = \begin{cases} 1 - C(\alpha) \int_0^1 x^{\theta - 1} \exp[-4N_e | s(\alpha, 0) | x] \left(1 - \frac{1}{\varphi(\alpha)}\right)^{2N_e x} dx & \text{if } 0 < \alpha < \alpha_{max} \\ 0 & \text{otherwise,} \end{cases}$$
(14)

where $C(\alpha) = \left(\frac{\gamma[\theta, 4N_e|s(\alpha, 0)|]}{(4N_e|s(\alpha, 0)|)^{\theta}}\right)^{-1}$.

When checked against Wright-Fisher simulations (see Methods for details), our analytical 280 approximation equation (14) performs generally very well (Figs. 1 and S3_1). The only 281 exception occurs when the background variation is high (large σ_g^2) and stabilizing selection 282 is weak (i.e., if σ_s^2 is large). In this case, equation (14) underestimates $P_{\text{SGV}}(\alpha)$ for small 283 $\alpha \sim 0.5\sigma_m$. The reason is that, under a constant optimum (i.e., before the environmental 284 change), the genetic background compensates for the deleterious effect of α (i.e., $\bar{z}_{\rm B} < 0$, 285 in violation of our assumption that $\bar{z}_{\rm B}(0) = 0$, effectively reducing the selection strength 286 against the deleterious mutant allele. Consequently, α is, on average, present at higher 287 initial frequencies than predicted by equation (A5). 288

Note that, if α is small compared to the genetic background variation (i.e., in the limit of

 $\alpha/\sigma_m \to 0$) and environmental change is slow (i.e., $v \ll 10^{-5}$), $P_{\rm SGV}(\alpha)$ will approach the probability of fixation from standing genetic variation for a neutral allele (i.e., $\alpha = 0$), which can be calculated as

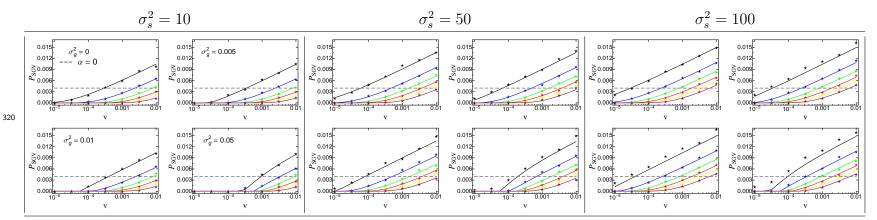
$$P_{\text{SGV, neutral}} = \int_0^1 x \rho(x) dx = \frac{H_\theta - 1}{\gamma + \psi(\theta)}.$$
(15)

where $\rho(x)$ is given by equation (A3), H_n denotes the n^{th} harmonic number, $\gamma \approx 0.577$ is Euler's gamma and $\psi(\cdot)$ is the polygamma function (see dashed lines in Figs. 1 and S3_1). Figures 1 and S3_1 show some general trends: First, the probability for a mutant allele to become fixed increases with the rate of environmental change, v, (irrespective of its effect size α , the per locus mutation rate θ and the width of the fitness landscape σ_s^2) since only the positive term in equation (8) depends (linearly) on v. Second, $P_{\text{SGV}}(\alpha)$ is proportional to θ as long as θ is small (compare $\theta = 0.004$ and $\theta = 0.04$ in Fig. S3_1), simply because the

probability that α is present in the population is linear in θ . Thus, Figure 1 is representative 300 for the limit $\theta \to 0$ which will be used below. Indeed, only if the per-locus mutation rate 301 is fairly large ($\theta > 0.1$), does the shape of the distribution of allele frequencies become 302 important, and the increase in $P_{\text{SGV}}(\alpha)$ with θ becomes less than linear (Fig. S3_1). Third, 303 changes in the width of the fitness landscape, σ_s^2 , have a dual effect: While increasing σ_s^2 304 promotes the initial frequency of the focal allele in the standing genetic variation (because 305 stabilizing selection is weaker), the selection coefficient increases more slowly after the onset 306 of environmental change (such that the allele is less likely to be picked up by selection; 307 see eq. 7). Our results, however, show that the former effect always outweighs the latter 308 (as $P_{\text{SGV}}(\alpha)$ increases with σ_s^2). Finally, if the genetic background variation σ_g^2 is below 309 a threshold value (e.g., $\sigma_g^2 < 0.005$; the exact threshold should depend on θ and σ_s^2) it 310 only marginally affects the fixation probability of the focal allele α . Once σ_g^2 surpasses this 311 value, however, it critically affects $P_{SGV}(\alpha)$ (in accordance with the results by CHEVIN and 312

HOSPITAL 2008). In particular, as σ_g^2 increases $P_{\text{SGV}}(\alpha)$ decreases, because most large-effect alleles remain deleterious even if environmental change is fast. Thus, enlarged background variation acts as if reducing the rate of environmental change v. In summary, our analytical results are in good agreement with the WF-simulation model, and will serve as an important first step towards deriving the distribution of adaptive substitutions from standing genetic variation. Figure 1 – The probability for a mutant allele to adapt from standing genetic variation as a function of the rate of environmental change v. Solid lines correspond to the analytical prediction (eq. 14), the grey dashed line shows the probability for a neutral allele ($\alpha = 0$; eq. 15), and symbols give results from Wright-Fisher simulations. The phenotypic effect size α of the mutant allele ranges from $0.5\sigma_m$ (top line; black) to $3\sigma_m$ (bottom line; purple) with increments of $0.5\sigma_m$. The figures in each parameter box (per locus mutation rate θ , width of fitness landscape σ_s^2) correspond to different values of the genetic background variation σ_g^2 with $\sigma_g^2 = 0$ (no background variation; top left), $\sigma_g^2 = 0.005$ (top right), $\sigma_g^2 = 0.01$ (bottom left) and $\sigma_g^2 = 0.05$ (bottom right). Other parameters: $N_e = 25000, \theta = 0.004, \sigma_m^2 = 0.05$.





³²¹ The distribution of adaptive substitutions from standing genetic variation

We now derive the distribution of adaptive substitutions from standing genetic variation 322 over all mutant effects α . In the previous section, we derived the fixation probability at 323 a focal locus (with a given effect α) by treating the genetic background variance σ_g^2 as an 324 independent model parameter. In the full model, this variance results from a balance of 325 mutation, selection and drift at all background loci. As such, it is a function of the basic 326 model parameters for these forces. Since we use an infinite-sites model, there is no recurrent 327 mutation and each allele originates from a single mutation. Consequently, the amount of 328 background variation σ_g^2 is accurately predicted by the Stochastic-House-of-Cards (SHC) 329 approximation (not shown; BÜRGER and LYNCH 1995) 330

$$\sigma_g^2 = \frac{\Theta \sigma_m^2}{1 + \frac{N_e \sigma_m^2}{\sigma^2}},\tag{16}$$

where mutation is parametrized by the total (per trait) mutation rate Θ and the mutational variance σ_m^2 , the width of the fitness landscape is given by σ_s^2 , and the effective population size N_e is a measure for genetic drift.

To derive the probability that an allele with a given phenotypic effect α contributes to adaptation, we first need to calculate the probability that such an allele segregates in the population at time 0. Following HERMISSON and PENNINGS (2005), the probability P_0 that the allele is *not* present can be approximated by integrating over the distribution of allele frequencies $\rho(x, \alpha)$ (eq. A5) from 0 to $\frac{1}{2N_e}$ yielding

$$P_0(\alpha) \approx \left(\frac{2N_e}{4N_e|s(\alpha,0)|+1}\right)^{-\theta}$$
$$= \exp\left[-\theta \log\left[\frac{2N_e}{4N_e|s(\alpha,0)|+1}\right]\right]$$
(17)

(eq. 7 and Appendix of HERMISSON and PENNINGS 2005). The fixation probability can then be calculated by conditioning on segregation of the allele in the limit $\theta \to 0$ (due to the infinite-sites assumption). Using equation (14), this probability reads

$$\Pi_{\text{seg}}(\alpha) = \lim_{\theta \to 0} \frac{P_{\text{SGV}}(\alpha)}{1 - P_0(\alpha)}$$
$$\approx \lim_{\theta \to 0} \frac{1 - C(\alpha) \int_0^1 x^{\theta - 1} \exp[-4N_e |s(\alpha, 0)|x] \left(1 - \frac{1}{\varphi(\alpha)}\right)^{2Nx} dx}{1 - \exp\left[-\theta \log\left[\frac{2N_e}{4N_e |s(\alpha, 0)| + 1}\right]\right]}, \quad (18)$$

where $C(\alpha) = \left(\frac{\gamma[\theta, 4N_e|s(\alpha, 0)|]}{(4N_e|s(\alpha, 0)|)^{\theta}}\right)^{-1}$ (see also eq. A5) and with $\varphi(\alpha)$ according to equation (13). The limit in equation (18) can be approximated numerically by setting θ to a very small, but positive value.

Multiplying by the rate of mutations with effect α (i.e., $\Theta p(\alpha)$), the distribution of adaptive substitutions from standing genetic variation is given by

$$p_{\text{SGV}}(\alpha) \approx \frac{\Theta p(\alpha) \Pi_{\text{seg}}(\alpha)}{\int_0^{\alpha_{max}} \Theta p(\alpha) \Pi_{\text{seg}}(\alpha) d\alpha}$$
$$= C_1(\alpha) p(\alpha) \Pi_{\text{seg}}(\alpha), \tag{19}$$

where $C_1(\alpha)$ is a normalization constant (black line in Figs. 2, 3 and Fig. 4). Note that equation (19) still depends on Θ through its effect on the background variance σ_g^2 (which affects $\Pi_{\text{seg}}(\alpha)$). In particular, in the SHC approximation (eq. 16), σ_g^2 scales linearly with Θ . Furthermore, equation (19) should be valid for any distribution of mutational effects $p(\alpha)$.

In the limit where the equilibrium lag is reached fast (i.e., when γ is large; eq. 6b), the moving-optimum model reduces to a model with constant selection for any focal allele (i.e., as in HERMISSON and PENNINGS 2005). Using equations (A6) and (17) the fixation probability

³⁵⁴ for a segregating allele can be calculated as

$$\Pi_{\text{seg,SGV},\delta_{\text{eq}}}(\alpha) \approx \lim_{\theta \to 0} \frac{1 - \exp\left[-\theta \log\left[1 + \frac{4N_e s(\alpha, \infty)}{4N_e |s(\alpha, 0)| + 1}\right]\right]}{1 - P_0(\alpha)}.$$
(20)

Plugging equation (20) into equation (19), the distribution of adaptive substitutions from standing genetic variation can be approximated by

$$p_{\text{SGV},\delta_{\text{eq}}}(\alpha) \approx C_2(\alpha) p(\alpha) \Pi_{\text{seg},\text{SGV},\delta_{\text{eq}}}(\alpha),$$
 (21)

where $C_2(\alpha)$ is a normalization constant (red line in Figs. 2, 3).

Similarly, the fixation probability of *de-novo* mutations under the equilibrium lag δ_{eq} can be derived (using 11 and eq. A2 with an initial frequency of 1/(2N)) as

$$\Pi_{\text{fix,DNM},\delta_{\text{eq}}}(\alpha) = \left(1 - \exp\left[-\frac{\alpha(2\delta_{\text{eq}} - \alpha)}{\sigma_s^2 + \sigma_g^2}\right]\right),\tag{22}$$

³⁶⁰ yielding the distribution of adaptive substitutions

$$p_{\text{DNM},\delta_{\text{eq}}}(\alpha) \approx p(\alpha)C_3(\alpha)\Pi_{\text{fix},\text{DNM},\delta_{\text{eq}}}(\alpha),$$
 (23)

where $C_3(\alpha)$ is a normalization constant (grey curve in Figs. 2, 3).

In contrast, if the environment changes very slowly, we can calculate the limit distribution of adaptive substitutions from standing genetic variation by approximating the fixation probability by that of a neutral allele (i.e., its allele frequency x). In this case,

$$\Pi_{\operatorname{seg},v\to0}(\alpha) \approx \lim_{\theta\to0} \frac{\mathcal{F}(\alpha)}{1 - P_0(\alpha)}$$
(24a)

365 with

$$\mathcal{F}(\alpha) = \int_0^1 \rho(x, \alpha) x dx = \frac{{}_1F(0, \theta + 1, 4N_e | s(\alpha, 0) |)_1}{{}_1F(0, \theta, 4N_e | s(\alpha, 0) |)_1},$$
(24b)

where $\rho(x, \alpha)$ is given by equation (A4) and the right-hand side is a ratio of hypergeometric functions.

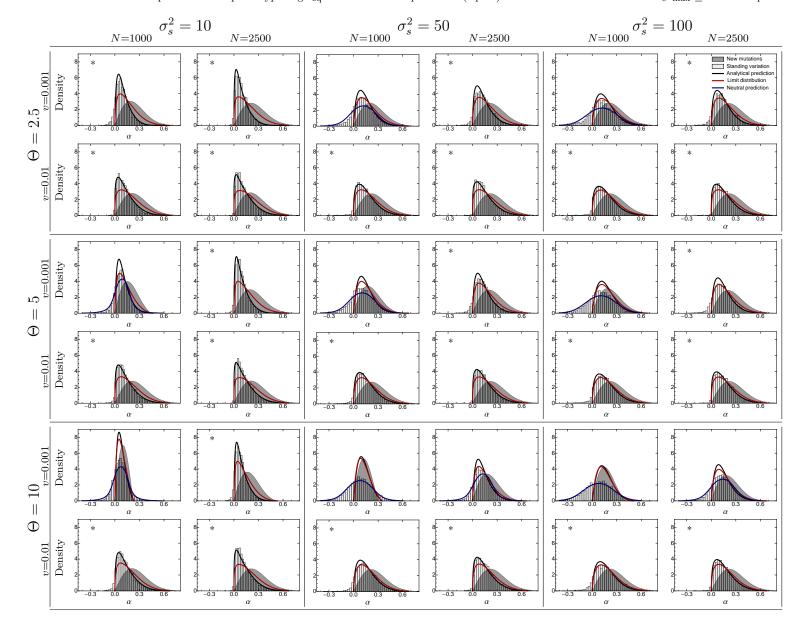
³⁶⁸ Using equation (24a) the distribution of substitutions from standing genetic variation reads

$$p_{\text{SGV},v\to0}(\alpha) \approx C_4(\alpha) p(\alpha) \Pi_{\text{seg},v\to0}(\alpha),$$
 (25)

where $C_4(\alpha)$ again denotes a normalization constant (blue line in Figs. 3, S3_2).

The accuracy of the approximation When compared to individual-based simulations, 370 our analytical approximation for the distribution of adaptive substitutions from standing 371 genetic variation (eq. 19) performs, in general, very well as long as selection is strong, that is, 372 the rate of environmental change v is high and/or the width of the fitness landscape σ_s^2 is not 373 too large (Fig. 2). Under weak selection, however, equation (19) fails to capture the fixation 374 of alleles with neutral or negative effects ("backward fixations"; $\alpha \leq 0$). The reason is that 375 equation (A7) only considers the fixation of alleles whose selection coefficient $s(\alpha, t)$ becomes 376 positive in the long term. But if the rate of environmental change is slow (or σ_s^2 is very 377 large), most alleles get fixed or lost simply by chance, that is, genetic drift. In particular, if 378 genetic drift is the main driver of phenotypic evolution (i.e., $N_e|s(\alpha, t)| < 1$), the distribution 379 of adaptive substitutions is almost symmetric around 0 (see Fig. $S3_2$). This distribution 380 is described very well by equation (25), which assumes that the fixation probability of an 381 allele is proportional to its initial frequency in the standing variation. In addition, even 382 for cases where environmental change imposes modest directional selection, equation (25)383 still captures the shape of the distribution of adaptive substitutions reasonably well, when 384 centered around the empirical mean (blue line in Figs. 2, 3). 385

Figure 2 – The distribution of adaptive substitutions from standing genetic variation. Histograms show results from individual-based simulations. The black line corresponds to the analytical prediction (eq. 19), with the genetic background variation σ_g^2 determined by the SHC approximation (eq. 16). The red line gives the analytical prediction for the limiting case where the equilibrium lag δ_{eq} is reached fast (eq. 21). The blue line is based on the analytical prediction eq. (25) – which assumes a neutral fixation probability – but has been shifted so that it is centered around the empirical mean. The grey curve gives the analytical prediction for substitutions from *de-novo* mutations under the assumption that the phenotypic lag δ_{eq} has reached its equilibrium (eq. 23). The asterisks indicate where $N_e s_{max} \ge 10$. Fixed parameter: $\sigma_m^2 = 0.05$.

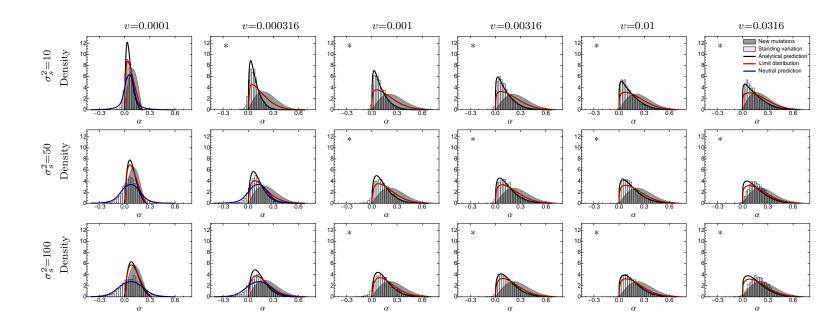


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Figure 3 – The distribution of adaptive substitutions from standing genetic variation for various rates of environmental change. For further details see Fig. 2. Fixed parameters: $\Theta = 2.5, N = 2500, \sigma_m^2 = 0.05$.



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With a moving phenotypic optimum, the selection coefficient (eq. 8) is initially very small. 386 Accordingly, there is always a phase during the adaptive process where genetic drift domi-387 nates, that is, where $N_e|s(\alpha,t)| < 1$ for all mutant alleles. The length of this phase (i.e., the 388 time it takes until selection becomes the main force of evolution) depends on the interplay of 389 multiple parameters, notably v, σ_s^2, N_e and Θ . A good heuristic to determine whether evo-390 lution will ultimately become dominated by selection is to calculate $N_e s_{\text{max}}$ (eq. 11), which 391 gives the maximal population-scaled selection coefficient. Since the selection coefficient of 392 most mutations will be smaller than this value, one can consider as a rule of thumb that 393 selection is the main driver of evolution as long as $N_e s_{\text{max}} \ge 10$. In this case, equation (19) 394 matches the individual-based simulations very well (see asterisks in Figs. 2, 3). In summary, 395 the accuracy of our approximation crucially depends on the efficacy of selection. 400

The effects of linkage on the distribution of adaptive substitutions from standing genetic variation are discussed in Supporting Information 1. The main result is that only tight linkage has a noticeable effect, namely to reduce the efficacy of selection and increase the proportion of "backward" fixations (moving the distribution closer to the prediction from eq. 25).

Biological interpretation As shown in Figures (2) and (3), adaptive substitutions from 406 standing genetic variation have, on average, smaller phenotypic effects than those from de-407 *novo* mutations. There are two reasons for this result. First, in the standing genetic variation, 408 small-effect alleles are more frequent than large-effect alleles and might already segregate 409 at appreciable frequency (increasing their fixation probability). Second, substitutions from 410 standing variation occur in the initial phase of the adaptive process, where the phenotypic 411 lag is small, whereas our approximation for *de-novo* mutations (eq. 23) assumes that the phe-412 notypic lag has reached its maximal (equilibrium) value (which need not be large, depending 413 on the amount of genetic background variation). The relative importance of these two effects 414 can be seen in Figures (2) and (3): Comparing the grey shaded area (eq. 23; de-novo muta-415

tions under the equilibrium lag) with the red line (eq. 21; standing genetic variation under 416 the equilibrium lag) shows the effect of larger starting frequencies of small-effect mutations 417 from the standing genetic variation. The difference of the black (eq. 19; standing genetic 418 variation) and red (eq. 21; standing genetic variation under the equilibrium lag) lines show 419 the effects of the initially smaller lag (i.e., the effect of the dynamical selection coefficient). 420 Note that the first effect is always important (even if Θ and σ_s^2 are large and v is small, where 421 the red line and the grey curve almost coincide—though this is only because the approxima-422 tion is bad). The second effect, however, becomes particularly important if $\gamma = \sigma_g^2/(\sigma_g^2 + \sigma_s^2)$ 423 is small (i.e., if the time to reach the equilibrium lag is large), such that selection coefficients 424 are dynamic and small-effect alleles are selected earlier than large-effect alleles, explaining 425 the relative lack of large-effect alleles in the distribution of adaptive substitutions. 426

Generally, the distribution of adaptive substitutions is unimodal and generally resembles a 427 log-normal distribution (Figs. 2, 3). Only if selection is very weak (i.e., when σ_s^2 is large 428 and/or v is small), does it contain a significant proportion of "backward fixations" (with 429 negative α ; Fig. 3; see "Accuracy of the Approximation"). As the rate of environmental 430 change v increases, the mean phenotypic effect of substitutions increases (Fig. 4, top row), 431 too, but the mode may actually decrease (Fig. 3), that is, the distribution becomes more 432 asymmetric and skewed, resembling the "almost exponential" distribution of substitutions 433 from *de-novo* mutations in the sudden change scenario (ORR 1998). A likely explanation 434 is that small-effect alleles, which are common in the standing variation, are under stronger 435 selection and have an increased fixation probability if v is large (see Fig. 1). 436

Interestingly, if the environment changes very fast the simulated distribution of adaptive substitutions from standing genetic variation almost exactly matches the one predicted by equation (23) for *de-novo* mutations (Fig. 5, see also Figs. 2, 3). However, this seems to be an artefact rather than a relevant biological phenomenon. The reason is that the environment changes so fast that the population quickly dies out. Thus, the resulting distribution of adaptive substitutions is that for a dying population and might not necessarily reflect the

adaptive process. In an experimental setup, though, where populations evolve until they
go extinct, the distribution of adaptive substitutions from standing genetic variation might
truly be indistinguishable from that from *de-novo* mutations.

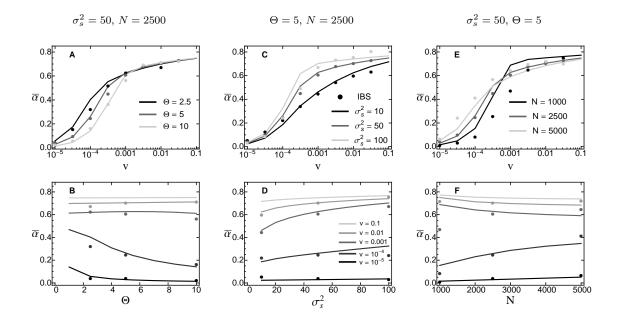


Figure 4 – The mean size of adaptive substitutions from standing genetic variation, measured in units of mutational standard deviations (σ_m) as a function of the rate of environmental change v (top row) and for various v as a function of the population-wide mutation rate Θ (bottom left), the width of the fitness landscape σ_s^2 (bottom middle) and the population size N (bottom right). Lines show the analytical prediction (the mean of the distribution eq. eq:pDistMoveOpt), and symbols give results from individual-based simulations. Error bars for standard errors are contained within the symbols. For v = 0.1, no simulation results are shown, as these constitute a degenerate case (for details see "The accuracy of the approximation"). Fixed parameter: $\sigma_m^2 = 0.05$.

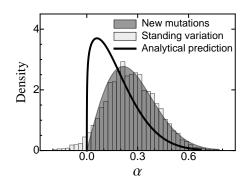


Figure 5 – The distribution of adaptive substitutions from standing genetic variation in the case of fast environmental change. For further details see Fig. 2. Fixed parameters: $\sigma_s^2 = 100$, $\Theta = 10$, N = 2500, v = 0.1, $\sigma_m^2 = 0.05$.

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In the following, we discuss the influence of the other model parameters (Θ , σ_s^2 and N) on the distribution of adaptive substitutions from standing genetic variation, and in particular,

450 its mean $\bar{\alpha}$ (Fig. 4).

The effect of the rate of mutational supply Θ depends strongly on the rate of environmental 451 change $v: \bar{\alpha}$ decreases with Θ if v is small but is independent of Θ if v is large (Fig. 4B). 452 Recall that Θ enters $p_{\text{SGV}}(\alpha)$ (eq. 19) only indirectly through the background variance σ_q^2 . 453 Accordingly, as Θ increases, so does σ_g^2 and, thus, γ (eq. 6b). In the limit $t \to \infty$, the 454 population will follow the optimum at a constant lag $\delta_{eq} = \frac{v}{\gamma}$. Thus, if v is large (such that, 455 even for large σ_g^2 , the lag is large relative to the mutational standard deviation σ_m) increasing 456 Θ does not affect $\bar{\alpha}$. In contrast, if v is small, increasing Θ (and, hence, σ_g^2) will reduce the 457 lag even further, such that most large-effect alleles will be deleterious. Consequently, for 458 small $v, \bar{\alpha}$ decreases as Θ increases. 459

The width of the fitness landscape σ_s^2 affects different aspects of the adaptive process, but its 460 net effect is an increase of the mean effect size of fixed alleles as σ_s^2 increases (i.e., as stabilizing 461 selection gets weaker), especially if the rate of environmental change is intermediate (Fig. 4D). 462 The reason is that weak stabilizing selection increases the frequency of large-effect alleles in 463 the standing variation. In addition, weak selection also increases the phenotypic lag (eq. 9; 464 see also KOPP and MATUSZEWSKI 2014), again favoring large effect alleles. Note that the 465 latter point holds true even though weak selection increases the background variance σ_q^2 . 466 Finally, the effect of σ_s^2 is strongest for intermediate v, because for small v, large-effect alleles 467 are never favored, whereas for large v, all alleles with positive effect have a high fixation 468 probability. 469

Similar arguments hold for N_e (when the rate of mutational supply, Θ , is held constant). First, increasing N_e will always increase the efficacy of selection, resulting in lower initial frequencies of mutant alleles (eq. A4) and decreased σ_g^2 (eq. 16). If the environment changes slowly, $\bar{\alpha}$ increases with N_e , because the equilibrium lag increases (caused by the decrease in σ_g^2). In contrast, if the rate of environmental change is fast, $\bar{\alpha}$ slightly decreases with N_e due to the lower starting frequency of large-effect alleles and because small-effect alleles are

selected more efficiently (i.e., they are less prone to get lost by genetic drift; Fig. 4F).

477 The potential for adaptation from standing genetic variation and the rate of 478 environmental change

So far, we have focussed on the distribution of adaptive substitutions for individual fixation events. We now address what can be said about the total progress that can be made from standing genetic variation following a moving phenotypic optimum. The overall potential for adaptation from standing genetic variation depends on the mean number of alleles segregating in the standing genetic variation, which can be accurately approximated as (FOLEY 1992)

$$|\mathcal{G}| = 1 + \Theta \log \left[\frac{2\sigma_s^2}{\sigma_m^2}\right] \tag{26}$$

(results not shown). The mean number of alleles that become fixed can then be calculated as

$$|\mathcal{G}|_{\text{fix}} = |\mathcal{G}| \int_0^{\alpha_{\text{max}}} p(\alpha) \Pi_{\text{seg}}(\alpha) d\alpha, \qquad (27)$$

where the integral equals the normalization constant in equation (19) (i.e., the proportion of fixed alleles). Finally, using equation (27), the average distance travelled in phenotype space before standing variation is exhausted is given by

$$z^* = 2|\mathcal{G}|_{\text{fix}} \,\bar{\alpha} = 2|\mathcal{G}| \int_0^{\alpha_{\text{max}}} \alpha p(\alpha) \Pi_{\text{seg}}(\alpha) \mathrm{d}\alpha, \tag{28}$$

where $\bar{\alpha}$ is the mean phenotypic effect size of adaptive substitutions from standing genetic variation, and the factor 2 in equation (28) comes from the fact that we are considering diploids (and α denotes the phenotypic effect per haplotype). Note that, once the shift of the optimum considerably exceeds z^* , the population will inevitably go extinct without the input of new mutations.

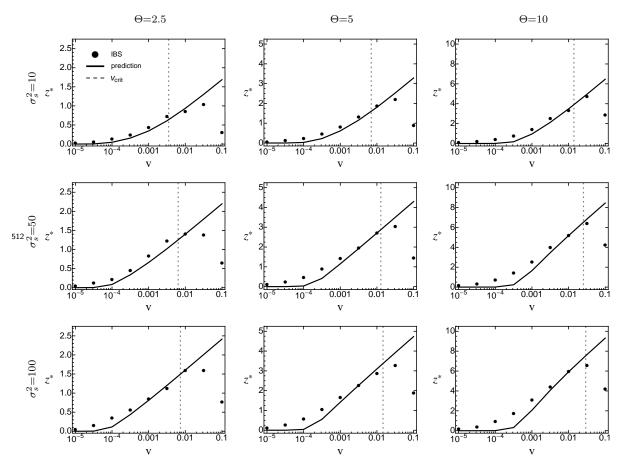
Figure 6 (see also Figs. S3 3, S3 4, S3 5 and Figs. S3 6, S3 7) illustrate these predictions 494 and compare them to results from individual-base simulations (where, unlike in the rest of 495 this paper, new mutational input was turned off after the onset of the environmental change). 496 Both the mean number of fixations $|\mathcal{G}|_{\text{fix}}$ and the mean phenotypic distance travelled z^* 497 increase with the rate of environmental change, reflecting the fact that more and larger-effect 498 alleles become fixed if the environment changes fast. Only for very large v, where the rate 490 of environmental change exceeds the "maximal sustainable rate of environmental change" 500 (BÜRGER and LYNCH 1995), which for our choice for the number of offspring B = 2 equals 501

$$v_{\rm crit} = \sigma_g^2 \sqrt{\frac{2\log\left[2\sqrt{\frac{\sigma_s^2}{\sigma_g^2 + \sigma_s^2}}\right]}{\sigma_g^2 + \sigma_s^2}},\tag{29}$$

do $|\mathcal{G}|_{\text{fix}}$ and z^* decrease sharply, because the population goes extinct before fixations can be 502 completed (grey-dashed line in Figs. 6, S3_6 and S3_7). At small values of v, $|\mathcal{G}|_{\text{fix}}$ matches 503 the "neutral" prediction (grey-dashed line in Figs. S3 3, S3 4 and S3 5). Note that these 504 fixations have almost no effect on z^* , because their average effect is zero. At intermediate 505 v, equation (28) slightly underestimates z^* for parameter values leading to large background 506 variance σ_q^2 (i.e., high Θ and σ_s^2). The likely reason is that the analytical approximation 507 assumes σ_g^2 to be constant, while it obviously decreases in the simulations (since there are 508 no *de-novo* mutations). All these results are qualitatively consistent across different values 509 of σ_s^2 and Θ (Figs. 6, S3_6, S3_7). 510

Figure 6 – The average distance traversed in phenotype space, z^* , as a function of the rate of environmental change v, when standing genetic variation is the sole source for adaptation. Symbols show results from individual-based simulations (averaged over 100 replicate runs). The black line gives the analytical prediction (eq. 28), with σ_g^2 taken from equation (16). The grey-dashed line gives the critical rate of environmental change (eq. 29). Error bars for standard errors are contained within the symbols. Fixed parameters: N = 2500, $\sigma_m^2 = 0.05$.





The relative importance of standing genetic variation and *de-novo* mutations over the course of adaptation

⁵¹⁵ Until now we have compared adaptation from standing genetic variation to that from *de-*⁵¹⁶ *novo* mutations in terms of their distribution of fixed phenotypic effects. We now turn to ⁵¹⁷ investigating their relative importance over the course of adaptation. For this purpose, we ⁵¹⁸ recorded (in individual-based simulations) the contributions of both sources of variation to ⁵¹⁹ the phenotypic mean and variance. An average time series for both measures is shown in ⁵²⁰ Figure 7. As expected, the initial response to selection is almost entirely based on standing ⁵²¹ variation, but the contribution of *de-novo* mutations increases over time. As a quantitative

measure for this transition, we define $t_{\text{DNM},50}(\bar{z})$ as the point in time where the cumulative 522 contribution of *de-novo* mutations has reached 50%. Indeed, we find that, beyond this time, 523 adaptation almost exclusively proceeds by the fixation of *de-novo* mutations (Fig. 7A). As 524 expected, $t_{\text{DNM},50}(\bar{z})$ decreases with v (Figs. 8, S3_8, first row), while the total phenotypic 525 response \bar{z} increases (Figs. 8, S3_8, second row). The reason is that faster environmental 526 change induces stronger directional selection and increases the phenotypic lag, such that 527 standing variation is depleted more quickly and *de-novo* mutations and contribute earlier. 528 Note that, as in Figure 6, the total phenotypic response at time $t_{\text{DNM},50}(\bar{z})$ decreases once 529 v exceeds the "maximal sustainable rate of environmental change", for the same reasons as 530 discussed above. Furthermore, $t_{\text{DNM},50}(\bar{z})$ increases with both Θ and σ_s^2 (due to the increased 531 standing variation; see eq. 16). Interestingly, the relative contribution of original standing 532 genetic variation to the total genetic variance at time $t_{\text{DNM},50}(\bar{z})$ remains largely constant (at 533 around 20%) over large range of v and does not show any dependence on Θ nor σ_s^2 (Figs. 8, 534 S3_8; third row). Deviations occur only if v is either very small or very large. In particular, 535 if v is small, standing variation is almost completely depleted before new mutations play 536 a significant role. Conversely, if v is very large, standing genetic variation still forms the 537 majority of the total genetic variance. As mentioned above, this is most likely because the 538 population goes extinct before fixations can be completed, that is, before the entire (standing) 539 adaptive potential is exhausted. All these results remain qualitatively unchanged if, instead 540 of $t_{\text{DNM},50}(\bar{z})$, we define $t_{\text{DNM},50}(\sigma_g^2)$ as the point in time where 50% of the current genetic 541 variance goes back to *de-novo* mutations (Figs. S3 9, S3 10). 542

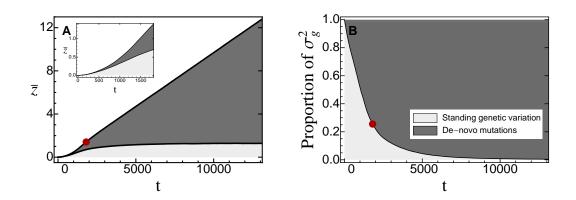


Figure 7 – The contributions of standing genetic variation (light grey) and *de-novo* mutations (dark grey) to the cumulative phenotypic response to selection \bar{z} (A) and the *current* genetic variance (B) over time. Plots show average trajectories over 1000 replicate simulations. The red dot marks the point in time where 50% of the total phenotypic response were due to *de-novo* mutations. The inset in (A) shows a more detailed plot of the dynamics of \bar{z} up to this point. Fixed parameters: $\sigma_s^2 = 50$, $\Theta = 5$, N = 2500, v = 0.001, $\sigma_m^2 = 0.05$.

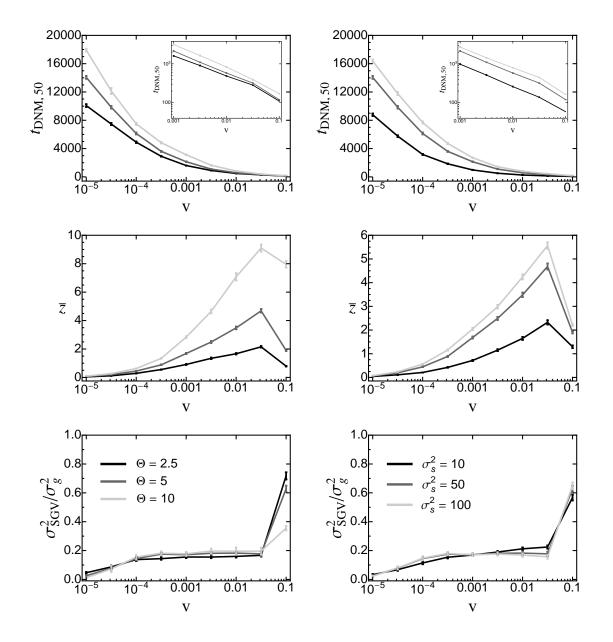


Figure 8 – First row: the point in time $t_{\rm DNM,50}$ (\bar{z}) where 50% of the phenotypic response to moving-optimum selection have been contributed by *de-novo* mutations as a function of the rate of environmental change for various values of Θ (left) and σ_s^2 (right). Insets show the results for large v on a log-scale. Second row: The mean total phenotypic response at this time. Third row: The relative contribution of original standing genetic variation to the total genetic variance at time $t_{\rm DNM,50}$ (\bar{z}). Data are means (and standard deviations) from 1000 replicate simulation runs. Fixed parameters (if not stated otherwise): $\sigma_s^2 = 50$, $\Theta = 5$, N = 2500, $\sigma_m^2 = 0.05$.

DISCUSSION

Global climate change has forced many populations to either go extinct or adapt to the 543 altered environmental condition. When studying the genetic basis of this process, most the-544 oretical work has focused on adaptation from new mutations (e.g., GILLESPIE 1984; ORR 545 1998, 2000; COLLINS et al. 2007; KOPP and HERMISSON 2007, 2009a,b; MATUSZEWSKI et al. 546 2014). Consequently, very little is known about the details of adaptation from standing ge-547 netic variation (but see ORR and BETANCOURT 2001; HERMISSON and PENNINGS 2005), 548 that is, which of the alleles segregating in a population will become fixed and contribute 549 to the evolutionary response. Here, we have used analytical approximations and stochastic 550 simulations to study the effects of standing genetic variation on the genetic basis of adap-551 tation in gradually changing environments. Supporting a verbal hypothesis by BARRETT 552 and SCHLUTER (2008), we show that, when comparing adaptation from standing genetic 553 variation to that from *de-novo* mutations, the former proceeds, on average, by the fixation of 554 more alleles of small effect. In both cases, however, the genetic basis of adaptation crucially 555 depends on the efficacy of selection, which in turn is determined by the population size, 556 the strength of (stabilizing) selection and the rate of environmental change. When standing 557 genetic variation is the sole source for adaptation, we find that fast environmental change en-558 ables the population to traverse larger distances in phenotype space than slow environmental 559 change, in contrast to studies that consider adaptation from new mutations only (PERRON 560 et al. 2008; Bell and GONZALEZ 2011; LINDSEY et al. 2013; Bell 2013). We now discuss 561 these results in greater detail. 562

⁵⁶³ The genetic basis of adaptation in the moving-optimum model

Introduced as a model for sustained environmental change, such as global warming (LYNCH et al. 1991; LYNCH and LANDE 1993), the moving-optimum model describes the evolution of a quantitative trait under stabilizing selection towards a time-dependent optimum (BÜRGER 2000). A large number of studies have analyzed both the basic model and several modifi-

cations, for example, models with a periodic or fluctuating optimum, or models for multi-568 ple traits (SLATKIN and LANDE 1976; CHARLESWORTH 1993; BÜRGER and LYNCH 1995; 569 LANDE and SHANNON 1996; KOPP and HERMISSON 2007, 2009a,b; GOMULKIEWICZ and 570 HOULE 2009; ZHANG 2012; CHEVIN 2013; MATUSZEWSKI et al. 2014). Following traditional 571 quantitative-genetic approaches, the majority of these studies assumed that the distribution 572 of genotypes (and phenotypes) is Gaussian with constant (time-invariant) genetic variance, 573 and they have mostly focussed on the evolution of the population mean phenotype and on 574 the conditions for population persistence (BÜRGER and LYNCH 1995; LANDE and SHANNON 575 1996: GOMULKIEWICZ and HOULE 2009). None of these models, however, allows to address 576 the fate of individual alleles (i.e., whether they become fixed or not). In a recent series of 577 papers on the moving-optimum model, KOPP and HERMISSON (2007, 2009a,b) studied the 578 genetic basis of adaptation from new mutations and derived the distribution of adaptive 579 substitutions (i.e., the distribution of the phenotypic effects of those mutations that arise 580 and become fixed in a population); this approach has recently been generalized to multiple 581 phenotypic traits by MATUSZEWSKI et al. (2014). The shape of this distribution resembles 582 a Gamma-distribution with an intermediate mode. Thus, most substitutions are of inter-583 mediate effect with only a few large-effect alleles contributing to adaptation. The reason is 584 that small-effect alleles – despite appearing more frequently than large-effect alleles – have 585 only small effects on fitness (and are, hence, often lost due to genetic drift), while large-effect 586 alleles might be removed because they "overshoot" the optimum (KOPP and HERMISSON 587 2009b). A detailed comparison and discussion of the distribution of adaptive substitutions 588 from *de-novo* mutations with (eq. 23) and without (KOPP and HERMISSON 2009b) genetic 589 background variation is given in Supporting Information 2. 590

Here, we have studied the genetic basis of adaptation from standing genetic variation. We find that the distribution of substitutions from standing genetic variation depends on the distribution of standing genetic variants (i.e., distribution of alleles segregating in the population prior to the environmental change) and the intensity of selection. The former is shaped

primarily by the distribution of new mutations and the strength of stabilizing selection, which 595 removes large-effect alleles. Depending on the speed of change v, we find two regimes that are 596 characterized by separate distributions of standing substitutions. If the environment changes 597 sufficiently fast, the distribution of adaptive substitutions resembles a lognormal distribu-598 tion with a strong contribution of small-effect alleles (eq. 19; Fig. 2). The reason is that, in 599 the standing genetic variation, small-effect alleles are more frequent than large-effect alleles 600 and might already segregate at appreciable frequency (so that they are not lost by genetic 601 drift). With a moving optimum, they furthermore are the first to become positively selected, 602 hence reducing the time they are under purifying selection. Finally, epistatic interactions 603 between co-segregating alleles (or between a focal allele and the genetic background) also 604 favor alleles of small effect. Consequently, when adapting from standing genetic variation, 605 most substitutions are of small phenotypic effect. 606

The second regime occurs if the rate of environmental change v is very small. In this case, allele-frequency dynamics are dominated by genetic drift, and the distribution of adaptive substitutions reflects the approximately Gaussian distribution of standing genetic variants (eq. 25; Fig. S3_2). It should be noted, however, that fixations under this regime take a very long time, similar to that of purely neutral substitutions (i.e., on the timescale of $4N_e$).

Finally, we have studied the relative importance of standing genetic variation and *de-novo* 612 mutations over the course of adaptation. As shown in Figures 7 and 8, the initial response 613 to selection is almost entirely based on standing variation, with *de-novo* mutations becoming 614 gradually more important. The time scale of this transition strongly depends on the rate 615 of environmental change, but for slow or moderately fast change, it typically occurs over at 616 least hundreds of generations (Figs. 8, S3 8 and Figs. S3 9, S3 10). This observation is 617 in contrast to results by HILL and RASBASH (1986b), who found that under strong artificial 618 (i.e., truncation) selection in small populations (N = 20), new mutations might contribute 619 up to one third of the total response after as little as 20 generations. Our results show 620 that the situation is very different for large populations under natural selection in gradually 621

changing environments. The likely reason for this difference is that truncation selection 622 induces strong directional selection (corresponding to large v) and only extreme phenotypes 623 reproduce. Thus, truncation selection is much more efficient in maintaining large-effect de-624 *novo* mutations, while eroding genetic variation more quickly (because it introduces a large 625 skew in the offspring distribution). However, the similarities and differences in the genetic 626 basis of responses to artificial versus natural selection is an interesting topic—in particular, 627 for the interpretation of the large amount of genetic data available from breeding programs 628 (STERN and ORGONZO 2009)—that should be addressed in future studies. 629

Throughout this study, we have focused on adaptation to a moving optimum, that is, a sce-630 nario of gradual environmental change. An obvious question is how our results would change 631 under the alternative scenario of a one-time sudden shift in the optimum (as assumed in 632 numerous studies, e.g., ORR 1998; HERMISSON and PENNINGS 2005; CHEVIN and HOSPI-633 TAL 2008). While beyond the scope of this paper, our approach should, in principle, still be 634 applicable. In particular, each focal allele still experiences a gradual change in its selection 635 coefficient, due to the evolution of the genetic background. Unlike in the moving-optimum 636 model, however, the selection coefficient *decreases*, as the mean phenotype gradually ap-637 proaches the new optimum. Hence, a suitably modified version of equation 13 would give 638 the probability that a focal allele *establishes* in the population (i.e., escapes stochastic loss), 639 but in the absence of continued environmental change, establishment does not guarantee 640 fixation. In other words, alleles need to "race for fixation" before other competing alleles get 641 fixed and they become deleterious (KOPP and HERMISSON 2007, 2009a). The dynamics of 642 a mutation along its trajectory should therefore be even more complex than in the moving-643 optimum model, and show an even stronger dependence on the genetic background (CHEVIN 644 and HOSPITAL 2008). 645

⁶⁴⁶ Extinction and the rate of environmental change

647 Recently, several experimental studies have explored how the rate of environmental change

affects the persistence of populations that rely on new mutations for adapting to a gradually 648 changing environment (PERRON et al. 2008; BELL and GONZALEZ 2011; LINDSEY et al. 649 2013). In line with theoretical predictions (BELL 2013), all studies found that "evolutionary 650 rescue" is contingent on a small rate of environmental change. In particular, LINDSEY et al. 651 (2013) evolved replicate populations of *E. coli* under different rates of increase in antibiotic 652 concentration and found that certain genotypes were evolutionarily inaccessible under rapid 653 environmental change, suggesting that "rapidly deteriorating environments not only limit 654 mutational opportunities by lowering population size, but [...] also eliminate sets of mutations 655 as evolutionary options". This is in stark contrast to our prediction that faster environmental 656 change can enable the population to remain better adapted and to traverse larger distances 657 in phenotype space when standing genetic variation is the sole source for adaptation (Fig. 6 658 and Figs S3 6, S3 7; in line with recent experimental observations; H. Teotonio, private 659 communication). The difference between these results arises from the availability of the 660 "adaptive material". While *de-novo* mutations first need to appear and survive stochastic loss 661 before becoming fixed, standing genetic variants are available right away and might already 662 be segregating at appreciable frequency. Thus, in both cases, the rate of environmental 663 change plays a critical, though antagonistic, role in determining the evolutionary options. 664 While fast environmental change eliminates sets of new mutations, it simultaneously helps 665 to preserve standing genetic variation until it can be picked-up by selection. Under slow 666 change, in contrast, most large-effect alleles from the standing variation, by the time they 667 are needed, are already eliminated by drift or stabilizing selection. 668

⁶⁶⁹ Our results also mean that, if the optimum stops moving at a given value $z_{opt,max}$, popula-⁶⁷⁰ tions will achieve a higher degree of adaptation (higher \bar{z}^*) if the final optimum is reached ⁶⁷¹ fast rather than slowly (see also UECKER and HERMISSON 2014), at least if standing genetic ⁶⁷² variation is the sole source for adaptation. While this assumption is an obvious simplification, ⁶⁷³ it may often be approximately true in natural populations. The same holds true in exper-⁶⁷⁴ imental populations, where selection is usually strong and the duration of the experiment short, such that *de-novo* mutations can frequently be neglected (see Fig. 8).

676 Testing the predictions

The predictions made by our model can in principle be tested empirically, even though suit-677 able data might be sparse and experiments challenging. There is, of course, ample evidence 678 for adaptation from standing genetic variation. For example, DOMINGUES et al. (2012) 679 showed that camouflaging pigmentation of oldfield mice (*Peromyscus polionotus*) that have 680 colonized Florida's Gulf Coast has evolved quite rapidly from a pre-existing mutation in the 681 Mc1r gene: LIMBORG et al. (2014) investigated selection in two allochronic but sympatric 682 lineages of pink salmon (Oncorhynchus gorbuscha) and identified 24 divergent loci that had 683 arisen from different pools of standing genetic variation, and TURCHIN et al. (2012) showed 684 that height-associated alleles in humans display a clear signal for widespread selection on 685 standing genetic variation. 686

However, testing the predictions of our model requires, in addition, detailed knowledge of 687 the genotype-phenotype relation. Currently, there is only a small (yet increasing) number of 688 systems for which both a set of functionally validated beneficial mutations and their selec-689 tion coefficients under different environmental conditions are available (JENSEN 2014). Thus, 690 estimating the distribution of standing substitutions will be challenging, because of the of-691 ten unknown phenotypic and fitness effects of beneficial mutations and the large number of 692 replicate experiments needed to obtain a reliable empirical distribution. Furthermore, even if 693 these problems were solved, small-effect alleles might not be detectable due to statistical lim-694 itations (OTTO and JONES 2000), and in certain limiting cases where the population quickly 695 goes extinct (i.e., when the environment changes very fast), the distribution of adaptive sub-696 stitutions from standing genetic variation might be indistinguishable to that from *de-novo* 697 substitutions (Fig. 5). 698

Recent developments in laboratory systems (MORRAN *et al.* 2009; PARTS *et al.* 2011), however, have created opportunities for experimental evolution studies in which population size,

the selective regime and the duration of selection can be manipulated, and adaptation from 701 *de-novo* mutations and standing genetic variation can be recorded (BURKE 2012). Applying 702 these techniques in experiments in the vein of LINDSEY et al. (2013), but starting from a 703 polymorphic population, should make it possible to test the relation between the rate of 704 environmental change and population persistence, and to assess the probability of adapta-705 tion from standing genetic variation. First experiments along these lines are currently being 706 carried out in populations of C. elegans, with the aim of determining the limits of adap-707 tation to different rates of increase in sodium chloride concentration (H. Teotonio, private 708 communication). Furthermore, PENNINGS (2012) recently applied the HERMISSON and PEN-709 NINGS (2005) framework to show that standing genetic variation plays an important role in 710 the evolution of drug-resistance in HIV, affecting up to 39% of patients (depending on treat-711 ment) and explaining why resistance mutations in patients who interrupt treatment are likely 712 to become established within the first year. A similar approach should also be applicable 713 to scenarios of gradual environmental change (e.g., evolution of resistance mutations under 714 gradually increasing antibiotic concentrations). 715

716 Conclusion

As global climate change continues to force populations to respond to the altered environ-717 mental conditions, studying adaptation to changing environments – both empirically and 718 theoretically – has become one of the main topics in evolutionary biology. Despite increased 719 efforts, however, very little is known about the genetic basis of adaptation from standing 720 genetic variation. Our analysis of the moving-optimum model shows that this process has, 721 indeed, a very different genetic basis than that of adaptation from *de-novo* mutations. In 722 particular, adaptation proceeds via the fixation many small-effect alleles (and just a few large 723 ones). In accordance with previous studies, the adaptive process critically depends on the 724 tempo of environmental change. Specifically, when populations adapt from standing genetic 725 variation only, the potential for adaptation increases as the environment changes faster. 726

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APPENDIX

⁸⁹² Appendix 1: Theoretical Background

⁸⁹³ In this Appendix, we briefly recapitulate results from previous studies that form the basis ⁸⁹⁴ for our analytical derivations.

⁸⁹⁵ The probability of adaptation from standing genetic variation for a single bi-⁸⁹⁶ allelic locus after a sudden environmental change

HERMISSON and PENNINGS (2005) studied the situation where the selection scheme at a single bi-allelic locus changes following a sudden environmental change. In particular, they derived the probability for a mutant allele to reach fixation that was neutral or deleterious prior to the change but has become beneficial in the new environment. In the continuum limit for allele frequencies this probability is given by

$$P_{\rm SGV} = \int_0^1 \rho(x) \Pi_x dx, \tag{A1}$$

where $\rho(x)$ is the density function for the allele frequency x of the mutant allele in mutationselection-drift balance and Π_x denotes its fixation probability.

For a mutant allele present at frequency x and with selective advantage s_b in the new environment, the fixation probability is given by (KIMURA 1957)

$$\Pi_x(s_b) \approx \frac{1 - \exp[-4N_e s_b x]}{1 - \exp[-4N_e s_b]}.$$
(A2)

There are two points to make here. First, mutational effects in the HERMISSON and PEN-NINGS (2005) model are directly proportional to fitness, whereas mutations in our model affect a phenotype under selection. Second, in our framework, s_b denotes the (beneficial)

⁹⁰⁹ selection coefficient for heterozygotes.

Approximations for $\rho(x)$ can be derived from standard diffusion theory (EWENS 2004; for details see HERMISSON and PENNINGS 2005). If the mutant allele was neutral prior to the change in the selection scheme

$$\rho(x) = Cx^{\Theta - 1} \frac{1 - x^{1 - \theta}}{x - 1}.$$
(A3)

Here, $C = (\gamma + \psi(\theta))^{-1}$ denotes a normalization constant where $\gamma \approx 0.577$ is Euler's gamma and $\psi(\cdot)$ is the polygamma function. Similarly, if the mutant allele was deleterious before the environmental change (with negative selection coefficient s_d) the allele-frequency distribution is given by

$$\rho(x) = C \frac{\left(1 - \exp\left[(1 - x)4N_e |s_d|\right]\right) x^{\theta - 1}}{x - 1},\tag{A4}$$

where $C = ({}_{1}F_{1}(0, \theta, 4N_{e}|s_{d}|))^{-1}$ denotes a normalization constant and ${}_{1}F_{1}(a, b, c)$ is the hypergeometric function. If the allele was sufficiently deleterious $(4N_{e}|s_{d}| \ge 10)$, equation (A4) can further be approximated as

$$\rho(x) = Cx^{\theta-1} \exp[-4N_e |s_d|x], \tag{A5}$$

where $C = \left(\frac{\gamma[\theta, 4N_e[s_d]]}{(4N_e[s_d])^{\theta}}\right)^{-1}$ again denotes a normalization constant with $\gamma[a, b] = \int_0^b t^{a-1} \exp[-t] dt$ denoting the lower incomplete gamma function.

Finally, the probability that a population successfully adapts from standing genetic variation can be derived as

$$P_{\rm SGV} = 1 - \left(1 + \frac{4N_e s_b}{4N_e |s_d| + 1}\right)^{-\theta} = 1 - \exp\left[-\theta \log\left[\frac{4N_e s_b}{4N_e |s_d| + 1}\right]\right].$$
 (A6)

⁹²⁴ Fixation probabilities under time-inhomogeneous selection

In gradually changing environments, the selection coefficient of a given (mutant) allele is not fixed but changes over time (i.e., as the position of the optimum changes). UECKER and HERMISSON (2011) recently developed a mathematical framework based on branchingprocess theory to describe the fixation process of a beneficial allele under temporal variation in population size and selection pressures. They showed that the probability of fixation of a mutation starting with n initial copies is given by

$$\Pi_{\text{fix}}(n) = 1 - \left(1 - \frac{1}{\varphi}\right)^n,\tag{A7a}$$

931 where

$$2\varphi = 1 + \int_0^\infty (N(0)/N_e(t)) \exp\left[-\int_0^t s(\tau)d\tau\right] \mathrm{d}t.$$
 (A7b)

932

Assuming that the population size remains constant and that the selection coefficient increases linearly in time, $s(t) = s_d + s_v t$, equation (A7a) becomes

$$\Pi_{\text{fix}} = 1 - \left(1 - \left[1 + \frac{1}{2}\sqrt{\frac{\pi}{2s_v}}\exp\left(\frac{s_d^2}{2s_v}\right)\operatorname{erfc}\left(\frac{s_d}{\sqrt{2s_v}}\right)\right]^{-1}\right)^n,\tag{A8}$$

where $\operatorname{erfc}(\cdot)$ denotes the complementary Gaussian error function.

SUPPORTING INFORMATION

⁹³⁶ Supporting Information 1: Limited Recombination

Figure S1_1 – The distribution of adaptive substitutions from standing genetic variation for free recombination (dark bins) compared to that for limited recombination (light bins). The black line corresponds to the analytical prediction (eq. 19). σ_q^2 is given by equation (16). Fixed parameters: $\sigma_s^2 = 50$, N = 2500, v = 0.001, $\sigma_m^2 = 0.05$.

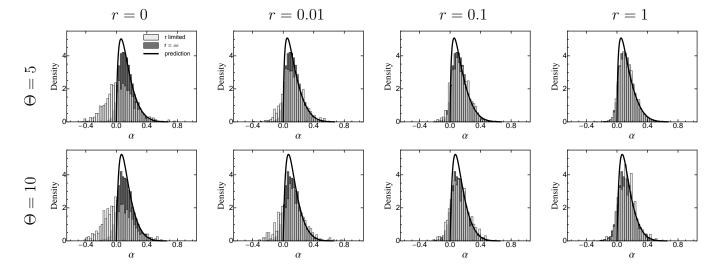
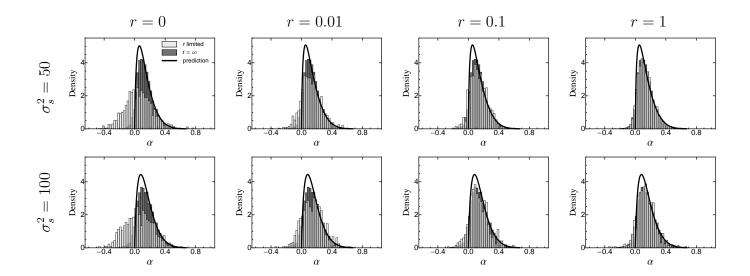


Figure S1_2 – The distribution of adaptive substitutions from standing genetic variation for free recombination (dark bins) compared to that for limited recombination (light bins). The black line corresponds to the analytical prediction (eq. 19). σ_g^2 is given by equation (16). Fixed parameters: $\Theta = 5$, N = 2500, v = 0.001, $\sigma_m^2 = 0.05$.



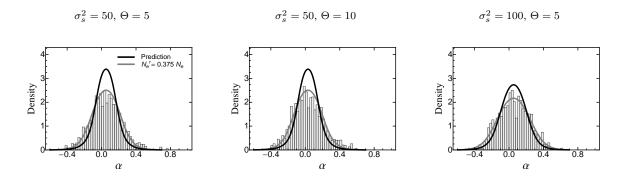


Figure S1_3 – The distribution of adaptive substitutions from standing genetic variation for complete linkage (no recombination). The black and the grey line corresponds to the analytical prediction (eq. 25) that are centred around the mean of the individual-based simulation. For the grey line N_e has been adjusted by a factor 0.385 to match the distribution from the individual-based simulations. Other parameters: $v = 0.001, r = 0, N = 2500, \sigma_m^2 = 0.05.$

The individual-based simulation results presented in the main text were obtained under the assumption of free recombination. In this Supplementary Information, we relax this assumption and study the effects of linkage (i.e., limited recombination).

We first clarify the meaning of the recombination parameter r, which determines the mean number of crossover events per meiosis. By definition, the simulated genome corresponds to a single chromosome of length $D_{\mathcal{G}} = r \cdot 100$ cM, and the mean distance between two randomly chosen sites is $\frac{1}{3}D_{\mathcal{G}}$. The mean distance between two adjacent polymorphic loci is $D_{\mathcal{G},adjacent} = \frac{1}{|\mathcal{G}|}D_{\mathcal{G}+1}$, where \mathcal{G} is the mean number of polymorphic loci, which depends on Θ and σ_s^2 (eq. 26).

The corresponding recombination rate \mathfrak{r} between two polymorphic loci is given by the inverse of Haldane's mapping function (SPEED 2005), that is,

$$\mathbf{r} = \frac{1}{2} \left(1 - \exp\left[-2D_{\mathcal{G}}\right] \right),\tag{S1}$$

 $_{950}$ see Table S1_1.

Table S1_1 – The classical population genetic recombination rate \mathfrak{r} (eq. S1) between two adjacent loci for different values of σ_s^2 , Θ and r. Other parameters: $\sigma_m^2 = 0.05$.

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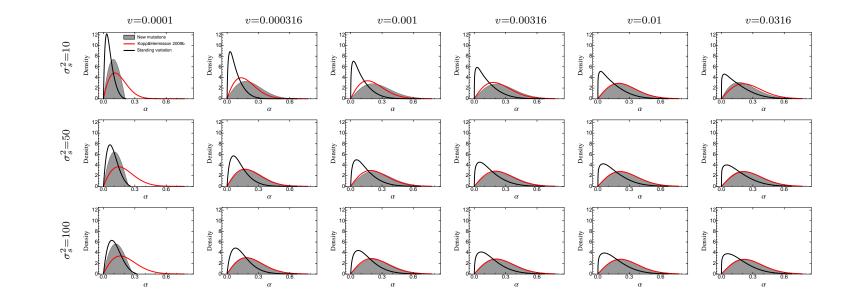
r

The effect of limited recombination on the distribution of adaptive substitutions from stand-953 ing genetic variation is illustrated in Figures S1_1 and S1_2. For r = 1 (corresponding to a 954 genome length of 100cM and an average recombination rate \mathfrak{r} of close to 0.5, see table S1 1), 955 the distribution is essentially identical to that for linkage equilibrium. As r decreases, the 956 distribution progressively shifts to the left, becomes more symmetric and includes more and 957 more alleles with negative phenotypic effect. For r = 0 (corresponding to complete linkage 958 or asexual reproduction), it resembles the distribution for "drift-driven" evolution (i.e., when 959 selection is not efficient; Fig. S3 2). The reason is that fixation involves entire haplotypes 960 carrying multiple mutations, whose (positive and negative) effects largely cancel. From a 961 different perspective, limited recombination leads to Hill-Robertson interference between co-962 segregating alleles (HILL and ROBERTSON 1966), which in many respects corresponds to a 963 decrease in effective population size N_e (COMERON et al. 2008), which in turn reduces the 964 efficacy of selection. Note, however, that unlike in the case of a slowly changing environment 965 (Fig. S3_2) reducing N_e also affects the equilibrium allele-frequency distribution $\rho(x, \alpha)$ (by 966 reducing the strength of selection against large-effect alleles). In line with previous simulation 967 results (COMERON et al. 2008), we find that equation (25) provides a very good fit, when N_e 968 is set to 38.5% of its original value. 969

⁹⁷⁰ Supporting Information 2: The distribution of adaptive substitutions from de-⁹⁷¹ novo mutations with and without genetic background variation

There are two ways in which the distribution of adaptive substitutions from standing genetic 972 variation can be compared to that from *de-novo* mutations. The first comparison consid-973 ers a population without genetic background variation. This is the situation studied by 974 KOPP and HERMISSON (2009a), where an essentially monomorphic population performs an 975 adaptive walk following a moving optimum. The second situation is the one described by 976 equation (23), where new mutations interact with a genetic background of constant variance 977 (this background is presumably itself constantly replenished by new mutations). Analytical 978 predictions for all three distributions are compared in Figures. S2_1, S2_2 and S2_3. It 979 can be seen that the adaptive-walk prediction (eq. 14 in KOPP and HERMISSON 2009b; red 980 line) is always shifted towards larger α compared to the distribution of adaptive substitutions 981 from standing genetic variation (eq. 19, black line). The predicted distribution from *de-novo* 982 mutations in the presence of genetic background variation (eq. 23, grey curve) shifts from 983 the latter to the former as v increases. The reason is that, for small v, the fixation of both 984 standing variants and new mutations in the presence of background variation is strongly 985 constrained by the equilibrium lag (eq. 9). For large v, in contrast, the lag is large and adap-986 tation is primarily limited by the available alleles, independent of their source and initial 987 frequency (mutation-limited regime *sensu* KOPP and HERMISSON 2009b). Note, however, 988 that in both limiting cases, equation (19) is a poor predictor for the simulated substitutions 989 from standing variation (Fig. 5, S3 2). Nevertheless, it remains true that adaptive substi-990 tutions from new mutations are generally smaller than those from new mutations, with or 991 without genetic background variation. 992

Figure S2_1 – Comparison of the analytical predictions for the distribution of adaptive substitutions from standing genetic variation and *de-novo* mutations. The black line corresponds to eq. (19), with the genetic background variation σ_g^2 determined by the SHC approximation (eq. 16). The grey curve gives the analytical prediction for substitutions from *de-novo* mutations under the assumption that the phenotypic lag δ_{eq} has reached an equilibrium (eq. 23). The red line gives the analytical prediction for the first substitution from *de-novo* mutations under the adaptive-walk assumption that there is no genetic background variation (KOPP and HERMISSON 2009b, eq. 14). Note that, for some parameter combinations, the simulated distribution from standing variation deviates from eq. (19). In particular, for small v, it approaches the "neutral" prediction eq. (25, see Fig. 3, and for large v, it may approach the distribution from new mutations, eq. (23), see Fig. 5. Fixed parameters: $\Theta = 2.5$, N = 2500, $\sigma_m^2 = 0.05$.



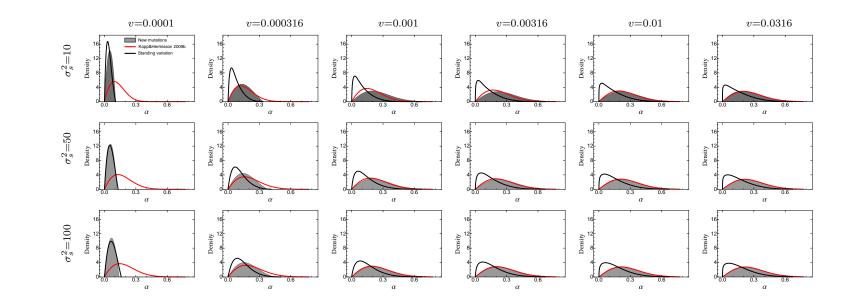
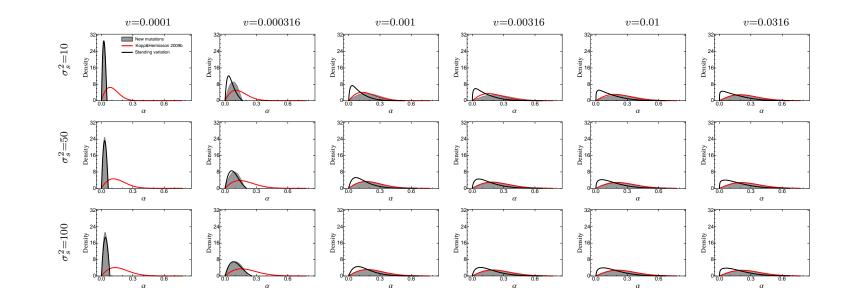
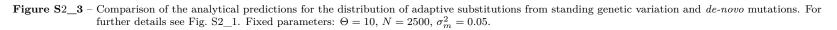


Figure S2_2 – Comparison of the analytical predictions for the distribution of adaptive substitutions from standing genetic variation and *de-novo* mutations. For further details see Fig. S2_1. Fixed parameters: $\Theta = 5$, N = 2500, $\sigma_m^2 = 0.05$.





⁹⁹⁹ Supporting Information 3: Supporting Figures

Figure S3_1 – The probability for a mutant allele to adapt from standing genetic variation as a function of the rate of environmental change v. Solid lines correspond to the analytical prediction (eq. 14), the grey dashed line shows the probability for a neutral allele ($\alpha = 0$; eq. 15), and symbols give results from Wright-Fisher simulations. The phenotypic effect size α of the mutant allele ranges from $0.5\sigma_m$ (top line; black) to $3\sigma_m$ (bottom line; purple) with increments of $0.5\sigma_m$. The figures in each parameter box (per locus mutation rate θ , width of fitness landscape σ_s^2) correspond to different values of the genetic background variation σ_g^2 with $\sigma_g^2 = 0$ (no background variation; top left), $\sigma_g^2 = 0.005$ (top right), $\sigma_g^2 = 0.01$ (bottom left) and $\sigma_g^2 = 0.05$ (bottom right). Other parameters: $N_e = 25000$, $\sigma_m^2 = 0.05$.

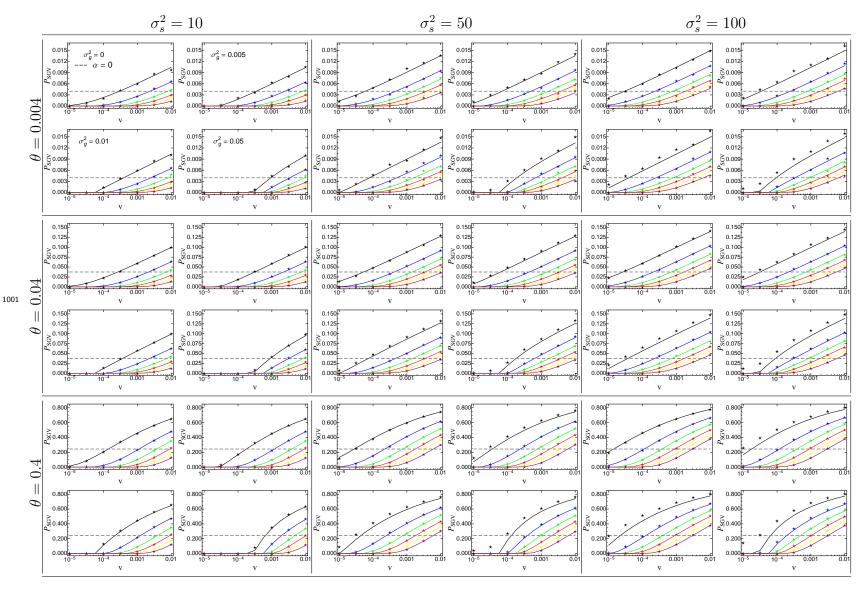


Figure S3_2 – The distribution of adaptive substitutions from standing genetic variation in the case of slow environmental change ($v = 10^{-5}$). Histograms show results from individual-based simulations. The blue line gives the analytical prediction (eq. 25), with σ_g^2 given by eq. 16), which assumes a neutral fixation probability. Fixed parameters: $\sigma_m^2 = 0.05$.

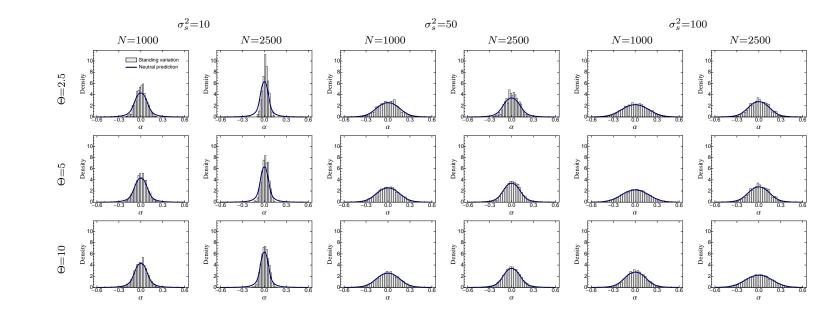


Figure S3_3 – The average number of fixed adaptive substitutions from standing genetic variation, $|\mathcal{G}|_{\text{fix}}$, as a function of the rate of environmental change v, when standing genetic variation is the sole source for adaptation. Symbols show results from individual-based simulations (averaged over 100 replicate runs). The black line gives the analytical prediction (eq. 27) and the grey line corresponds to the average number of neutral fixations ($|\mathcal{G}|_{\text{fix},v\to 0} = |\mathcal{G}| \int_{-\infty}^{\infty} p(\alpha) \prod_{\text{seg},v\to 0}(\alpha) d\alpha$.). In both cases, σ_g^2 was taken from equation (16). Error bars for standard errors are contained within the symbols. Fixed parameters: $N = 1000, \sigma_m^2 = 0.05$.

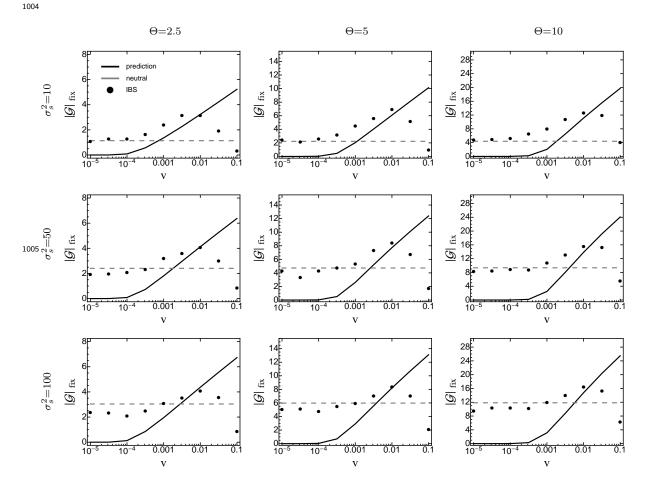


Figure S3_4 – The average number of fixed adaptive substitutions from standing genetic variation, $|\mathcal{G}|_{\text{fix}}$, as a function of the rate of environmental change v, when standing genetic variation is the sole source for adaptation. Symbols show results from individual-based simulations (averaged over 100 replicate runs). The black line gives the analytical prediction (eq. 27) and the grey line corresponds to the average number of neutral fixations ($|\mathcal{G}|_{\text{fix},v\to 0} = |\mathcal{G}| \int_{-\infty}^{\infty} p(\alpha) \prod_{\text{seg},v\to 0}(\alpha) d\alpha$.). In both cases, σ_g^2 was taken from equation (16). Error bars for standard errors are contained within the symbols. Fixed parameters: $N = 2500, \sigma_m^2 = 0.05$.

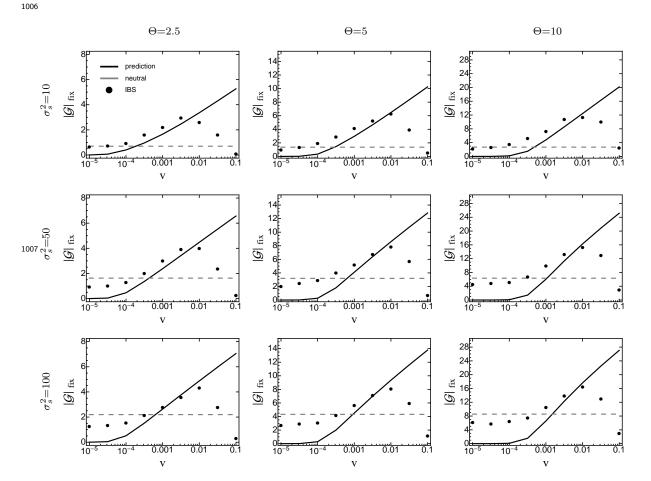


Figure S3_5 – The average number of fixed adaptive substitutions from standing genetic variation, $|\mathcal{G}|_{\text{fix}}$, as a function of the rate of environmental change v, when standing genetic variation is the sole source for adaptation. Symbols show results from individual-based simulations (averaged over 100 replicate runs). The black line gives the analytical prediction (eq. 27) and the grey line corresponds to the average number of neutral fixations ($|\mathcal{G}|_{\text{fix},v\to 0} = |\mathcal{G}| \int_{-\infty}^{\infty} p(\alpha) \prod_{\text{seg},v\to 0}(\alpha) d\alpha$.). In both cases, σ_g^2 was taken from equation (16). Error bars for standard errors are contained within the symbols. Fixed parameters: $N = 5000, \sigma_m^2 = 0.05$.

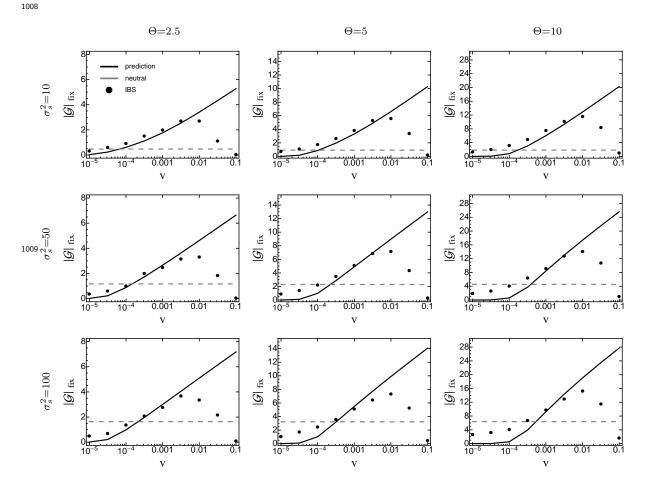


Figure S3_6 – The average distance traversed in phenotype space, z^* , as a function of the rate of environmental change v, when standing genetic variation is the sole source for adaptation. Symbols show results from individual-based simulations (averaged over 100 replicate runs). The black line gives the analytical prediction (eq. 28), with σ_g^2 taken from equation (16). The grey-dashed line gives the critical rate of environmental change (eq. 29). Error bars for standard errors are contained within the symbols. Fixed parameters: N = 1000, $\sigma_m^2 = 0.05$.

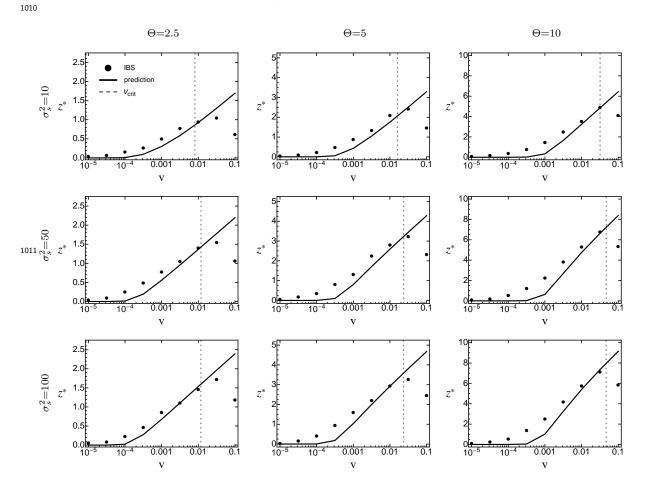
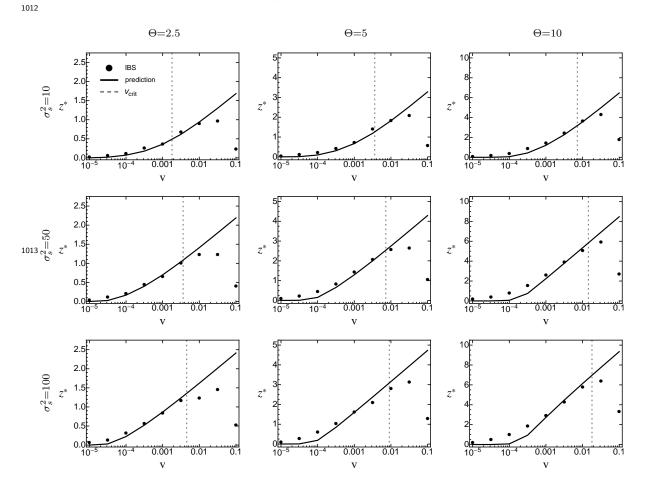


Figure S3_7 – The average distance traversed in phenotype space, z^* , as a function of the rate of environmental change v, when standing genetic variation is the sole source for adaptation. Symbols show results from individual-based simulations (averaged over 100 replicate runs). The black line gives the analytical prediction (eq. 28), with σ_g^2 taken from equation (16). The grey-dashed line gives the critical rate of environmental change (eq. 29). Error bars for standard errors are contained within the symbols. Fixed parameters: N = 5000, $\sigma_m^2 = 0.05$.



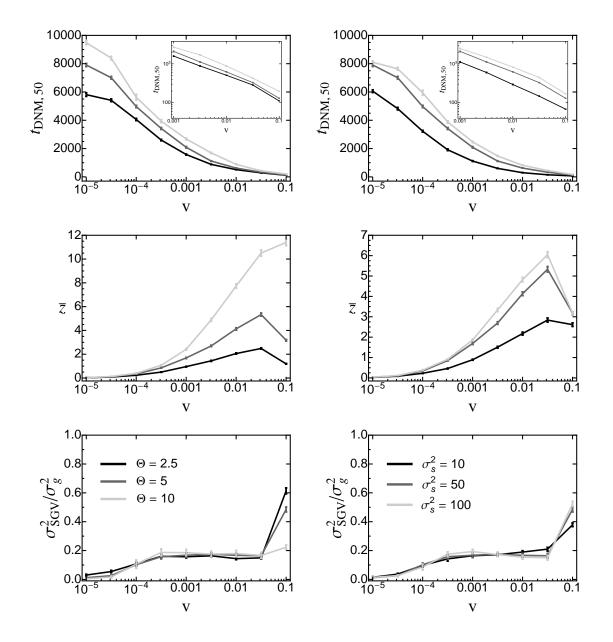


Figure S3_8 – First row: the point in time $t_{\text{DNM},50}(\bar{z})$ where 50% of the phenotypic response to moving-optimum selection have been contributed by *de-novo* mutations as a function of the rate of environmental change for various values of Θ (left) and σ_s^2 (right). Insets show the results for large v on a log-scale. Second row: The mean total phenotypic response at this time. Third row: The relative contribution of original standing genetic variation to the total genetic variance at time $t_{\text{DNM},50}(\bar{z})$. Data are means and standard errors from 1000 replicate simulation runs. Fixed parameters (if not stated otherwise): $\sigma_s^2 = 50, \, \Theta = 5, \, N = 1000, \, \sigma_m^2 = 0.05.$

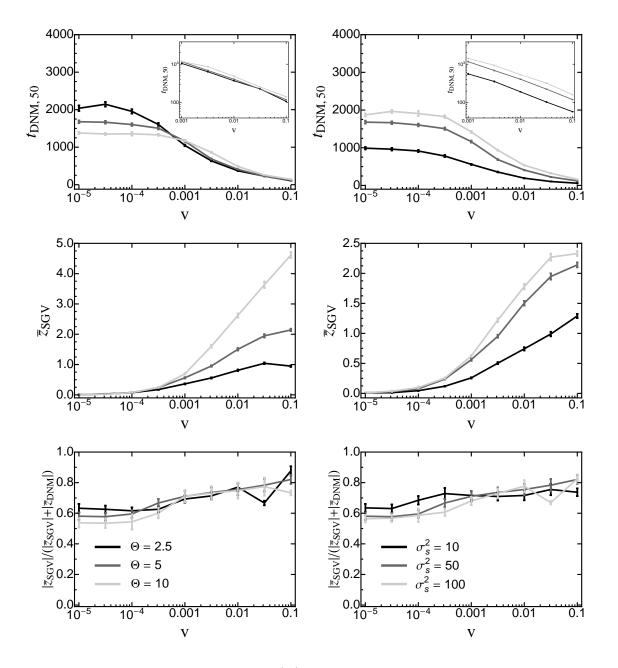


Figure S3_9 – First row: the point in time $t_{\text{DNM},50} \left(\sigma_g^2\right)$ where 50% of the genetic variance is composed of *de-novo* mutations as a function of the rate of environmental change for various values of Θ (left) and σ_s^2 (right). Insets show the results for large v on a log-scale. Second row: The mean total phenotypic response from standing genetic variation at this time. Third row: The relative contribution of original standing genetic variation to the total genetic variance at time $t_{\text{DNM},50} \left(\sigma_g^2\right)$. Data are means and standard errors from 1000 replicate simulation runs. Fixed parameters (if not stated otherwise): $\sigma_s^2 = 50$, $\Theta = 5$, N = 1000, $\sigma_m^2 = 0.05$.

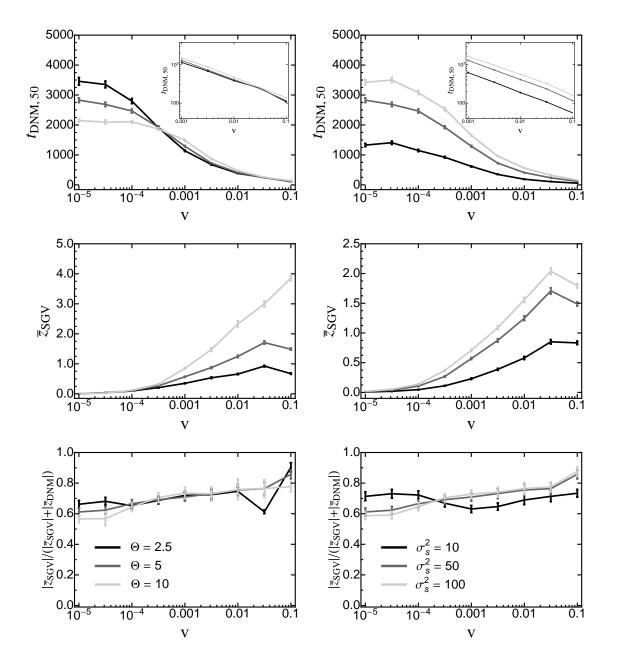


Figure S3_10 – First row: the point in time $t_{\text{DNM},50} \left(\sigma_g^2\right)$ where 50% of the genetic variance is composed of *de-novo* mutations as a function of the rate of environmental change for various values of Θ (left) and σ_s^2 (right). Insets show the results for large v on a log-scale. Second row: The mean total phenotypic response from standing genetic variation at this time. Third row: The relative contribution of original standing genetic variation to the total genetic variance at time $t_{\text{DNM},50} \left(\sigma_g^2\right)$. Data are means and standard errors from 1000 replicate simulation runs. Fixed parameters (if not stated otherwise): $\sigma_s^2 = 50, \, \Theta = 5, \, N = 2500, \, \sigma_m^2 = 0.05.$